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COMMUNICATIONS

TEMPERATURE MEASUREMENT BY CALIBRATED SPRT IN ACCORDANCE WITH ITS–90

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Procedure of the evaluation of temperature measured by a standard platinum resistance thermometer (SPRT) is presented in this contribution. It is based on the estimation of the parameters of the deviation function and their uncertainties and covariances. Application of the proposed procedure is illustrated on an example of temperature measurement by the SPRT calibrated in the temperature range from 0 °C to 660 °C.

K e y w o r d s: calibration model, temperature measurement, uncertainty, covariance, temperature scale

1 INTRODUCTION

The procedure of the evaluation of temperature measurements by a standard platinum resistance thermometer (SPRT) is presented in this contribution. Such temperature measurements are needed for the determination of temperatures in the calibration baths or calibration furnaces for the calibration of resistance thermometers by comparative methods. The SPRT must comply with the conditions in the document International Temperature Scale 1990 (ITS-90). We assume that the SPRT was calibrated in accordance with the ITS-90 in the defining fixed points in the temperature range from 0 °C to 660 °C. It means that it was calibrated in the following defining fixed points (DFP): triple point of water (TPW), freezing points of tin (Sn), zinc (Zn) and aluminium (Al) [11]. Then this SPRT was used for temperature measurement. Proposal of the procedure of evaluation of temperature and uncertainties is the main topic of this contribution. The proposed procedure of evaluation of the uncertainties follows the documents [5, 6] and links to [2, 3, 4, 8, 10].

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2 EVALUATION OF THE SPRT CALIBRATION IN THE DEFINING FIXED POINTS

The inverse function is valid for the evaluation of the measured temperatures T by the SPRT in accordance with the ITS-90 in the corresponding temperature range [8]

$$T - 273.15 = D_0 + \sum_{i=1}^{9} D_i \left[\frac{W_{rT} - 2.64}{1.64}\right]^i \tag{1}$$

where the values of constants Di are given in table 4 in [11] and the values W_{rT} are determined from the deviation function for the corresponding temperature range (eq. (14) in [11] for d=0):

$$\begin{aligned} W_T - W_{rT} = \Delta W_T = a[W_T - 1] \\ + b[W_T - 1]^2 + c[W_T - 1]^3 \,, \end{aligned} \tag{2}$$

it means from equation

$$W_{rT} = W_T - \Delta W_T \,, \tag{3}$$

where ΔW_T is the deviation function from the calibration of the SPRT, and W_T is the ratio of resistances defined as follows

$$W_T = \frac{R_T}{R_{TPW}}. \tag{4}$$

Here R_T is the resistance of the SPRT corresponding to temperature T, and R_{TPW} is the resistance of the SPRT corresponding to the temperature T_{TPW} of TPW from the calibration (arithmetic mean of the resistances in the TPW measured after the measurements in the DFPs (Al, Zn, Sn) which should be taken from the calibration certificate of the SPRT).

The uncertainty of the evaluated temperature from the measurements of the resistance of the SPRT is given by

$$u(T) = A_{rT} u(W_{rT}) \,, \tag{5}$$

where A_{rT} is the sensitivity coefficient,
$u(W_{rT})$ is the combined standard uncertainty of the resistance ratio W_{rT}.

The sensitivity coefficient A_{rT} is given by the derivative of the inverse function (1):

$$A_{rT} = \frac{\partial f(T)}{\partial W_{rT}} = \sum_{i=1}^{9} \frac{iD_i}{1.64} \left[\frac{W_{rT} - 2.64}{1.64}\right]^{i-1}. \tag{6}$$

Using eq. (2), the uncertainty $u(W_{rT})$ is

$$u^2\big(W_{rT}\big) = u^2\big(W_T\big) + u^2\big(\Delta W_T\big) - 2u\big(W_T, \Delta W_T\big) \,, \tag{7}$$

where $u(W_T)$ is the uncertainty of determination of the resistance ratio W_T on the basis of resistance measurement (which corresponds to measured T) by calibrated SPRT and of the value of resistance of the SPRT in the TPW taken from the calibration certificate of the SPRT,
$u\big(\Delta W_T\big)$ is the uncertainty of the determination of the SPRT resistance ratio deviation ΔW_T from its calibration,
$u\big(W_T, \Delta W_T\big)$ is the covariance between the determination of resistance ratio W_T for the measured temperature T and the determination of the resistance ratio deviation ΔW_T of the SPRT from the calibration.

It is assumed that the uncertainties $u(a)$, $u(b)$, $u(c)$ of coefficients a, b, c and the covariances between them $u(a,b)$, $u(a,c)$, $u(b,c)$ and the value of the SPRT resistance measured in the TPW are known from the calibration certificate of SPRT as well.

According to the law of propagation of uncertainty [6] for $u\big(W_T\big)$ we get

$$u^2\big(W_T\big) = \frac{R_{TPW}^2 u^2(R_T) + R_T^2 u^2(R_{TPW})}{R_{TPW}^4}. \tag{8}$$

We do not consider the correlation between the resistance measurement of the SPRT corresponding to the measured temperature T and the measured resistance of the SPRT in the TPW from calibration.

Uncertainty $u(R_T)$ is a combined standard uncertainty of the resistance measurement of the SPRT corresponding to the measured temperature T by corresponding equipment. Uncertainty $u\big(R_{TPW}\big)$ is the uncertainty of the resistance measurement in TPW from calibration of the SPRT (taken from the calibration certificate).

Then the uncertainty $u(\Delta W_T)$ of the resistance ratio deviation ΔW_T according to eq. (2) for the calibrated SPRT is [12,7], here denoting $w = W_T - 1$

$$\begin{aligned} u^2(\Delta W_T) = w^2 u^2(a) + w^4 u^2(b) + w^6 u^2(c) + \\ + 2w^3 u(a,b) + 2w^4 u(a,c) + 2w^5 u(b,c) \end{aligned} \tag{9}$$

Covariance $u\big(W_T, \Delta W_T\big)$ is due to W_T and ΔW_T, the measured resistance R_T and resistance of SPRT in the TPW taken from the calibration, are both involved - and it is determined from

$$\begin{aligned} & u(W_T, \Delta W_T) = \\ & (d_1 R_{Sn} + d_2 R_{Zn} + d_3 R_{Al}) \frac{R_T}{R_{TPW}^4} u^2(R_{TPW}) \end{aligned} \tag{10}$$

where, d_1, d_2, d_3 are given by

$$\begin{aligned} \frac{\partial \Delta W_T}{\partial W_{Sn}} &= b_{11} w + b_{21} w^2 + b_{31} w^3 = d_1 \\ \frac{\partial \Delta W_T}{\partial W_{Zn}} &= b_{12} w + b_{22} w^2 + b_{32} w^3 = d_2 \\ \frac{\partial \Delta W_T}{\partial W_{An}} &= b_{13} w + b_{23} w^2 + b_{33} w^3 = d_3 \end{aligned} \tag{11}$$

and where coefficients $b_{11}, b_{12}, \ldots, b_{33}$ are elements of matrix

$$\mathbf{M}_{DFP}^{-1} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \tag{12}$$

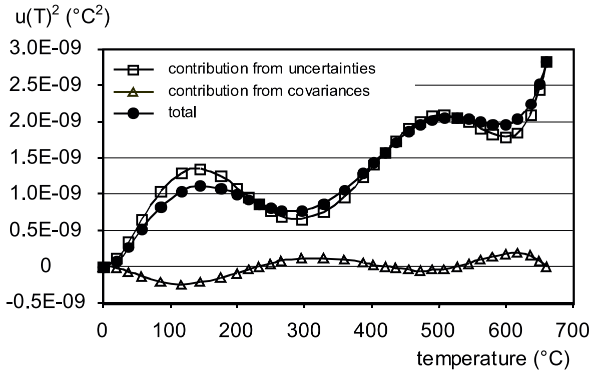

Fig. 1. Uncertainties of SFRT calibration.

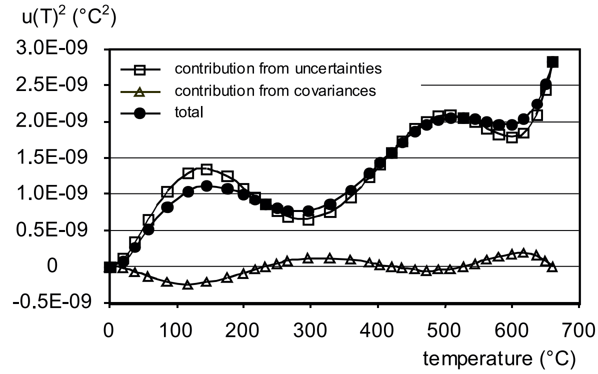

Fig. 2. Total uncertainty of the temperature measurement with and without taking into account the covariances between the DFPs.

The values of the resistances in the DFPs from the calibration are in the calibration certificate of the SPRT. Coefficients a, b, c, are determined from the three known solutions of (2),

$$W_{Sn} - W_{rSn} = \Delta W_{Sn} = a w_{Sn} + b w_{Sn}^2 + c w_{Sn}^3$$
$$W_{Zn} - W_{rZn} = \Delta W_{Zn} = a w_{Zn} + b w_{Zn}^2 + c w_{Zn}^3$$
$$W_{Al} - W_{rAl} = \Delta W_{Al} = a w_{Al} + b w_{Al}^2 + c w_{Al}^3$$

for $w_{Sn} = W_{Sn} - 1$, $w_{Zn} = W_{Zn} - 1$ and $w_{Al} = W_{Al} - 1$, where W_{Sn}, W_{Zn}, W_{Al}, are obtained by substituting R_{Sn}, R_{Zn}, R_{Al} for R_T and W_{rSn}, W_{rZn}, W_{rAl}, [11], see Tab. 2.

We can rewrite (9) into a matrix form $\Delta\mathbf{W}_{DFP} = \mathbf{M}_{DFP}\mathbf{A}$, where $\Delta\mathbf{W}_{DFP} = (\Delta W_{Sn}, \Delta W_{Zn}, \Delta W_{Al})^T$, $\mathbf{A} = (a, b, c)^T$ and where

$$\mathbf{M}_{DFP} = \begin{pmatrix} w_{Sn} & w_{Sn}^2 & w_{Sn}^3 \\ w_{Zn} & w_{Zn}^2 & w_{Zn}^3 \\ w_{Al} & w_{Al}^2 & w_{Al}^3 \end{pmatrix}.$$

Table 1. Values of the resistances in the DFPs and their uncertainties

DFP	Resistance R/Ω	Standard uncertainty of resistance $u(R)/\Omega$
TPW_{Sn}	24.8002001	1.17×10^{-5}
TPW_{Zn}	24.8001927	1.17×10^{-5}
TPW_{Al}	24.8001872	1.17×10^{-5}
Sn	46.9397533	3.85×10^{-5}
Zn	63.7056752	4.98×10^{-5}
Al	83.7191875	6.32×10^{-5}

Table 2. SPRT resistance ratios in the DFPs

DFP	W_{DFP}	W_{rDFP}
Sn	1.892716716	1.89279768
Zn	2.568757266	2.56891730
Al	3.375748208	3,37600860

3 EXAMPLE

The calibrated SPRT has coefficients according to its calibration certificate

$$a = -6.88168 \times 10^{-5} \qquad u(a) = 6.2 \times 10^{-6}$$
$$b = 2.89225 \times 10^{-5} \qquad u(b) = 7.8 \times 10^{-6}$$
$$c = 0.49476 \times 10^{-5} \qquad u(c) = 2.2 \times 10^{-6}$$
$$u(a,b) = -4.7 \times 10^{-11} \qquad u(a,c) = 1.3 \times 10^{-11}$$
$$u(b,c) = -1.8 \times 10^{-11}$$

Evaluation of the temperature and its uncertainty of the measurement by the SPRT

The resistance $R_T = 71.76548\,\Omega$ was measured by the SPRT with the combined standard uncertainty $u(R_T) = 0.00013\,\Omega$. $W_T = 2.8937468$ in accordance with eq. (4) and from Table 1 was evaluated and its uncertainty $u(W_T) = 5.418 \times 10^{-6}$ in accordance with eq. (8).

Then according to eq. (9) there was determined $\Delta W_T = -2.00444 \times 10^{-4}$ and in accordance with eq. (10) $u(\Delta W_T) = 1.8622 \times 10^{-6}$.

From eq. (3) there was determined $W_{rT} = 2.89395$. After substitution of this value into eq. (1) the value of temperature $T = 514.01782\,^\circ\mathrm{C}$ was determined.

Combined standard uncertainty $u(T)$ was evaluated from eq. (5) for the sensitivity coefficient in accordance with eq. (6) $A_{rT} = 295.5298\,\mathrm{K}$.

In accordance with eq. (11) correlation $u(W_T, \Delta W_T) = 1.84827 \times 10^{-12}$ was determined and in accordance with eq. (7) the uncertainty $u(W_{rT}) = 5.397 \times 10^{-6}$ for evaluated values of $u(W_T)$ and $u(\Delta W_T)$. Then in accordance with eq. (5) $u(T) = 0.001595\,^\circ\mathrm{C}$ was evaluated.

Result:
By the SPRT the resistance $R_T = 71.76548\,\Omega$ was measured and $u(R_T) = 0.00013\,\Omega$ to which corresponds the temperature $T = 514.0178\,^\circ\mathrm{C}$ and $u(T) = 0.0016\,^\circ\mathrm{C}$.

We suppose the knowledge of the parameters of the deviation function from the calibration of the SPRT and also the knowledge of the uncertainties and covariances, the SPRT resistance value of the TPW and its uncetrtainty. Covariances between the measurements of the

SPRT resistances in DFPs must be taken into account too.

If we do not know the parameters of the deviation function and their uncertainties and covariances between them then we have to evaluate them in accordance with the procedure in [10, 13 14].

The temperature dependence of uncertainties of SPRT calibration $u(\Delta W_T)$, with and without taking into account the covariance between the SPRT resistances measurements in the DFPs are presented in Fig. 1. This uncertainty consists of two components: a component caused by the uncertainty of the resistance values (resistance ratios) in the DFPs and a component caused by covariances between the resistance ratios in the DFPs. This covariance will not be equal to zero even if the covariance between the resistance values is zero. This is caused by using the same value of R_{TPW} in the resistance ratios in DFP.

Two curves of the total uncertainty of the temperature measurement (expressed in Ohms) are in Fig. 2. The upper line is the case of not taking into account the covariances (the same is also in Fig. 1). The lower line is with taking into account the covariances between the resistance values in the DFPs.

In the case presented in this contribution the value of the uncertainty was reduced by taking into account the covariances between the resistance values in the DFPs. It means the uncertainty of the temperature measurement was reduced too.

4 CONCLUSION

The determination of uncertainties of the calibrated SPRT in the DFPs (in the temperature range from 0 °C to 660 °C) for the measurement of temperature by this SPRT is presented in this contribution. The procedure of temperature determination by the SPRT is presented, too. In this procedure the knowledge is supposed of the resistances of the SPRT corresponding to the calibration in the DFPs and the uncertainties of these resistances and covariances between them as well. The procedure takes into account possible covariances. In many cases a matrix form is used for simplification of the mathematical expressions.

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