Journal of ELECTRICAL ENGINEERING, VOL. 51, NO. 1-2, 2000, 3–12

DECENTRALISED EXCITATION CONTROL FOR A MULTIMACHINE POWER SYSTEM

Vojtech Veselý * — Igor Zvara * — Vladimr A. Stroev **

In this paper, a robust decentralised nonlinear stabiliser design method is proposed to increase the transient stability of a multimachine power system. The method is based on the input-output feedback linearisation and the design of a robust controller which guarantee of the exponential stability with a prescribed decay rate.

K e y w o r d s: robust controllers, nonlinear stabiliser, exponential stability, multi-machine power systems

1 INTRODUCTION

After the power systems VEAG and the Centrel had been connected to the interconnected power system UCPTE in 1995, poorly damped interarea oscillations occurred in early 1997. The interconnected power system has become less stable. On the basis of the study concerning the interarea oscillation damping the following behavior can be determined: large area power transits from the satellite systems to the center of the UCPTE/Centrel power system generally lead to the worsening of damping. The power transits from the Spanish area already reach nearly undamped interarea oscillations at 1 GW and even unstable oscillations at 2 GW transit. Interarea oscillations damping can be improved by equipping the power plants in the satellite part with Power Systems Stabilisers (PSS)[14]. The damping of sustained oscillations has been studied by a number of investigators[1], [2], [4], [11]. The approach of a linear power system model has been used as the basis for designing excitation controllers and PSS. The disadvantages of these kinds of design approaches include that the controllers depend on the operating conditions of the plant and may not work properly for large disturbances. Power system models are large- scale nonlinear systems. The very nonlinear nature of the power system models and severe resulting disturbances preclude the use of classical linear control design techniques. Only decentralised control schemes based on a nonlinear power system model that guarantee the robustness of the control system and provide reasonable algorithms and performance can be accepted.

The design of a decentralised controller for a power system with signal stability can be found in [3], [11]. The design of a decentralised governor controller that stabilised a power system for disturbances anywhere in the power system with a power system nonlinear model can be found in [7], [8], [11]. The control presented in the above-mentioned paper is derived from the stability proof of a multimachine power system based on Lyapunov's direct method. The proposed controller may be adaptive and with decentralised control schemes. Feedback linearisation applied to nonlinear control design attracted a great deal of interest in recent years. The control idea of the approach is to algebraically transform a nonlinear system dynamics into a linear one. The chief drawback of the method is that it relies on an exact cancellation of nonlinear terms in order to get a linear input-output behavior. Consequently, if there are errors or uncertainty in the model of nonlinear terms, the cancellation is no longer exact. In [3] the combination of a feedback linearisation and sliding mode control are presented. In [9] a unified approach to the design of a nonlinear excitation controller and system stabiliser for a synchronous generator operating on an infinite bus is presented. The proposed approach is based on a form of state feedback linearisation. Due to this state transformation, the terminal voltage becomes a linear function of the control input. In [16], [17] a robust decentralised nonlinear excitation controller design is presented. At first, a direct feedback linearisation through the excitation loop is designed to eliminate the nonlinearities, then a robust decentralised controller is proposed to guarantee the asymptotic stability. In [7] a robust decentralised excitation control of power system is proposed. Stability has been proved using the Lyapunov direct method and partial linearisation.

For practical implementation, in this paper a robust decentralised nonlinear excitation controller design is proposed for a nonlinear N-machine power system model. First, the feedback linearisation is designed to eliminate the nonlinearties in subsystem models. With this transformation the terminal active power of a synchronous

* Slovak University of Technology, Faculty of Electrical Engineering & Information Technology, Department of Automatic Control Systems, Ilkoviova 3, 812 19 Bratislava, Slovakia

** Moscow Power Engineering Institute, Electrical Power System Department, Krasnokazarmenaja 14, iii 250 Moscow E-250, Russia

ISSN 1335-3632

generator becomes a linear function of the control input. Then, a robust decentralised controller is proposed to guarantee the exponential stability of the multimachine power system with a prescribed decay rate. The robust exponential stability of the nonlinear power system model plays an important role to guarantee robustness and dynamic performance quality of a system [15].

The paper is organised as follows. In Section 2 the problem formulation and some preliminary results are presented. The main results — the design of the nonlinear excitation controller which guarantees the exponential stability of the complex power system with a prescribed decay rate is given in Section 3. In Section 4 the obtained theoretical results are applied to the design of a nonlinear excitation controller and the corresponding simulations for a two machine power system are provided.

2 PROBLEM FORMULATION AND PRELIMINARIES

Consider the following power system uncertain model:

$$\dot{x}_i = f_i(x_i) + \delta f_i(x_i) + (b_i(x_i) + \delta b_i(x_i))u_i + \sum_{i\neq j}^{N} h_{ij}(x_j)\,,$$
$$y_i = c_i(x_i)\,, \quad i \in N \tag{1}$$

where $x_i \in R^{n_i}$, $u_i \in R^{m_i}$, $y_i \in R^{l_i}$ are the state, control input and output of the ith subsystem, respectively, $f_i(x_i)$, $\delta f_i(x_i)\colon R^{n_i} \to R^{n_i}$, $b_i(x_i)$, $\delta b_i(x_i)\colon R^{n_i} \to R^{n_i\times m_i}$, $h_{ij}(x_j)\colon R^{n_j} \to R^{n_i}$, $C_i(x_i) \in R^{n_i} \to R^{l_i}$, are continuous and uniformly bounded vector functions of class C^k, $k>0$ is sufficiently large, differentiable on the set R_{ρ_i} with respect to system variables x_i, u_i

$$R_{\rho_i} = \{x_i \in R^{n_i}\colon \|x_i\| < \rho_i\}\,,\ \rho_i > 0\,, i \in N \tag{2}$$

and $f_i(0) = f_i(0) = 0$, $b_i(0) + \delta b_i(0) \neq 0$, $x^\top = \left[x_1^\top, \dots, x_N^\top\right]$ are state variables of the complex system,

$$n = \sum_{i=1}^{N} n_i,\ m = \sum_{i=1}^{N} m_i,\ l = \sum_{i=1}^{N} l_i,\ N = \{1,2,3,\dots,N\}\,.$$

The functions $f_i(x_i)$, $b_i(x_i)$, $h_{ij}(x_j)$, are supposed to be known, for $i \in N$. The unknown functions $\delta f_i(x_i)$, and $\delta b_i(x_i)$ represent the internal uncertainties of the ith subsystem. It is assumed that the above uncertainties and functions $h_{ij}(x_j)$ for $i \neq j \in N$ are norm bounded. The problem studied in this paper can be stated as follows. On the local level it is necessary to find the robust decentralised controller in the form

$$u_i = u_i(y_i)\,. \tag{3}$$

The designed controller has to guarantee the robust exponential stability of the closed-loop system comprising the plant (1) and the decentralised controller (3). Throughout this paper, the concept of robust exponential stability of a complex system will be used in the sense of Krasovskij as follows:

Definition 1. The uncertain closed-loop system (1), (3) is said to be robustly exponential stable with a decay rate $\alpha > 0$ if for all initial conditions $x(t_0) = x_0$, and for all admissible uncertainties given by (1) there exists a Lyapunov function $V(x)\colon R^n \to R^+$, $V(x) \in C^1$ for a nominal closed-loop system (without uncertainties) such that the following inequalities hold along the solution of (1) and (3)

$$\begin{aligned} \frac{\mathrm{d}V(x)}{\mathrm{d}t} &\leq -\alpha V(x) \\ c_1\|x\|^2 \leq V(x) &\leq c_2\|x\|^2 \\ \dot{V}(x) &\leq -c_3\|x\|^3 \\ \|\operatorname{grad} V(x)\| &\leq c_4\|x\| \end{aligned} \tag{4}$$

where $(\operatorname{grad} V(x))^\top = \left[\frac{\delta V}{\delta x_1}, \dots, \frac{\delta V}{\delta x_n}\right]$ and $\|\bullet\|$ denotes Euclidean norm and $c_i > 0$, $i = 1,2,3,4$.

3 DECENTRALISED ROBUST CONTROL SYSTEM DESIGN

In this section the input-output feedback linearisation and an original approach is used for the design of the output feedback decentralised nonlinear controller (3) that guarantees the exponential stability of the closed-loop system (1) and (3) with the bounded uncertainties. The nonlinear controller design procedure is divided into two steps. First, the feedback linearisation is used to eliminate the nonlinearties in subsystem model such that the terminal active power of ith synchronous generator becomes a linear function of the control input, then a robust decentralised controller with an output feedback is proposed to guarantee the exponential stability with a prescribed decay rate of the multimachine power system.

3.1 FEEDBACK LINEARISATION

As we mentioned above, feedback linearisation is used for the elimination of nonlinearties on subsystem level. The aim of the Input-Output linearisation method is to find an integer ρ and a state feedback

$$u = \alpha(x) + \beta(x)v \tag{5}$$

where $\alpha(\cdot)$ and $\beta(\cdot)$ are smooth functions defined in a neighborhood of some point $x_0 \in R^n$ and $\beta(x_0) \neq 0$ such that on subsystem level the nominal model of subsystem

$$\begin{aligned} \dot{x}(t) &= f(x) + b(x)\alpha(x) + b(x)\beta(x)v(t) \\ y(t) &= c(x) \end{aligned} \tag{6}$$

has the property that the ρth-order derivative of the output is given by

$$y^{(\rho)}(t) = v(t)\,, \quad t \in \Gamma$$

where Γ is an open interval including $t = 0$. The point x_0 is the equilibrium point around which linearisation is performed. For simplification, subsystem model indices are omitted in the above equations. To facilitate the formulation of differentiating the output $y(t)$ the following notation is introduced.

DEFINITION 2. Suppose that function $c(x)\colon R^n \to R^1$ is a smooth scalar function and $f\colon R^n \to R^n$, $b\colon R^n \to R^{n\times m}$ are vector fields, then the following notations is introduced

$$
\begin{aligned}
&\frac{\delta c}{\delta x} = \left[\frac{\delta c}{\delta x_1}, \dots, \frac{\delta c}{\delta x_n}\right] \\
&L_f^0 c = c \\
&L_f c = \sum_{j=1}^{n} \frac{\delta c}{\delta x_j} f_j(x) = \frac{\delta c}{\delta x} f(x) \\
&\vdots \\
&L_f^k c = L_f\big(L_f^{k-1} c(x)\big) = \frac{\delta L_f^{(k-1)} c}{\mathrm{d}x} f(x)
\end{aligned}
\tag{7}
$$

and for $k = 0, 1, \dots$

$$
L_b L_f^k c(x) = L_b\big(L_f^k c(x)\big) = \frac{\delta L_f^k c}{\delta x} b(x)\,.
$$

Now differentiating $y(t)$ gives

$$
\begin{aligned}
y^{(1)}(t) = \frac{\delta c}{\delta x}\frac{\mathrm{d}x}{\mathrm{d}t} &= \frac{\delta c}{\delta x}\big(f(x(t)) + b(x(t))u(t)\big) \\
&= L_f c(x(t)) + L_b c(x(t))u(t)
\end{aligned}
$$

and the ith differentiating $y^{(i)}(t)$ has the form

$$
y^{(i)}(t) = L_f^i c(x(t)) + L_b L_f^{(i-1)} c(x(t))u(t)\,.
$$

Differentiating $y(t)$ is repeated until we find an integer ρ such that for all x in the neighborhood of x_0 the following conditions hold

I. $\quad L_b L_f^i c(x) = 0\,, \qquad i = \rho\,,$

II. $\quad L_b L_f^{(\rho-1)} c(x) \neq 0\,, \qquad \forall i < \rho - 1\,.$

If such an integer exists we say that the system has a relative degree ρ at the point x_0. Thus, we arrive at the necessary and sufficient conditions for the local input-output linearisation problem [5].

THEOREM 1. *The system* (1) *is locally input-output linearisable around* x_0 *if and only if it has a relative degree* ρ *at the point* x_0 *and the desired feedback transform is given by*

$$
u = \frac{-L_f^\rho c(x(t)) + v(t)}{L_b L_f^{(\rho-1)} c(x(t))}\,. \tag{8}
$$

R e m a r k . Using eq. (8) with respect to nominal model of power system (1) control input (8) eliminates the nonlinearties on subsystem model and if $y_i(t)$ is equal to the active power of the subsystem, the active power becomes a linear function of the control input.

Stability of the achieved solutions is determined by the following theorem [5].

THEOREM 2. *The equilibrium of the closed-loop system consisting of equations (6) and (8) is asymptotically stable if all the zeros of the input-output feedback linearisation of the system (6) have negative real parts, and is unstable if at least one of the zeros of input-output feedback linearisation of the system (6) has a positive real part.*

3.2 DESIGN OF ROBUST CONTROLLER WITH OUTPUT FEEDBACK

The result of section 3.1 is the uncertain complex system model described as follows

$$
\begin{aligned}
\dot{x}_i &= (A_i + \delta A_i)x_i + (B_i + \delta B_i)u_i + \sum_{i\neq j}^{N} h_{ij}(x_j) \\
y_i &= C_i x_i \qquad\qquad i \in N
\end{aligned}
\tag{9}
$$

where A_i, B_i, C_i are known constant matrices of appropriate dimensions which represent the nominal model of the ith subsystem, δA_i, and δB_i are the matrices of appropriate dimensions which represent internal uncertainties in each subsystem. The entries of the above unknown matrices are piecewise continuous and bounded, the unknown vector function $h_{ij}(x_j)$ is continuous, sufficiently smooth in x_j and bounded so that the system (9) has a unique continuous solutions for all initial conditions $x_i(0) = x_{i0} \in R_{\rho i}$. The isolated subsystem nominal model is

$$
\begin{aligned}
\dot{x}_i &= A_i x_i + B_i u_i \\
y_i &= C_i x_i
\end{aligned}
\qquad i \in N\,. \tag{10}
$$

The following assumptions are made for system (9).

ASSUMPTION 1. The internal uncertainties in each subsystems and the unknown vector function $h_{ij}(x_j)$ satisfy the following assumptions:

$$
\begin{aligned}
\delta A_i &= \sum_{k=1}^{p} \varepsilon_{ik} A_{ik} \\
\delta B_i &= \sum_{j=1}^{q} \gamma_{ij} B_{ij} \\
h_{ij}(x_j) &\leq D_{ij} x_j
\end{aligned}
\tag{11}
$$

where A_{ik}, B_{ij} and D_{ij} are known matrices, $\varepsilon_{ik} \in \langle \underline{\varepsilon}_{ik}, \overline{\varepsilon} ik\rangle$, $\gamma_{ij} \in \langle \underline{\gamma}_{ij}, \overline{\gamma} ij\rangle$ are unknown constants.

For the next we need the following lemma [17].

LEMMA 1. *The nominal ith subsystem* (10) *is static output feedback stabilisable if and only if there exist a symmetric and positive definite matrix P and a matrix K satisfying the following equality*

$$(A+BKC)^\top P + P(A+BKC) + Q_0 = 0 \tag{12}$$

where $Q_0 = Q_0^\top > 0$.

One can choose the K by minimization of the following criterion

$$L = \min_K \operatorname{trace}\big\{(A+BKC)^\top P + P(A+BKC) + Q + C^\top K^\top RKC\big\} \tag{13}$$

where $Q_0 = Q + C^\top K^\top RKC$, $R = R^\top > 0$, $Q = Q^\top \geq 0$ some matrices of appropriate dimensions.

The solution of (13) is as follows

$$K = -R^{-1}B^\top PC^\top (CC^\top)^{-1}. \tag{14}$$

For Eq. (12) after some transformation we obtain

$$A^\top P + PA + Q - PBR^{-1}B^\top P + \big(R^{-1/2}B^\top P + R^{1/2}KC\big)^\top (R^{-1/2}B^\top P + R^{1/2}KC) = 0\,. \tag{15}$$

Equation (15) inspired a similar algorithm to that given in [6] for the calculation of matrix K.

ALGORITHM A

Step 1. Set $l = 0$ and $G_l = 0$.

Step 2. Solve the equation

$$A^\top P_l + P_l A + Q - P_l BR^{-1}B^\top P_l + G_l^\top G_l = 0 \tag{16}$$

for P_l symmetric nonnegative definite.

Step 3.

$$G_{l+1} = R^{-1/2}B^\top P_l - R^{-1/2}B^\top P_l C^\top \big(CC^\top\big)^{-1} C \tag{17}$$

increase l by one and go to step 2.

If the sequence $P_0, P_1, \ldots$ converges say to P, the K is given by (14). The convergence remains to be proved, however.

In order to guarantee the exponential stability of the uncertain closed loop of the ith subsystem without interconnections due to *Definition* 1 the following inequality must hold

$$(A_i + B_iK_iC_i)^\top P_i + P_i(A_i + B_iK_iC_i) + (\delta A_i + \delta B_iK_iC_i)^\top P_i + P_i(\delta A_i + \delta B_iK_iC_i) \leq -\alpha_i P_i \tag{18}$$

where α_i is a required value of the decay rate for exponential stability.

Adding the term of $C_i^\top K_i^\top R_iK_iC_i$ to (18) and introducing the matrix for estimate the uncertainties Q_i

$$Q_i = I\Bigg(\sum_{k=1}^{p} \varepsilon_{ikm}\big\|A_k^\top P_i + P_iA_{ik}\big\| + \sum_{j=1}^{s}\gamma_{ijm}\big\|(B_{ij}K_iC_i)^\top P_i + P_iB_{ij}K_iC_i\big\|\Bigg) \tag{19}$$

where I — identity matrix of corresponding dimension.

After some manipulations one can get

$$\Big(A_i + \frac{\alpha_i}{2}I\Big)^\top P_i + P_i\Big(A_i + \frac{\alpha_i}{2}I\Big) + Q_i + G_i^\top G_i - P_iB_iR_i^{-1}B_i^\top P_i^\top \leq 0 \tag{20}$$

where $\varepsilon_{ikm} = \max\big(\operatorname{abs}(\varepsilon_{ik})\big)$, $\gamma_{ijm} = \max\big(\operatorname{abs}(ij)\big)$, and G_i is given by (17). Matrix K_i is calculated by *Algorithm* A.

The following theorem summarizes the above results.

THEOREM 3. *The uncertain closed-loop ith subsystem with uncertainty* (11) *is exponentially stable with a decay rate α_i if there exist matrices $P_i = P_i^\top > 0$ and $R_i = R_i^\top > 0$ satisfying the matrix inequality* (20) *and the gain matrix K_i calculated by Algorithm* **A**.

P r o o f . Proof of sufficiency is clear from eqs. (18)–(20).

Let the candidate Lyapunov function of the complex uncertain system be

$$V = \sum_{i=1}^{N} V_i\,. \tag{21}$$

Using (18) and (19) for the time derivative of (21) with respect to uncertain model (9) one obtains

$$\frac{\mathrm{d}V}{\mathrm{d}t} \leq \sum_{i=1}^{N} x_i^\top \Big[(A_i + B_iK_iC_i)^\top P_i + P_i(A_i + B_iK_iC_i) + Q_i\Big]x_i + 2\lambda_m(P_i)\|x_i\|\sum_{j\neq i}^{N}\xi_{ij}\|x_j\| \tag{22}$$

and using the same arguments as in equations (18), (19) and (20) we obtain

$$\frac{\mathrm{d}V}{\mathrm{d}t} \leq \sum_{i=1}^{N} x_i^\top \Big[\ (A_i + \frac{\alpha_i}{2}I)^\top P_i + P_i\big(A_i + \frac{\alpha_i}{2}I\big) + Q_i + G_i^\top G_i - P_iB_iR_i^{-1}B_i^\top P_i^\top\Big]x_i + 2\lambda_m(P_i)\|x_i\|\sum_{j\neq i}^{N}\xi_{ij}\|x_j\|\,.$$

Finally, for the time derivative of the Lyapunov function (22) one obtains

$$\frac{\mathrm{d}V}{\mathrm{d}t} \leq -\alpha V + \sum_{i=1}^{N} x_i^\top\Big[-(\alpha_i - \alpha)P_i - C_i^\top K_i^\top R_iK_iC_i + I_i\vartheta_i\Big]x_i \tag{23}$$

where $I_i \in R^{n_i \times n_i}$ is the identity matrix,

$$\vartheta_i = \sum_{j\neq i}^{N} \Big\{ \lambda_M^{1/2}\big(D_{ij}^\top D_{ij}\big)\lambda_M(P_i) + \lambda_M^{1/2}\big(D_{ji}^\top D_{ji}\big)\lambda_M\big(P_j\big)\Big\},$$

where α is the required value of exponential stability, the decay rate of the complex system. If the matrix

$$M_i = -(\alpha_i - \alpha)P_i - C_i^\top K_i^\top R_i K_i C_i + I_i \vartheta_i \tag{24}$$

is non-positive for all $i = 1, 2, \dots, N$ then the sufficient condition for the exponential stabilinzability of the complex system with decay rate α is guaranteed. The above results are summarized in the following theorem.

THEOREM 4. *The uncertain closed-loop complex system with uncertainty* (11) *is exponentially stable with a decay rate* $\alpha \le \alpha_i$ *if the following conditions are satisfied.*

(i) *Theorem 3 holds for some* $\alpha_i \ge \alpha$ *and* K_i, $i = 1, 2, \dots, N$

(ii) *there exists a non-positive matrix* M_i *for all* $i = 1, 2, \dots, N$, *and* $\alpha \le \alpha_i$.

P r o o f . Proof of sufficiency is clear from the eqs. (21)–(24).

4 DECENTRALISED ROBUST CONTROL DESIGN FOR POWER SYSTEMS

The actual dynamic response of a synchronous generator in a practical power system is very complex when a fault occurs and is very difficult to be dealt with in the controller design unless some simplifications are made. Under some standard assumptions, the motion of the interconnected generators can be described by a third order, nonlinear, time invariant, state space model. For turbine/governor model one can use a second-order linearized differential equation. The mathematical model of ith synchronous generator is given by (1) [10]

$$\begin{aligned}
\dot\delta_i &= \omega_i \\
\dot\omega_i &= \frac{1}{T_{ji}}\big(P_{Ti} - P_i - D_i\omega_i\big) \\
\dot E'_{qi} &= \frac{1}{T'_{d0_i}}\big(E_{fi} - E_{qi}\big) \\
\dot P_{Ti} &= T_i \\
\dot T_i &= \frac{1}{T_{Ti}T_{Gi}}\left[u_{Ti} - P_{Ti} - T_i(T_{Ti} + T_{Gi})\right]
\end{aligned} \tag{25}$$

where $u_{Ti} = -\frac{\omega_i}{\sigma_i} + P_{Di}$

T_{Ti}, T_{Gi} – turbine and governor time constants, in sec.;
P_{Di} – power controller output in p.u, which is constant;
σ_i – gain of conventional governor controller;
δ_i – the load angle of the ith generator in radians;
ω_i – the relative speed of the ith generator, in rad/s;
P_{Ti} – the mechanical turbine power in p.u;
P_i – the electrical terminal power in p.u;
D_i – damping constant, in p.u;
$T_{ji} = \frac{T_{Mi}}{\omega_0}$; T_{Mi} – inertia time constant, in sec.;
ω_0 – the synchronous machine speed, in rad/s;
E'_{qi} – the transient EMF in the quadrature axis of the ith generator, in p.u;
T'_{doi} – the direct axis transient time constant, in sec.;
$u_i = E_{fi}$ control input variable in p.u;

Let us take the model of the ith synchronous generator as the voltage $E_i = E_{Qi}$, $i \in N$ behind the quadrature axis reactance x_{qi} then the electrical equations of the ith synchronous generator are well known and summarized as follows

$$\begin{aligned}
E'_{qi} &= E_i + I_{di}(x_{qi} - x'_{di}) \\
E_{qi} &= E_i - I_{di}(x_{qi} - x_{di}) \\
I_{di} &= -\frac{E_i}{z_{ii}}\sin\varphi_{ii} + \sum_{\substack{k=1\\k\neq i}}^{N}\frac{E_k}{z_{ik}}\sin\big(\varphi_{ik} + \delta_{ik}\big) \\
I_{qi} &= \frac{E_i}{z_{ii}}\cos\varphi_{ii} - \sum_{\substack{k=1\\k\neq i}}^{N}\frac{E_k}{z_{ik}}\cos\big(\varphi_{ik} + \delta_{ik}\big) \\
P_i &= E_i I_{qi} \\
Q_i &= -E_i I_{di} - I_i^2 x_{qi} \qquad I_i = \sqrt{I_{di}^2 + I_{qi}^2}
\end{aligned} \tag{26}$$

where
x_{di} – the direct axis reactance, in *pu*;
x'_{di} – the direct axis transient reactance, in *pu*;
$\delta_{ik} = \delta_i - \delta_k$
Q_i – reactive terminal power, in *pu*;
I_{di} - the direct axis current, in *pu*;
I_{qi} – the quadrature axis current, in *pu*;
$\overline{z}_{ij} = z_{ij}e^{j\varphi_{ij}}$, $i, j \in N$ – the complex impedance, which determinates the current in the ith branch when $E_k = 0$, $k \neq j$, $k \in N$;
I_i – current of ith generator, in p.u;

From the model discussed above we can see that the multimachine power system is highly nonlinear and interconnected by the transmission network. At first, a direct feedback linearisation compensator will be designed to cancel the non-linearities with respect to active power of the ith generator. With this transformation the active power P_i, $i \in N$ becomes a linear function of the control input. This approach has been proposed for a more simple model of a power system in [16].

Owing to results of section 3.1, the steps of the design of the feedback linearization compensator are the following:

1. From (26) calculate the time derivative of active power

$$\dot P_i = \dot E_i I_{qi} + E_i \dot I_{qi}\,.$$

2. Calculate the time derivatives of $\dot{I}_{qi}$ and $\dot{E}_i$ and substitute them with equations $\dot{E}'_q$ (25) to the above equation for $\dot{P}_i$, finally we obtain

$$\dot{P}'_i = \frac{1}{a_i T'_{doi}}\Big\{\Big[E_{fi} - E_i + I_{di}(x_{di} - x_{qi}) + T'_{doi}(x_{qi} - x'_{di})\Big(I_{qi} - \frac{E_i}{z_{ii}}\cos(\varphi_{ii})\Big)\omega_i\Big]\Big(\frac{E_i}{z_{ii}}\cos(\varphi_{ii}) - I_{qi}\Big) + \sum_{k\neq i}^{N} \frac{\dot{E}_k\cos(\varphi_{ik}+\delta_{ik})\omega_k}{z_{ik}}\Big[T'_{doi}(x_{qi} - x'_{di})\Big(\frac{E_i}{z_{ii}}\cos(\varphi_{ii}) - I_{qi}\Big) - E_i a_i T'_{doi}\Big]\Big\}.$$

3. If the algorithm of feedback linearization compensator is given as follows

$$E_{fi} = \frac{1}{\frac{E_i}{z_{ii}}\cos\varphi_{ii} + I_{qi}}\Big\{\frac{E_i^2}{z_{ii}}\cos\varphi_{ii} - \Big[I_{di}(x_{di} - x_{qi}) + T'_{doi}(x_{qi} - x'_{di})\Big(I_{qi} - \frac{E_i}{z_{ii}}\cos\varphi_{ii}\Big)\omega_i\Big]\Big(\frac{E_i}{z_{ii}}\cos\varphi_{ii} + I_{qi}\Big) + u_{i1}\Big\} \quad (27)$$

then the above first order differential equation is

$$\dot{P}_i = \frac{1}{a_i T'_{doi}}(-P_i + u_{i1}) + h_{i3}\,,$$

where

$$h_{i3} = \sum_{k\neq i}^{N} h_{ik}$$

$$h_{ik} = \frac{\dot{E}_k\cos(\varphi_{ik}+\delta_{ik}) + E_k\sin(\varphi_{ik}+\delta_{ik})\omega_k}{z_{ik}}\Big[\frac{x_{qi} - x'_{di}}{a_i} \times \Big(\frac{E_i}{z_{ii}}\cos\varphi_{ii} + I_{qi}\Big) - E_i\Big]$$

Finally one can obtain a mathematical model of the ith synchronous generator in the following form

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + h_i \\ y_i &= C_i x_i \end{aligned} \qquad i \in N \qquad (28)$$

where

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{D_i}{T_{ji}} & -\frac{1}{T_{ji}} & \frac{1}{T_{ji}} & 0 \\ 0 & 0 & -\frac{1}{a_i T'_{doi}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -\frac{1}{\sigma_i T_{Ti} T_{Gi}} & 0 & -\frac{1}{T_{Ti}T_{Gi}} & -\frac{T_{Ti}+T_{Gi}}{T_{Ti}T_{Gi}} \end{bmatrix}$$

$$B_i^\top = \begin{bmatrix} 0 & 0 & -\frac{1}{a_i T'_{doi}} & 0 & 0 \end{bmatrix}$$

$$h_i^\top = \begin{bmatrix} 0 & 0 & \sum_{k\neq i}^{N} h_{ik} & 0 & 0 \end{bmatrix}$$

$$x_i = \big[\,(\delta_i - \delta_{io})\ \omega_i\ (P_i - P_{i0})\ (P_{Ti} - P_{Ti0})\ \dot{P}_{Ti}\,\big]\,,\ i \in N$$

$$a_i = 1 - \frac{x_{qi} - x'_{di}}{z_{ii}}\sin\varphi_{ii}$$

δ_{io} — the load angle of the ith generator in the operating point. Further we assume that $P_{Di} = P_{Tio}$.

One of the compulsory functions of excitation controllers is the control of the generator terminal voltage U_{Gi}. Let us assume that the above controller is proportional, that is

$$u_{i1} = k_{pi}(U_{si} - U_{Gi}) + u_{i2} = -k_{pi}\Delta U_{Gi} + u_{i2} \quad i \in N \qquad (29)$$

where

U_{si} – set point of proportional ith voltage controller,
k_{pi} – gain of controller,
u_{i2} – additional input which guarantees the robustness and exponential stability of complex system.

Obviously the gain k_{pi}, $i \in N$ is about 50 and more, therefore under small and large disturbances ΔU_{Gi} is rather small, which allows the following relation

$$\Delta U_{Gi} = \frac{\partial U_{Gi}}{\partial \delta_i}\Delta\delta_i + \sum_{k\neq i}^{N}\frac{\partial U_{Gi}}{\partial \delta_k}\Delta\delta_k + \frac{\partial U_{Gi}}{\partial P_i}\Delta P_i + \sum_{k\neq i}^{N}\frac{\partial U_{Gi}}{\partial P_k}\Delta P_k\,. \qquad (30)$$

Substituting (29) and (30) to (28) one gets

$$\dot{x}_i = A_{pi}x_i + B_i u_{i2} + h_{pi}\,, \qquad (31)$$

(31) where

$$A_{pi} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{D_i}{T_{ji}} & -\frac{1}{T_{ji}} & \frac{1}{T_{ji}} & 0 \\ -k_{pi}\frac{\partial U_{Gi}}{\partial \delta_i} & 0 & \frac{-1-k_{pi}\frac{\partial U_{Gi}}{\partial P_i}}{a_i T'_{doi}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -\frac{1}{\sigma_i T_{Ti} T_{Gi}} & 0 & -\frac{1}{T_{Ti}T_{Gi}} & -\frac{T_{Ti}+T_{Gi}}{T_{Ti}T_{Gi}} \end{bmatrix}$$

$$h_{pi} = [0 \quad 0 \quad \textstyle\sum h_{pik} \quad 0 \quad 0]^\top\,,$$

$$h_{pik} = \Big[h_{ik} - \frac{k_{pi}}{a_i T'_{doi}}\Big(\frac{\partial U_{Gi}}{\partial \delta_k}\Delta\delta_k + \frac{\partial U_{Gi}}{\partial P_k}\Delta P_k\Big)\Big]$$

$$\Delta\delta_i = \delta_i - \delta_{io}\,,\quad \Delta P_i = P_i - P_{io}\,,\quad i \in N\,.$$

We assume that uncertainties in system (31) may be created by not exactly known time constants of the ith generator/turbine, the parameter a_i, $i \in N$ may be changed with changing the operating point of the ith generator, one may not exactly implement the algorithm of feedback linearisation (27), and so on. Let us assume that all the above uncertainties may be included in two matrices δA_i

and δB_i and in the vector function δh_{pi}. Equation (31) with $h_{pi} = 0$ may be referred to as a nominal model of the ith subsystem

$$\dot{x}_i = (A_{pi} + \delta A_i)x_i + (B_{pi} + \delta B_i)u_{i2}\,, \quad i \in N\,.$$

Due to linearisation of terminal voltage U_{Gi} (30) yields linear model (31) with different dynamic matrices A_{pi} and B_i for each operating condition. Let r be the number of operating conditions, and A^j_{pi}, B^j_i, $j = 1, 2, \ldots, r$, be

$$A^j_{pi} = \{a^j_{lk}\}\,, \quad B^j_i = \{b^j_{uv}\}\,.$$

Let us take the maximum and minimum values of each element

$$\overline{a}_{lk} = \max_j\{a^j_{lk}\}\,, \quad \underline{a}_{lk} = \min_j\{a^j_{lk}\}\,,$$
$$j = 1, 2, \ldots, r\,; \quad l, k = 1, 2, \ldots, n\,;$$

and from matrices $A\max_i$ and $A\min_i$ one obtains the nominal matrix A_{ni}

$$A_{ni} = \frac{A\max_i + A\min_i}{2} \tag{32}$$

and the uncertainty δA_i

$$\delta A_i = \frac{A\max_i - A\min_i}{2}\,. \tag{33}$$

Uncertainties in the input matrix δB_i can be represented in the same way. The function h_{pik} in steady state $h_{pik} = 0$. Due to the large value of time constants T'_{doi}, $i \in N$ the following inequality holds around the origin

$$h_{pik} \le c_{ik1}\Delta\delta_k + c_{ik2}\omega_k + c_{ik3}\Delta P_k\,, \quad c_{ikj} \ge 0\,, \ j = 1, 2, 3\,.$$

Then the vector h_{pi} could be written as follows

$$h_{pi} \le \sum_{k \ne i}^{N} D_{ik}x_k\,,$$

where

$$D_{ik} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ c_{ik1} + d_{ik1} & c_{ik2} & c_{ik3} + d_{ik3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$d_{ik1} = -\frac{k_{pi}}{a_i T'_{doi}}\Big(\frac{\partial U_{Gi}}{\partial \delta_k}\Big)\Big|_m \quad d_{ik3} = -\frac{k_{pi}}{a_i T'_{doi}}\Big(\frac{\partial U_{Gi}}{\partial P_k}\Big)\Big|_m$$

where

$$\Big(\frac{\partial U_{Gi}}{\partial \delta_k}\Big)\Big|_m = \max_j\Big(\frac{\partial U_{Gi}}{\partial \delta_k}\Big) \quad \Big(\frac{\partial U_{Gi}}{\partial P_k}\Big)\Big|_m = \max_j\Big(\frac{\partial U_{Gi}}{\partial P_k}\Big)$$
$$j = 1, 2, \ldots, r\,.$$

On the basis of results of subsection 3.2 and proposed *Algorithm* A one may design the robust decentralised controller which guarantees the exponential stability of the complex system with a decay rate α. Due to the principle of dominant subsystems [11], there exist such values of exponential stability decay rate α_i $i = 1, 2 \ldots, N$ of subsystems, which guarantee the robustness properties and exponential stability of the complex system with decay rate $\alpha \le \alpha_i$ $i = 1, 2, \ldots, N$. Therefore in our practical example given below we assume that the interconnections between subsystems are not known. The exponential stability of the complex power system with the demanded decay rate α can be reached by correspondingly increasing α_i, $i = 1, 2, \ldots, N$ to the values which one can obtain for example by simulating the power system. To simplify the simulation, a two machine power system model is chosen to demonstrate the efficiency of the proposed controller. The system parameters and model uncertainties are given in *Appendix*. Following the design procedure the output feedback matrix we will use are

$$C_i = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad i = 1, 2$$

that is we will use only the speed of the ith machine ω_i and the time derivative of the turbine power $\dot{P}_{Ti}$ as feedback signals. The results of calculations for different α_i for the first and second synchronous generator are given below

$$\alpha_i = 1\,\mathrm{s}^{-1} \qquad \begin{matrix} K_1 = [\,10.7349 \quad 34.1914\,] \\ K_2 = [\,8.9055 \quad 30.9741\,] \end{matrix}$$

$$\alpha_i = 2\,\mathrm{s}^{-1} \qquad \begin{matrix} K_1 = [\,15.5167 \quad 40.2425\,] \\ K_2 = [\,13.5612 \quad 39.1285\,] \end{matrix}$$

$$\alpha_i = 3\,\mathrm{s}^{-1} \qquad \begin{matrix} K_1 = [\,20.5024 \quad 34.2002\,] \\ K_2 = [\,18.2968 \quad 36.3611\,] \end{matrix}$$

The simulation results are given in Figs. 1–3. The initial conditions for two load angles are assumed to be $\delta_{io} = 1.5$ rad $i = 1, 2$. At $t = 10$ s a three-phase shortcut circuit has occurred at the 3rd node for $t_p = 0.2$ s. Figure 1 shows the transient responses of the load angles of a two machine power system under the proportional controller and the designed output feedback controller for the nominal model and decay rate $\alpha = 2\,\mathrm{s}^{-1}$, $\alpha = 3\,\mathrm{s}^{-1}$. Figure 2 shows the transient process of the two-machine power system under the designed output controller and decay rate $\alpha = 3\,s^{-1}$. Figure 3 shows the transient process of the power system in operating point 0.5 *pu*of P_i, $i = 1, 2$.

Figure 1 shows a strong influence of the increasing decay rate α_i prescribed in the controller design in comparison with responses of a model controlled only by a proportional controller. Increasing of the coefficient α_i contracts transient responses and increase damping.

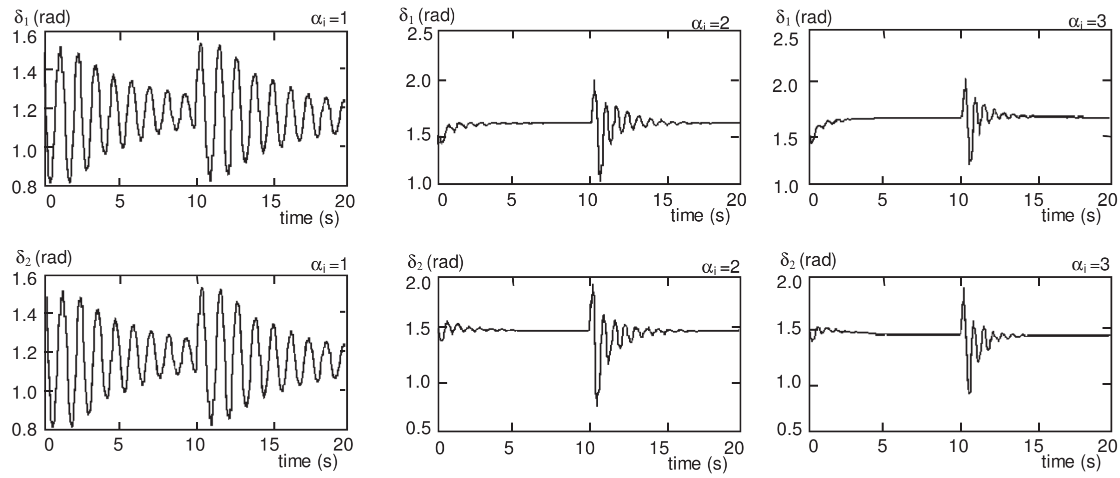

Fig. 1. The transient responses of the load angles of a two-machine power system under the proportional controller and output feedback controller for the nominal model and prescribed decay rate coef. $\alpha_i = 2\text{s}^{-1}$ and $\alpha_i = 3\text{s}^{-1}$.

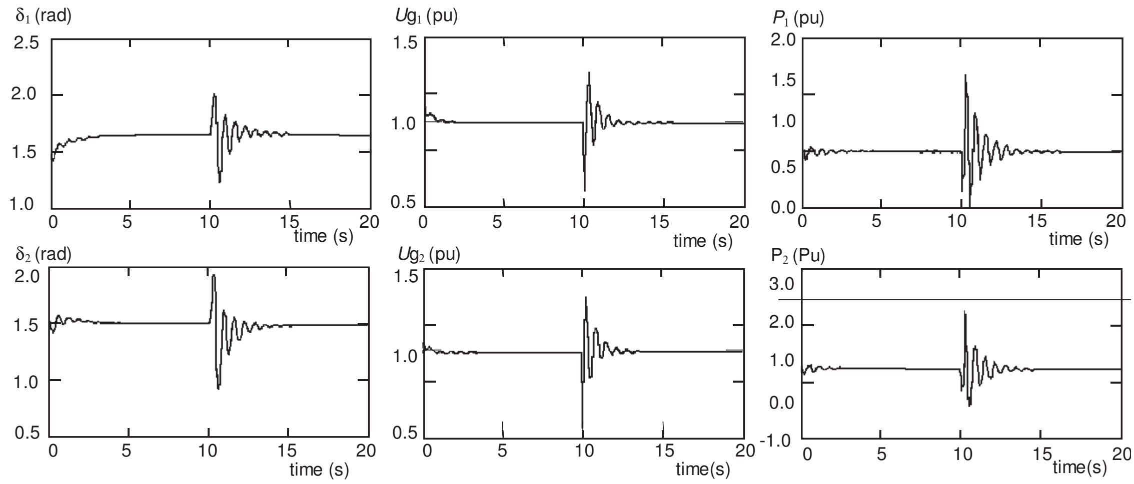

Fig. 2. The transient process of a two-machine power system under the designed output controller ($\alpha_i = 3\text{s}^{-1}$).

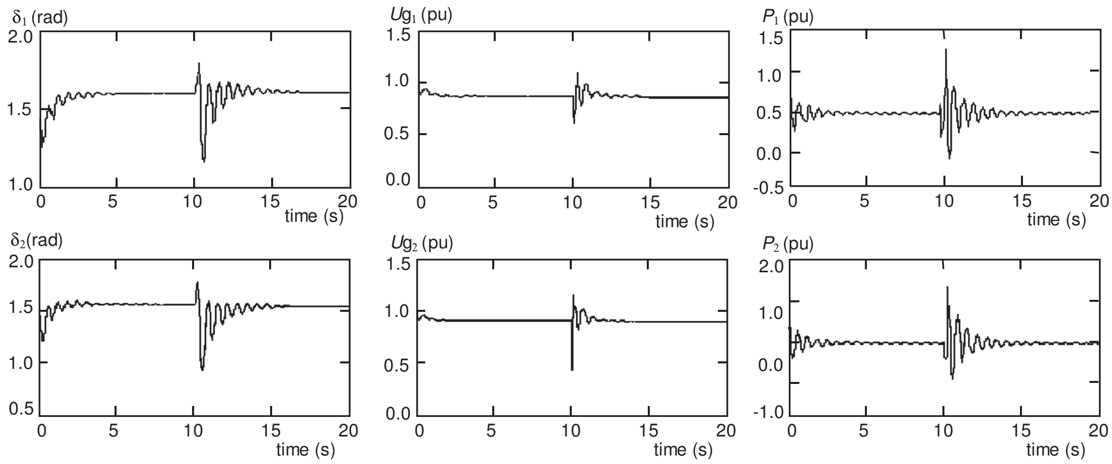

Fig. 3. The transient process of a two-machine power system under the designed output controller in operating point $P_i = 0.5\ pu(\alpha_i = 3\text{s}^{-1})$.

High performance of the controller, short and well-damped transient responses of the state variables δ_i, U_{gi} and P_i can be seen in Fig. 2.

Figure 3 shows well-damped responses of the state variables δ_i, U_{gi} and P_i in a controller operating region under the uncertainty influence of operating point 1.

5 CONCLUSIONS

The direct feedback linearizing control discussed in this paper is a powerful aid to the nonlinear control design. The nonlinear excitation controllers designed for multimachine power system by using this approach are composed of two parts:

the feedback linearizing compensator, and the nonlinear voltage stabilizer. The first part makes the nonlinear part linear in operating regions of the generators, which is important in improving the system stability under large disturbances. The second part secures exponential stability with a prescribed decay rate, via Lyapunov theory of stability, under the influence of uncertainties. Simulations depict good stability, fast damping and response of the proposed control design. The proposed control algorithm suits for practical implementation.

Acknowledgement

This research was partially supported by Grant from the Scientific Grant Agency of the Slovak Republic.

References

[1] BOURLES, H.—COLLEDANI, F.—HARWY, M. P.: Robust Continuos Yeed Governor Control for Small-Signal and Transient Stability, IEEE Trans. on Power Systems, Feb. 1997, Vol. 12, No. 1, pp. 129–135.

[2] CAI, H.—QU, Z.—DORSEY, J. F.: Robust Decentralised Excitation Controller for Large Scale Power Systems, In 13th Triennial World Congres, San Francisco, USA, 1996, IFAC, pp. 217–222, 706–073.

[3] CELA, A.—HERNAM, Y.—MIELSEN, J. R.—CHRISTENSEN, L. T.: Nonlinear Approach to the Control of Multimachine Power Systems, Nonlinear Analysis, Theory, Method and Applications, 1997, Vol. 30, No. 7, pp. 4301–4308.

[4] GAO, L.—CHEN, L.—FAN, Y.—MA, H.: A Nonlinear Control Design For Power Systems, Automatica, 1992, Vol. 28, No. 5, pp. 975–979, GB.

[5] CHING-FANG, L.: Advanced Control Systems Design, PTR Prentice Hall, Englewood Cliffts, USA, 1994.

[6] GEROMEL, J. C.—PERES, P. L. D.: Decentralized Load-Frequency Control, IEE proc, 1985, Ptd 132, 225–230.

[7] JIANG, H.—CAI, H.—DORSEY, J. F.—QU, Z.: Toward a Globally Robust Decentralised Control for Large-Scale Power Systems, IEEE Trans. on Control Systems, May. 1997, Vol. 5, No. 3, pp. 129–135.

[8] JIANG, H.—DORSEY, J. F.—QU, Z.—BOND, J.—Mc CALLEY, J. M.: Global Robust Adaptive Control of Power Systems, IEE Proc. Gener. Transm. Distrib., Sep. 1994, Vol. 141, No. 5, pp. 429–436.

[9] KENEDY, D.—MILLER, D.—QUINTERMA, V.: A Nonlinear Geometric Approach to Power System Excitation Control and Sabilization, Electrical Power and Energy Systems, 1998, Vol. 20, No. 8, pp. 501–515.

[10] MACHOWSKI, J—BIALEK, J. W.—BUMBY, J. R.: Power System Dynamics and Stability, John Wiley & Sons Ltd, Baffins Lane, Chichester, West Sussex PO19 1Ud, England, 1997.

[11] MUDRONK, D.—VESEL, V.: Power System Stabilization Using the Dominant Subsystem Principle, IEE Proc. Generation Transmission and Distribution, (1994), 141, No. 3, 243–247.

[12] MURGA, J—MIKLOVIČOVÁ, E.: Power System Stabilizers with Heuristic Pole Shift Factor, 14th World Congress of IFAC, 1999, N L-5a01-4.

[13] SIMOES COSTA, A. J. A.—FREITES, F. D.—e SILVA, A. S.: Design of Decentralised Controllers for Large Power Systems Considering Spervity, IEEE Trans. on Power Systems, Feb. 1997, Vol. 12, No. 1, pp. 144–152.

[14] SPENCER, M.—WELFONDER, E.—TILLMANN, H. B.—ZERNYI, D.: Load Flow Dependence of Slow Interarea Oscillations Occurring in the Extended European Power Systems, Cigre Session 1998, pp. 38–103.

[15] VESEL, V.—ROSINOVÁ, D.: Robust Exponential Stability of a Class of Nonlinear Systems, Kybernetika, 1998, Vol. 34, No. 5, pp. 579–584.

[16] WANG, Y.—GVO, G.—HILL, D. J.: Robust Decentralised Nonlinear Controller Design for Multimachine Power Systems, Automatica, 1997, Vol. 33, No. 9, pp. 1725–1733.

[17] YOUNG, Y. C—YOU, X. S.: Static Output Feedback Smultaneous Stabilitsation, ILMI approach, Int. J. Control, 1998, Vol. 70, No 5, 803–815.

[18] ZHU, CH.—ZHAN, R.—WANG, Y.: A New Decentralised Nonlinear Voltage Controller for Multimachine Power Systems, IEEE Trans. on Power Systems, Feb. 1998, Vol. 13, No. 1, pp. 211–216.

Received 4 October 1999

Vojtech Veselý (Prof, Ing, DrSc) was born in 1940. Since 1964 he has been working at the Department of Automatic Control Systems at the Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava, where he has supervised up today 15 CSc (PhD) students. Since 1986 he has been Full Professor. His research interests include the areas of power control, decentralized control of large-scale systems, process control and optimization. Prof Vesel spent several study and research visits in England, Russia, Bulgaria. He is author of more than 130 scientific papers.

Igor Zvara (Ing) was born in Banská Bystrica, in 1974. He graduated in control of continuous processes from the Faculty of Electrical Engineering and Information Technology, Slovak University of Technology, Bratislava, in 1997. Currently he is a PhD student at the Department of Automatic Control Systems. His postgraduate study is directed towards decentralized control and robust control of large-scale systems.

Vladimír A. Stroev (Prof, Ing, DrTn) was born in 1938. He received his PhD degree in 1969 and DrTn degree in 1988. He has been working as Head of Electrical Power Systems Department, Moscow Power Engineering Institute, where he has supervised, up today, 18 PhD students. His research interests include the areas of Power System Control, Modelling od Power Systems, design od PSS, Reactive power planning and voltage control. He is author and co-author of more than 15 terxt-books and 150 scientific papers.

APPENDIX

Generators parameters

$$\begin{array}{llll}
T_{j1} = 0.0207\,\mathrm{s}^2 & \sigma_1 = 25 & T_{j2} = 0.0223\,\mathrm{s}^2 & \sigma_2 = 25 \\
T'_{do1} = 6.5\,\mathrm{s} & x_{d1} = 1.43\,pu & T'_{do2} = 6.5\,\mathrm{s} & x_{d2} = 1.43\,pu \\
T_{G1} = 0.2\,\mathrm{s} & x_{q1} = 1.42\,pu & T_{G1} = 0.2\,\mathrm{s} & x_{q2} = 1.42\,pu \\
T_{T1} = 0.25\,\mathrm{s} & x'_{d1} = 0.31\,pu & T_{T1} = 0.25\,\mathrm{s} & x'_{d2} = 0.31\,pu \\
k_{p1} = 100 & \delta_{1o} = 0.2626\,rad & k_{p2} = 100 & \delta_{2o} = 0.2543\,rad
\end{array}$$

Operating points 1,2,3:

$$\begin{array}{llll}
D_{11} = 0.0096\,\mathrm{s} & D_{21} = 0.0098\,\mathrm{s} & P_{d11} = 0.5\,pu & P_{d21} = 0.5\,pu \\
D_{12} = 0.012\,\mathrm{s} & D_{22} = 0.01\,\mathrm{s} & P_{d12} = 0.75\,pu & P_{d22} = 0.75\,pu \\
D_{13} = 0.0144\,\mathrm{s} & D_{23} = 0.012\,\mathrm{s} & P_{d13} = 1\,pu & P_{d23} = 1\,pu
\end{array}$$

Network impedances

$$\begin{array}{ll}
z_{11} = 1.9366\,pu & z_{22} = 1.8407\,pu \\
z_{21} = 9.295\,pu & z_{31} = 2.442\,pu \\
z_{32} = 2.2916\,pu & z_{32} = 2.2916\,pu
\end{array}$$
$$z_{ij} = z_{ji} \;\; i \neq j \quad \varphi_{ij} = \pi/2\,rad,\; i,j = 1,2,3$$

The parameters listed above are used in constructing matrices of the state space model and matrices of uncertainties as follows.

At first the subsystems state variables δ_{ix} and E_{ix} were measured in the steady state of three operating points by setting parameters D_{1x}, D_{2x}, P_{d1x} and P_{d2x} as in paragraph Operating points ($x = 1,2,3$). Then the linearization coefficients of terminal voltage ΔU_{Gix} (30) were calculated. Next the state space matrices of operating points in form (31) were obtained. The final nominal models and matrices of uncertainties of subsystems were calculated from matrices of maximal and minimal elements of state space models of operating points, equations (32) and (33).

Nominal model and model of uncertainty subsystem 1

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -0.5797 & -48.3092 & 48.3092 & 0 \\ 30.1060 & 0 & -43.7348 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -0.8 & 0 & -20 & -9 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0.3603 \\ 0 \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad D_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\delta A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1159 & 0 & 0 & 0 \\ 10.1160 & 0 & 4.6925 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \delta B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Nominal model and model of uncertainty subsystem 2

$$A_2 = \begin{bmatrix} 0 & -0.4888 & -44.8430 & 44.8430 & 0 \\ 26.3655 & 0 & -45.1800 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -0.8 & 0 & -20 & -9 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0.3875 \\ 0 \\ 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad D_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\delta A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0493 & 0 & 0 & 0 \\ 9.0335 & 0 & 3.5165 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \delta B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The subsystem uncertainty (11) is given by three matrices A_{ij}, $i = 1,2$, $j = 1,2,3$, where each matrix A_{ij} has only one nonzero element, the position of which is that of δA_i. We assume that each entry of matrix δA_i changes independently of others. The ε_i can vary in time arbitrarily, provided that each element is within the given boundary.