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COMMUNICATIONS

MINIMUM TIME POLE ASSIGNMENT PWM CONTROLLERS FOR LINEAR SECOND ORDER TYPE 0 SYSTEM

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This paper presents the design of nonlinear minimum time pole assignment P-controllers for the linear second order type 0 systems with a pulse width modulated (PWM) input. It extends some recent results. These may be considered as the minimum time controllers. The controller design represents a new form of generalization of the algorithm with two nonzero poles.

K e y w o r d s: pulse width modulated (PWM) system, constrained control, control signal saturation

1 INTRODUCTION

The importance of the control signal constraints is generally well known. Some attempts to treat its influence can already be found in the beginning of control. Feldbaum in [5] mentions the idea of two Russian engineers to improve the steel rolling mills control by a quadratic velocity feedback — an idea from 1935 which was later rigorously elucidated by the theory of the relay minimum time systems in 50's. As a one of the main general results of this early period, the Feldbaum's theorem about n-intervals of the optimal continuous time control (1949) should be mentioned precised later by conclusions of the well known Pontrjagin's minimum (maximum) principle (1956). In the field of the minimum time control of discrete time systems, some interesting and generally valid results have been presented by Desoer and Wing [6] at the beginning of the 60s. They have shown that the n interval of optimal control (with control signal taking just the extreme values) can be separated by non-saturated control values. The same can occur during the change from the last saturated train of pulses to the steady state. The linear minimum time (dead-beat) control is dealing just with these intermediate control steps. In this period, also the first papers about minimum time PWM systems arrived Nelson [7], Polak [8].

2 PLANT DIFFERENCE EQUATIONS UNDER PWM CONTROL

In this chapter the controller design is derived with saturating input taking the limit values $u = U_1$ or U_2 and with the actual pulse width $d_n \in \langle 0, T\rangle$. The active parts of sampling periods are complemented by passive periods (Fig. 1) of duration $T - d_n$ with $u = 0$. The task is to find a switching strategy guaranteeing the minimum time transient fulfilling some additional constraints expressed by the closed loop poles.

The method is illustrated by a nonlinear P-controller design for the linear 2^{nd} order type 0 system

$$G(s) = \frac{1}{(T_1 s + 1)(T_2 s + 1)}\,, \tag{1}$$

where the ratio of the time constants $T_1/T_2 = 1/2$.

The state space representation of this system is described by

$$\dot{x} = Ax + bu\,, \quad y = c^{\top} x \tag{2}$$

for

$$T_1 = 1\,,\ T_2 = 2\,, \quad A = \begin{pmatrix} 0 & 1 \\ -1/2 & -3/2 \end{pmatrix}, \qquad b = \begin{pmatrix} 0 \\ 1/2 \end{pmatrix} \quad \text{and} \quad c^{\top} = (\,1 \quad 0\,)\,. \tag{3}$$

The state-transition matrix of the system is

$$e^{At} = L^{-1}\left\{[sI - A]^{-1}\right\} = L^{-1}\left\{\frac{\operatorname{adj}[sI - A]}{\det[sI - A]}\right\} \tag{4}$$

and the solution of the state-space representation is

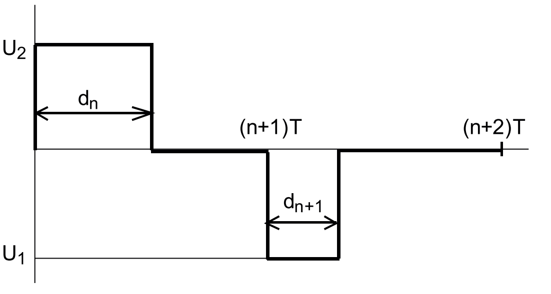

Fig. 1. Pulse-width modulated signal

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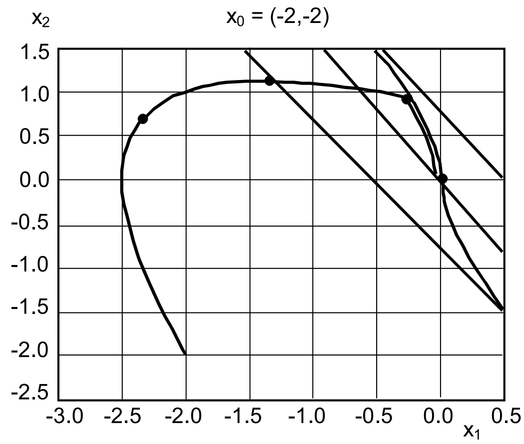

Fig. 2. Phase plane with the curve OBC, ZC, SSC.

$$x(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{(t-\tau)A} bu \mathrm{d}\tau$$
$$= \begin{pmatrix} 2e^{-t/2} - e^{-t} & 2\left(e^{-t/2} - e^{-t}\right) \\ -e^{-t/2} + e^{-t} & 2\left(-\frac{1}{2}e^{-t/2} + e^{-t}\right) \end{pmatrix} x(t_0) + \begin{pmatrix} 1 + e^{-t} - 2e^{-t/3} \\ -e^{-t} + e^{-t/2} \end{pmatrix} U_p \,. \quad (5)$$

Considering control with the sampling period T, active input (see *eg* $[1,3,4]$) and the plant state vector $x_n = [x_n, \dot{x}_n]$; $\dot{x}_n = [\mathrm{d}x/\mathrm{d}t]_{t=nT}$, the system behavior is described by the state difference equations

$$x_{n+1} = \begin{pmatrix} -e^{-T} + 2e^{-T/2} & -2e^{-T} + 2e^{-T/2} \\ e^{-T} - e^{-T/2} & 2e^{-T} - e^{-T/2} \end{pmatrix} x_n + \begin{pmatrix} 2e^{(-T+d_n)/2} - e^{(-T+d_n)} + e^{-T} - 2e^{-T/2} \\ e^{-T/2} - e^{(-T+d_n)/2} + e^{(-T+d_n)} - e^{-T} \end{pmatrix} U_P \,. \quad (6)$$

The goal is to transfer the system's state to the origin $(0,0)$ from an arbitrary initial state.

3 DETERMINATION OF THE BRAKING CURVE (BC)

For the desired state shifted to the origin, let us consider an optimal braking phase corresponding to active control pulses U_b (U_b is equal to one of the limit values U_p) applied with variable pulse width d_n. The corresponding phase-plane trajectory will be called **the optimal braking curve** (OBC), here.

It can be obtained after eliminating d_n from (6), where vector $x_n = (x_1, x_2)^\top$, for $x_{a+1} = 0$ and $U_p = U_b$:

$$x_1 = \frac{1}{2}U_b - x_2 \pm \frac{1}{2}\sqrt{U_b^2 - 4x_2U_b} \,. \quad (7)$$

The coordinates of the point x_A, from which the system reaches the origin with the full braking ($d_n = T$) are

$$x_A = \begin{pmatrix} U_b e^T(1 + e^{-T} - 2e^{-T/2}) \\ U_b e^T(-1 + e^{-T/2}) \end{pmatrix} . \quad (8)$$

For higher velocities, just fully active braking is considered ($d_n = 0$) along the trajectory finishing at x_A, then according to (6)

$$x_A = \begin{pmatrix} 2e^{-t/2} - e^{-t} & 2\left(e^{-t/2} - e^{-t}\right) \\ -e^{-t/2} + e^{-t} & 2\left(-\frac{1}{2}e^{-t/2} + e^{-t}\right) \end{pmatrix} x + \begin{pmatrix} 1 + e^{-t} - 2e^{-t/3} \\ -e^{-t} + e^{-t/2} \end{pmatrix} U_b \,. \quad (9)$$

4 DETERMINATION OF THE ZERO CURVE (ZC)

After eliminating t from these equations one gets under requirement of the regular ("pole assignment") velocity decrease specified *eg* by

$$\dot{x}_{n+1} = \lambda_1 \dot{x}_n \,. \quad (10)$$

Let us define a new curve called here as the "zero" curve (ZC) as a set of points from which the system will reach OBC in one control step without any control. That means that points of ZC have to satisfy condition (6) for $d_n = 0$. To the left of ZC, it is necessary to chose $U_p = U_2$ and to the right of ZC is $U_p = U_1$. Connecting (8) and (6) for $d_n = 0$ two parts of the zero curve are obtained again:

$$x_1 = \frac{1}{2D}\Big[2U_P - U_P D - 2x_2 D \pm \sqrt{U_P^2 D^2 - 4U_P^2 D - 4x_2 U_P D^2 - 4U_P^2}\Big] , \quad (11)$$

where $D = e^{-T/2}$.

5 DETERMINATION OF THE SHIFTED SWITCHING CURVE (SSC)

The 2nd new curve called the shifted switching curve (SSC) will trace out a border of the proportional control. It will corresponds to the points, at which $U_P = -U_b$ and $d_n = T$. Connecting (8) and (6) two parts of the shifted switching curve are obtained again:

$$x_1 = -\frac{1}{D}\Big[\frac{3}{2}U_b D + x_2 D - 2U_b \pm \frac{1}{2}\sqrt{8U_b^2 - 8U_b^2 D + U_b^2 D^2 - 4U_b x_2 D^2}\Big] . \quad (12)$$

For all points lying between OBC and ZC is $U_p = U_b$, between SSC and ZC is $U_p = -U_b$, $d \in \langle 0, T\rangle$. Else $U_p = -U_b$ and $d_n = T$.

6 DETERMINATION OF d_n

From comparison of relations (8) and (6), after eliminating time t, the relations for computing the impulse width d can be obtained for different polarity of the actuating variable U_p.

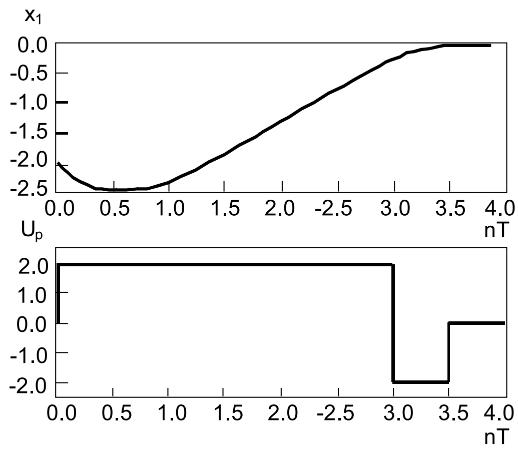

Fig. 3. Phase plane trajectory for $x_0 = [-2; 2]^\top$; $U_1 = -2$; $U_2 = 2$

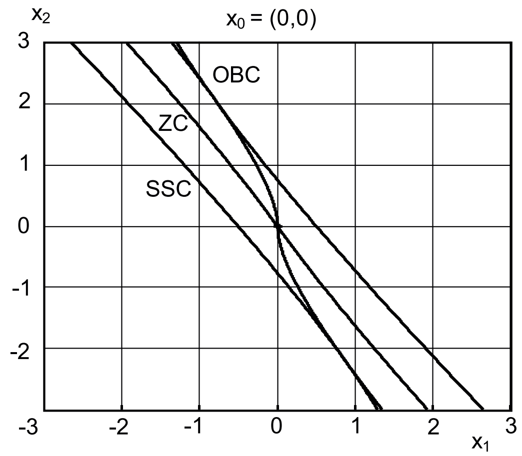

Fig. 4. Time response of the output variable and the active control variable for $x_0 = [-2; 2]^\top$; $U_1 = -2$; $U_2 = 2$

$$\begin{pmatrix} U_b e^t(1+e^{-t}-2e^{-t/2}) \\ U_b e^t(-1+e^{-t/2}) \end{pmatrix} = \begin{pmatrix} -e^{-T}+2e^{-T/2} & -2e^{-T}+2e^{-T/2} \\ e^{-T}-e^{-T/2} & 2e^{-T}-e^{-T/2} \end{pmatrix} x_n + \begin{pmatrix} 2e^{(-T+d_n)/2}-e^{(-T+d_n)}+e^{-T}-2e^{-T/2} \\ e^{-T/2}-e^{(-T+d_n)/2}+e^{(-T+d_n)}-e^{-T} \end{pmatrix} U_P\,. \quad (13)$$

So, the pulse width d_n is determined as solution of (13). For $U_p = U_b$ the impulse width d_n is obtained in the ulterior fashion

$$d_n = 2\ln\Big(\big(x_1^2 D + 2x_1x_2D + x_2^2D + 2U_b^2D + 3U_bx_1D + 4U_bx_2D - 2U_bx_1 - 2U_bx_2 - 2U_b^2\big)\big/\big(2U_b(U_bD + x_1D + x_2D - U_b)\big)\Big)\,. \quad (14)$$

In a similar manner for $U_P = -U_b$ is

$$d_n = T + 2\ln\bigg(\frac{1}{2U_b}\Big(U_b - x_2D + U_bD - x_1D \pm \big\{(-2U_bx_2 - x_2^2 - 2x_1x_2 + U_b^2 - x_1^2)D^2 + (x_1 + x_2 - U_b)(2U_bD) + U_b^2\big\}^{1/2}\Big)\bigg)\,. \quad (15)$$

In Fig. 3–4 there are transient responses of a minimum time pole assignment control of the linear 2nd type 0 system (2) for $T = 1$, $U_1 = -2$, $U_2 = 2$, $x_0 = [-2, -2]^\top$.

7 CONCLUSIONS

The above procedure for determination of the PWM control may be considered as an analogy to the pole assignment control of systems with pulse amplitude modulation.

The given approach to the problem of discrete t-optimal systems is analytical solvable in a closed fashion only for some second order systems. It is necessary to consider the influence of the time delays to which t-optimal control is particularly sensitive.

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