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COMMUNICATIONS

PARAMETRIZATION OF WIENER MODELS

Jozef Vörös *

The paper deals with the parametrization of block-oriented nonlinear dynamic models of Wiener type. Application of a compound mapping decomposition technique provides special forms of the Wiener models with separated parameters for polynomial, asymmetric and discontinuous nonlinear characteristics. These are linear in parameters and proper for identification and control purposes.

K e y w o r d s: nonlinear systems, Wiener model, parametrization.

1 INTRODUCTION

Many nonlinear dynamic systems can be represented by block-oriented models composed of nonlinear static and linear dynamic blocks [2]. The so-called Hammerstein and Wiener models are the simplest cases consisting of only two blocks. While the Hammerstein model is of N_1 class according to the known Zadeh classification, the Wiener model is of N_∞ class [10]. It means that the Wiener model is more complex.

The main issues by the parametrization of Wiener model are the same as by that of Hammerstein model [9]. They concern the description of nonlinear block and that of overall model, assuming the linear dynamic block can be uniquely characterized by the polynomials describing its transfer function. The characteristics of nonlinear blocks can be generally approximated by single polynomials of proper degree, which are integrated into the model description. In some cases the nonlinearity can be described in special forms using proper switching function. However, in all cases the way of nonlinear block integration can significantly influence the overall model description and consequently the algorithms using this model.

In this paper an original approach to the parametrization of Wiener models with different types of nonlinear characteristics is presented. A compound mapping decomposition technique [4], [6] is applied to the block operators with the aim to gain nonlinear model descriptions, which are linear in all the model parameters and the number of parameters is minimal. The resulting model descriptions with separated parameters provide a broader framework in searching for the optimal structure of Wiener model, *eg*, for the optimal degrees of the polynomials characterizing the linear and nonlinear blocks.

2 WIENER MODEL

The Wiener model is given by the cascade connection of a linear dynamic system followed by a static nonlinearity block (Fig. 1). The difference equation model of its linear dynamic block can be given as

$$x(t) = A(q^{-1})u(t) + [1 - B(q^{-1})]x(t)\,, \tag{2.1}$$

where $u(t)$ and $x(t)$ are the inputs and outputs, respectively, $A(q^{-1})$ and $B(q^{-1})$ are scalar polynomials in the unit delay operator q^{-1}

$$A(q^{-1}) = a_0 + a_1 q^{-1} + \cdots + a_m q^{-m}\,, \tag{2.2}$$

$$B(q^{-1}) = 1 + b_1 q^{-1} + \cdots + b_n q^{-n}\,. \tag{2.3}$$

The nonlinear block can be described by the equation

$$y(t) = G[x(t)]\,, \tag{2.4}$$

where $x(t)$ is the input, $y(t)$ is the corresponding output. Hence the Wiener model is characterized by a compound mapping from the set of model inputs $u(t)$ into the set of model outputs $y(t)$ and $x(t)$ is an internal variable, which is generally unmeasurable.

The description of overall nonlinear model depends on the characteristics of nonlinear block, *ie*, on the mapping $G(\cdot)$. In the following sections some types of nonlinearities are parametrized and the corresponding Wiener model equations are derived.

3 POLYNOMIAL NONLINEARITIES

Assume the nonlinear mapping G(.) is approximable by the polynomial of proper degree as

$$y(t) = \sum_{k=1}^{r} g_k x^k(t)\,. \tag{3.1}$$

V After a direct substitution of (2.1) into (3.1), the com-

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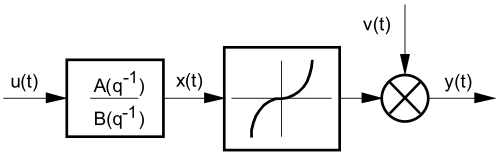

Fig. 1.

posite description of the Wiener model output is given as

$$y(t)=\sum_{k=1}^{r} g_k\left[\sum_{i=1}^{m} a_i u(t-i)-\sum_{j=1}^{n} b_j x(t-j)\right]^k . \qquad (3.2)$$

It is predominantly nonlinear because of its data and parameter combinations. Moreover the internal variable $x(t)$ is generally not measurable. The total number of parameters is $N=(m+n+1)\cdot r$.

To simplify this problem the so-called key term separation principle can be applied [6]. This concerns compound mappings and consists of two steps: 1) a key term is separated in the outer mapping, and then 2) the inner mapping is substituted only for the separated term.

Assuming that $g_1=1$ (this is always possible in the given model), (3.1) can be rewritten as follows:

$$y(t)=x(t)+\sum_{k=2}^{r} g_k x^k(t) . \qquad (3.3)$$

After choosing $x(t)$ as the key term and then substituting (2.1) only for this separated $x(t)$, the model output with an additive noise $\nu(t)$ will be

$$y(t)=\sum_{i=0}^{r} a_i u(t-i)-\sum_{j=1}^{n} b_j x(t-j)+\sum_{k=2}^{r} g_k x^k(t)+\nu(t) . \qquad (3.4)$$

Equation (3.4) and (2.1) defining the internal variable $x(t)$ represent a special form of the Wiener model with polynomial nonlinearity where all system parameters are given explicitly. The total number of parameters is less than in (3.2), since $N=m+n+r$.

The system description is nonlinear in data, but linear in the parameters of (3.4) and (2.1). As the internal variable is unmeasurable, the model parameter estimation can be performed by a proper iterative method [4].

This new form of the Wiener model is parametrically irredundant and all the model parameters appear separately. It provides sufficient flexibility for experimenting with the model structure. The polynomial degrees in linear and nonlinear blocks, respectively, can be adjusted separately without any influence on the other parameters.

4 ASYMMETRIC NONLINEARITIES

Assume, as the simplest case of asymmetric nonlinearity, the two-segment piecewise-linear nonlinearity according to Fig. 2. Then the output of nonlinear block depends on the sign of input and can be written as

$$y(t)=\begin{cases} m_1 x(t) , & \text{if } x(t)>0 ,\\ m_2 x(t) , & \text{if } x(t)<0 , \end{cases} \qquad (4.1)$$

assuming that $0<m_1<\infty$ and $0<m_2<\infty$ are the corresponding segment slope constants. However, it is possible to introduce a switching sequence $\{h(t)\}$ [3], which is defined as

$$h(t)=\begin{cases} 0 , \text{if } x(t)>0 ,\\ 1 , \text{if } x(t)<0 . \end{cases} \qquad (4.2)$$

Then the relation between the inputs $\{x(t)\}$ and outputs $\{y(t)\}$ of asymmetric nonlinearity can be written as

$$y(t)=m_1 x(t)+(m_2-m_1)h(t)x(t) , \qquad (4.3)$$

and in this equation the two segment slopes are incorporated as parameters.

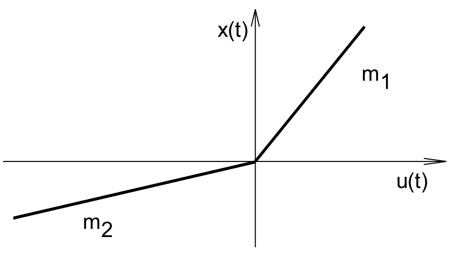

Fig. 2.

The Wiener model with above defined two-segment piecewise-linear asymmetric nonlinearity can be obtained in similar way as in the previous section. It is always possible to put $m_1=1$ and to gain the nonlinear block description with separated variable $x(t)$. Then after substituting (2.1) into (4.3) only for the separated $x(t)$ the output equation of the model will be:

$$y(t)=A\big(q^{-1}\big)u(t)+\big[1-B\big(q^{-1}\big)\big]x(t)+(m_2-1)h(t)x(t) + \nu(t) . \qquad (4.4)$$

The resulting form of nonlinear model given by (4.4) and (2.1) is linear in all the model parameters. As the internal variable $x(t)$ is unmeasurable, the model parameter estimation can be performed by an iterative method [5].

Note that the asymmetric characteristics of nonlinear blocks may be much more complex than the mentioned two-segment piecewise-linear nonlinearity. Similarly as in the case of Hammerstein model [8], the nonlinear block of Wiener model may be given by the input/output relation as follows:

$$y(t)=\begin{cases} f[x(t)] , & \text{if } x(t)>0 ,\\ g[x(t)] , & \text{if } x(t)<0 , \end{cases} \qquad (4.5)$$

where $f(\cdot)$ and $g(\cdot)$ are the corresponding mappings for positive and negative inputs, respectively. As the output of nonlinear block again depends on the sign of input, it is possible to use the switching sequence $\{h(t)\}$ defined by (4.2) and the relation between inputs x(t) and outputs $\{y(t)\}$ of general asymmetric nonlinearity can be written as

$$y(t) = f[x(t)] + \{g[x(t)] - f[x(t)]\}h(t)\,, \tag{4.6}$$

where both mappings are incorporated into a single equation. A special case of discontinuous asymmetric nonlinearity is presented in the following section.

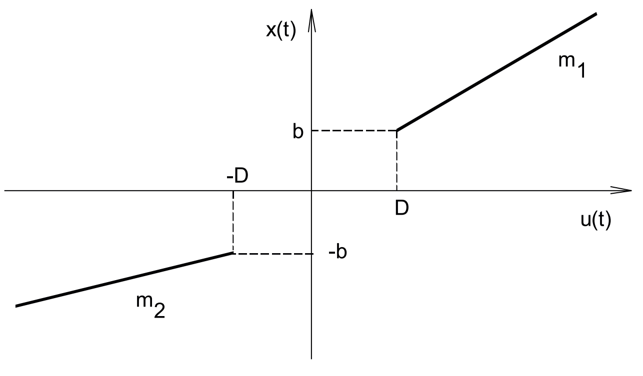

Fig. 3.

5 DISCONTINUOUS NONLINEARITIES

The parametrization of Wiener model is much more complex when the nonlinear block characteristics are discontinuous, *eg*, two-segment piecewise-linear asymmetric functions with preloads and dead zones according to Fig. 3 [1].

The output of assumed nonlinearity $y(t)$ depends on the sign and magnitude of input $x(t)$ and can be written as

$$y(t) = \begin{cases} \underline{y}(t)\,, & \text{if } |x(t)| > D\,, \\ 0\,, & \text{if } |x(t)| < D\,, \end{cases} \tag{5.1}$$

$$\underline{y}(t) = K(t)\{x(t) - D\,\mathrm{sgn}[x(t)]\} + b\,\mathrm{sgn}[x(t)]\,, \tag{5.2}$$

$$K(t) = m_1 + (m_2 - m_1)h(t)\,, \tag{5.3}$$

assuming that $|m1| < \infty$, $|m_2| < \infty$ are the corresponding segment slopes, $0 \le D < \infty$ is the dead zone and $|b| < \infty$ is the preloaded constant. The switching function $h(t)$, or more precisely $h[x(t)]$, is defined as previously.

However, a similar kind of switching appears for the output $y(t)$ in (5.1) because of the dead zone D and we can write

$$y(t) = h[D - |x(t)|]y(t) \tag{5.4}$$

instead of (5.1). Assuming that the following equivalence for (5.4), *ie*,

$$h[x(t)] = 0.5\{1 - \mathrm{sgn}[x(t)]\} \tag{5.5}$$

(for $x(t) \neq 0$) is analogously applicable for (5.4), we can write (5.4) as:

$$y(t) = g[x(t), \underline{y}(t)] = 0.5[1 - \mathrm{sgn}(D - |x(t)|)]\underline{y}(t)\,. \tag{5.6}$$

As g() in (5.6) is a composite mapping we can apply the key term separation principle. First, we separate the internal variable y(t) rewriting (5.6) as follows:

$$y(t) = \underline{y}(t) - 0.5[1 + \mathrm{sgn}(D - |x(t)|)]y(t)\,. \tag{5.7}$$

Then we half-substitute (5.2) for the separated term only, *ie*,

$$\begin{aligned} y(t) = K(t)\{x(t) - D\,\mathrm{sgn}[x(t)]\} + b\,\mathrm{sgn}[x(t)] \\ - 0.5[1 + \mathrm{sgn}(D - |x(t)|)]y(t)\,. \end{aligned} \tag{5.8}$$

To avoid multiplication of parameters in (5.8) we can again apply the key term separation principle. We separate $K(t)x(t)$ in (5.2) as the key term and then assigning

$$\xi(t) = K(t)x(t) = m_1 x(t) + (m_2 - m_1)h(t)x(t)\,, \tag{5.9}$$

we half-substitute (5.9) for the separated (first) term into (5.2)

$$\begin{aligned} \underline{y}(t) = m_1 x(t) + (m_2 - m_1)h(t)x(t) \\ - D\,\mathrm{sgn}[x(t)]\xi(t)/x(t) + b\,\mathrm{sgn}[x(t)]\,. \end{aligned} \tag{5.10}$$

Finally, we can rewrite (5.8) as follows:

$$\begin{aligned} y(t) = m_1 x(t) + (m_2 - m_1)h(t)x(t) - D\,\mathrm{sgn}[x(t)]\xi(t)/x(t) \\ + b\,\mathrm{sgn}[x(t)] - 0.5[1 + \mathrm{sgn}(D - |x(t)|)]y(t)\,. \end{aligned} \tag{5.11}$$

After two successive applications of the key term separation principle we have replaced the original composite mapping, describing the discontinuous characteristics of Fig. 3, with the equation (5.11) and those of (5.9) and (5.10) defining the imposed internal variables $\underline{y}(t)$ and $\xi(t)$, respectively.

To obtain the Wiener model description the above described nonlinearity has to be connected with the linear block. However, a direct substitution of $x(t)$ from (2.1) into (5.11) would result in a very complex expression. Therefore, the key term separation principle can be applied again to simplify the model equation.

Assuming that $m_1 = 1$ (this is always possible in the given system description) we separate the variable $x(t)$ as the key term of the nonlinear mapping (5.11). Then after the substitution of $x(t)$ from (2.1) into (5.11) for the key term only, the system output is given in the form

$$\begin{aligned} y(t) = A\big(q^{-1}\big)u(t) + \big[1 - B\big(q^{-1}\big)\big]\, x(t) + mh(t)x(t) \\ - D\,\mathrm{sgn}[x(t)]\xi(t)/x(t) + b\,\mathrm{sgn}[x(t)] \\ - 0.5\{1 + \mathrm{sgn}[D - |x(t)|]\}\underline{y}(t)\,, \end{aligned} \quad (5.12)$$

where $m = m_2 - 1$. The equation (5.12) and those of (2.1), (5.9) and (5.10) defining the imposed internal variables $x(t)$, $\xi(t)$, and $\underline{y}(t)$ represent a special form of Wiener model with discontinuous nonlinearity as the result of multiple application of the key term separation principle. Again the parameters of linear and nonlinear blocks are separated enabling experimentation with the structure of model by changing the degrees of polynomials for the linear block.

This new form of Wiener model has a minimum number of parameters and all of them enter the model expressions linearly, except D, which appears both linearly and nonlinearly. As all the internal variables are unmeasurable, the model parameter estimation can be performed by an iterative method similarly as for the Hammerstein model [7]. Finally note that putting the corresponding parameters of the above model to zero, models for the nonlinear dynamic systems with the combinations of pure preloaded nonlinearities, dead zone or two-segment piecewise-linear asymmetric nonlinearities can be obtained.

6 CONCLUSIONS

An original approach to the parametrization of block-oriented nonlinear models of Wiener type has been presented. Application of a compound mapping decomposition technique resulted in parametrically irredundant forms of the Wiener models with polynomial, asymmetric and discontinuous nonlinearities.

The proposed Wiener model descriptions with separated parameters of linear and nonlinear blocks provide sufficient flexibility for experimenting with the model structures. Namely, a change of polynomial degrees in the first and second case, or an inclusion/omission of parameters in the third case will not influence the other model parameters. The resulting models are linear in parameters and proper for identification and control purposes.

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References

[1] GU, X.—BAO, Y.—LANG, Z.: A Parameter Identification Method for a Class of Discrete Time Nonlinear Systems, Proceedings 12th IMACS World Congress, Paris, Vol. 4, 1988, pp. 627-629.

[2] HABER, R.—KEVICZKY, L.: Identification of Nonlinear Dynamic Systems, Preprint 4th IFAC Symp. Identification and Sys. Parameter Estimation, Tbilisi, 1976, pp. 62–112.

[3] KUNG, M. C.—WOMACK, B. F.: Discrete Time Adaptive Control of Linear Dynamic Systems with a Two-Segment Piecewise-Linear Asymmetric Nonlinearity, IEEE Trans. Automatic Control **AC-29** No. 2 (1984), 1170–172.

[4] VÖRÖS, J.: Nonlinear System Identification with Internal Variable Estimation, Proceedings 7th IFAC Symp. Identification and Sys. Parameter Estimation, York, 1985, pp. 439–443.

[5] VÖRÖS, J.: Identification and Control of Block-Orientated Systems with Special Nonlinearities, Preprints 9th IFAC Symp. on Identification and Sys. Parameter Estimation, Budapest, Vol. 1, 1991, pp. 622–627.

[6] VÖRÖS, J.: Identification of Nonlinear Dynamic Systems Using Extended Hammerstein and Wiener Models, Control-Theory and Advanced Technology **10** No. 4, Part 2 (1995), 1203–1212.

[7] VÖRÖS, J.: Parameter Identification of Discontinuous Hammerstein Systems, Automatica **33** No. 6 (1997), 1141–1146.

[8] VÖRÖS, J.: Iterative Algorithm for Parameter Identification of Hammerstein Systems with Two-Segment Nonlinearities, IEEE Trans. Automatic Control **44** No. 11 (1999), 2145–2149.

[9] VÖRÖS, J.: On Parametrization of Hammerstein Models J. Electrical Engineering.

[10] ZADEH, L. A.: A Contribution to the Theory of Nonlinear Systems, J. Franklin Institute Vol. **255** (1953), 387–408.

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