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COMMUNICATIONS

DECENTRALIZED CONTROLLER DESIGN WITH GUARANTEED CLOSED–LOOP PERFORMANCE

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The paper presents an original frequency domain method to design a decentralized controller for MIMO systems. The design is carried out on the subsystem level for the so-called equivalent subsystems which are in fact mathematical models of isolated subsystems modified using characteristic functions (CF) of the plant interaction matrix. Local controllers designed for equivalent subsystems guarantee the fulfilment of performance requirements imposed on the global system without any performance deterioration brought about by the effect of interactions. Theoretical results are supported with results obtained by solving several examples, one of which is included.

K e y w o r d s: decentralized controller, frequency domain, generalized Nyquist stability criterion, characteristic functions/loci

1 INTRODUCTION

Complex systems are typical by multiple inputs and multiple outputs (MIMO systems). In this paper, a complex system will be understood as a set of a finite number of subsystems and interactions between them, even though the latter represent the most important problem in MIMO system control. The control design technique proposed has been motivated by the following practical aspects:

1. Control design within the frequency-domain (FD).
FD techniques are probably the most popular among control engineers as they are insightful and use data represented in a non-parametric form, which allows to deal with systems of arbitrary order.

2. Decentralized control (DC) structure.
Multivariable controllers are used if strong interactions within the plant subsystems are to be compensated for. However, there are practical reasons that make restrictions on the controller structure necessary or reasonable. In the extreme case, the controller is split into several local feedbacks and represents a decentralized controller. The plant is then considered and modelled as a composition of a number of subsystems which, having independent inputs, are either stabilized and/or optimized using just local feedbacks. Constraints on the controller structure lead to a certain performance deterioration when compared with centralized (full-controller) systems. This drawback is, however, weighted against by important benefits such as hardware simplicity (reduction of cost information exchange), operation simplicity and reliability improvement (failure tolerance, possibility of a gradual start-up and shutdown of the system) and design simplicity (fewer controller parameters to be chosen).

Major important multivariable frequency-response Nyquist-type design techniques of sequential type based on the concept of the return difference generalized for MIMO systems were developed in the late 60s and throughout the 70's: the Inverse-Nyquist Array (INA) and Direct Nyquist Array (DNA) methods by Rosenbrock, as well as the sequential design technique by Mayne, were followed almost simultaneously by the non-interacting Characteristic locus (CL) technique by MacFarlane and Belletrutti [1]. Development of decentralized control (DC) techniques dates back to the 70's, too, and the research, though not so excessive, is still going on. The FD decentralized control techniques rely on achieving dominance (diagonal, block diagonal), reduction of sensitivity or improvement of performance measures in subsystems. The decentralized control design techniques differ from the multivariable ones in that the latter ones but the CL are actually two-step procedures (design of a compensator to cope with plant interactions followed by a diagonal controller design using SISO methods) while in the DC design the effect of plant interactions on the global system is assessed and then transformed into bounds imposed on individual subsystem closed-loops.

To account for plant interactions, the proposed frequency-domain DC design technique applies a novel approach based upon representing them by their characteristic functions (CF) [1], [2] and introducing the term *equivalent subsystems* for mathematical models of subsystems modified by these CF's and employed in the design procedure for the prespecified performance. Thus, local controllers designed for equivalent subsystems guarantee fulfilment of requirements imposed on the global system (in terms of the decay rate) without any performance deterioration brought about by the effect of interactions.

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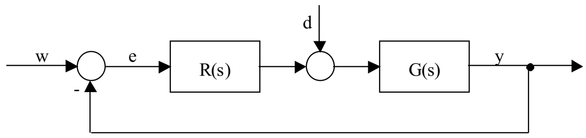

Fig. 1. Standard feedback configuration

The paper is organized as follows: theoretical preliminaries of the proposed technique are surveyed in Section 2, main results are presented in Section 3 and verified on an example in Section 4. Conclusions are given at the end of the paper.

2 PRELIMINARIES

Consider a MIMO system $G(s)$ and the controller $R(s)$ in a standard feedback configuration (Fig. 1) where $R(s) \in R^{l\times m}$ and $G(s) \in R^{m\times l}$ are transfer matrices and w, u, y, e, d are, respectively, vectors of reference variable, control variable, output, control error and disturbance of corresponding dimensions.

According to the fundamental multivariable stability theorem on which relies the Nyquist encirclement criterion for MIMO systems ([1], [4], and references therein),

$$F(s) = [I + Q(s)] = \frac{\det Q(s)}{\det H(s)} \tag{1}$$

where $F(s)$ is the system return-difference matrix, $Q(s)$ is the $m\times m$ (open) loop transfer function (for the system in Fig. 1) $Q(s) = G(s)R(s)$, $H(s)$ is the closed-loop transfer function.

It is important to keep in mind, that the poles of $H(s)$ are at the same time poles of $[I+Q(s)]^{-1}$, and the zeros of $\det[I + Q]$ are poles of $H(s)$ (which, however does not hold conversely), poles of $H(s)$ are either zeros of $\det[I+Q(s)]$ or poles of $Q(s)$.

A necessary and sufficient condition for a feedback system to be well-posed is

$$\det F(s) \neq 0\,. \tag{2}$$

Let D be the *Nyquist D-contour* which is a large contour in the complex plane consisting of the imaginary axis $s = j\omega$ and an infinite semi-circle into the RHP. The Nyquist D-contour must also avoid locations where $Q(s)$ has $j\omega$-axis poles (*eg*, if $R(s)$ includes integrators) by making small indentations around these points. Thus for practical reasons, unstable poles of $Q(s)$ will be considered as poles in the *open* RHP (the $j\omega$-axis poles being included in the LHP).

The characteristic polynomial of the system in Fig. 1 is

$$\Phi(s) = \det[I + G(s)R(s)]\,. \tag{3}$$

By the *Nyquist plot* of $\Phi(s)$ we mean the image of $\Phi(s)$ as s goes clockwise around the Nyquist D-contour. Let $N[k,\Phi(s)]$ denote the number of clockwise encirclements of the point $(k, j0)$ by the Nyquist plot of $\Phi(s)$. If there are n_p open-loop RHP poles, the closed-loop stability can be determined according to the generalized Nyquist stability criterion [1], [3].

Theorem 1 (Generalized Nyquist Stability Criterion). *The feedback system in Fig. 1 is stable if and only if*

1. $\Phi(s) \neq 0$
2. $N[0,\Phi(s)] = -n_p$

For every specific value, say $s = s_1$, the square $m\times m$ matrix function $Q(s)$ of a complex variable s is a matrix having complex number entries and therefore also a set of m complex eigenvalues $q_i(s_1), i = 1, 2, \dots, m$ called *characteristic functions* (CTF) [1]. Put simply, eigenvalues of a matrix function of a complex variable are themselves functions of a complex variable. Then

$$\det[I + Q(s)] = \prod_{i=1}^{m} [1 + q_i(s)]\,. \tag{4}$$

. The CTF $q_i(s)$, $i = 1, 2, \dots, m$ map the usual Nyquist D-contour into the set of m *characteristic loci* denoted $q_i(j\omega)$, $i = 1, 2, \dots, m$. The following theorem holds [1].

Theorem 2. *The closed-loop system with the open-loop transfer function $Q(s)$ is stable if and only if the net sum of clockwise encirclements of the critical point $(-1, 0j)$ contributed by the characteristic loci of $Q(s)$ equals n_p (the number of RHP poles of $Q(s)$), ie*

$$\sum_{i=1}^{m} N[-1,\ q_i(s)] = -n_p\,. \tag{5}$$

3 PROBLEM FORMULATION

Suppose $G(s)$ to be a plant transfer function that can be partitioned into the diagonal and the off-diagonal parts (transfer functions of isolated subsystems and interactions, respectively)

$$G(s) = G_d(s) + G_m(s) \tag{6}$$

where $G_d(s) = \text{diag}\{G_i(s)\}$, $i = 1, \dots, m$ and $G_m(s) = G(s) - G_d(s)$.

A decentralized controller

$$R(s) = \text{diag}\{R_i(s)\}\,, \quad i = 1, \dots, m \tag{7}$$

is to be designed ensuring the overall system stability and guaranteeing a specified performance in terms of the global system decay rate.

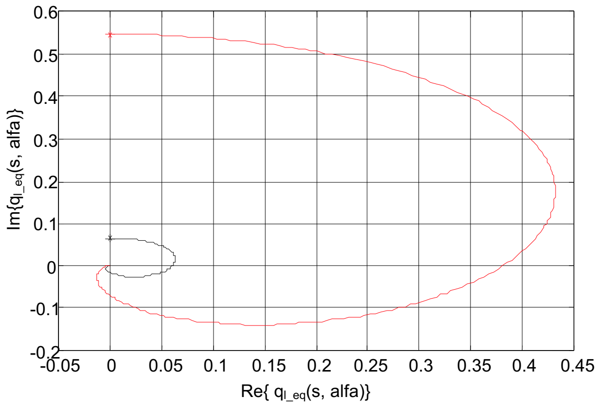

Fig. 2. Equivalent characteristic loci of both subsystems for $\alpha = 0.1$

4 MAIN RESULTS

The global system characteristic polynomial (3) can be factorized with respect to (6)

$$\begin{aligned}\Phi(s) &= \det F(s) = \det\Big\{I + R(s)\big[G_d(s) + G_m(s)\big]\Big\}\\ &= \det[R^{-1}(s) + G_d(s) + G_m(s)]\det[R(s)] = \Phi_1(s)\Phi_2(s)\end{aligned} \tag{8}$$

The assumption that $\det[R(s)] \neq 0$ implies existence $R^{-1}(s)$. Moreover, if $R(s)$ is stable, the closed loop stability is ensured if $\Phi_1(s)$ is stable, whereby

$$\Phi_1(s) = \det[R^{-1}(s) + G_d(s) + G_m(s)]\,. \tag{9}$$

In other words, the global system under DC will be stable if and only if

$$N[0, \Phi_1(s)] = -n_p\,. \tag{10}$$

Consider the diagonal term in (6) having the form

$$R^{-1}(s) + G_d(s) = p(s)I \tag{11}$$

where $p(s)$ is an unknown function of the complex variable s, to be determined such that the global closed-loop system is stable, I is a $m \times m$ identity matrix. From (11) results

$$I + R(s)[G_d(s) - p(s)I] = 0 \tag{12}$$

which for individual subsystems yields

$$1 + R_{ii}(s)G_i^{eq}(s) = 0\,, \quad i = 1, 2, \dots, m \tag{13}$$

where

$$G_i^{eq}(s) = G_{ii}(s) - p(s)\,, \quad i = 1, 2, \dots, m \tag{14}$$

will be called the equivalent transfer function of the i-th subsystem. In general, $p(s)$ can be specified arbitrarily, however since the aim is to stabilize the global system $G(s)$, it should in an appropriate manner account for the interaction term $G_m(s)$. Considering instead of $G_m(s)$ its characteristic functions $\lambda_\ell(s)$, $\ell = 1, \dots, m$ yields the following modification of (9)

$$\Phi_1(s) = \det[p(s)I + G_m(s)] = \prod_{\ell=1}^{m}[p(s) + \lambda_\ell(s)]\,. \tag{15}$$

Should $p(s)$ in (11) be specified as any j-th of m characteristic functions of $G_m(s)$

$$p(s) = -\lambda_j(s) \quad j \in \{1, \dots, m\}$$

then $\Phi_1(s) = 0$ (recalling the definition $\det[\lambda_i(V) - V] = 0$), thus the system is not stable (*Thm* 1).

The novel approach underlying the DC design presented in this paper proposes to choose $p(s)$ equal to any of the m characteristic loci of $G_m(s)$ evaluated for the generalized frequency, *ie*

$$\begin{aligned}p(s) = p(s,\alpha) &= -\lambda_j(s,\alpha)\,, \quad j \in \{1, \dots, m\}\,,\\ s &= -\alpha + j\omega\,, \quad \alpha \geq 0\,,\ \omega \in (-\infty, \infty)\,.\end{aligned} \tag{16}$$

Then (15) becomes

$$\Phi_1(s) = \prod_{\ell=1}^{m}[p(s,\alpha) + \lambda_\ell(s)] \tag{17}$$

and the corresponding transfer function of the i-th equivalent subsystem is

$$G_{ii}^{eq}(s,\alpha) = G_{ii}(s,\alpha) - p(s,\alpha)\,, \quad i = 1, \dots, m\,. \tag{18}$$

In such a case, if the equivalent characteristic functions

$$\begin{aligned}q_\ell^{eq}(s,\alpha) &= p_j(s,\alpha) + \lambda_\ell(s)\,,\\ \ell &= 1, \dots, m\,,\ j \in \{1, \dots, m\}\end{aligned} \tag{19}$$

satisfy the encirclement criterion (*Thm* 2) for n_m RHP poles of $G_m(s)$ and the critical point $(0,0)$, *ie*if

$$\sum_{\ell=1}^{m} N[0, q_\ell^{eq}(s)] = -n_m \tag{20}$$

the closed-loop system is stable with the stability degree α.

5 DECENTRALIZED CONTROL DESIGN PROCEDURE

The main theoretical result from the preceding section can be summarized as follows: If each local controller $R_i(s)$, $i = 1, \dots, m$ simultaneously stabilizes pertinent equivalent subsystems (those obtained by sequentially applying all $p_j(s,\alpha)$, $j = 1, 2, \dots, m$ for the i-th subsystem (18)), *ie* if the equivalent characteristic polynomial

$$\begin{aligned}CLCP_{i\ell}^{eq}(s,\alpha) &= 1 + R_i(s,\alpha)G_{ii\ell}^{eq}(s,\alpha)\,,\\ i &= 1, 2, \dots, m\,;\ \ell = 1, 2, \dots, m\end{aligned} \tag{21}$$

is stable and the equivalent characteristic loci $q_\ell^{eq}(s,\alpha)$, $\ell = 1, \dots, m$, $j \in \{1, \dots, m\}$ defined by (19) satisfy the encirclement condition of *Thm* 2, then the global system is stable with decay rate α.

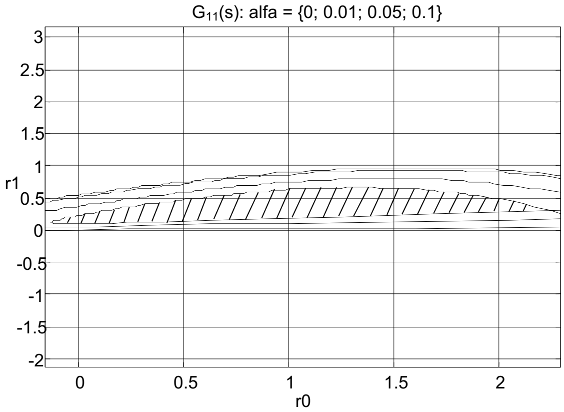

Fig. 3. Neymark D-plots for the 1st subsystem

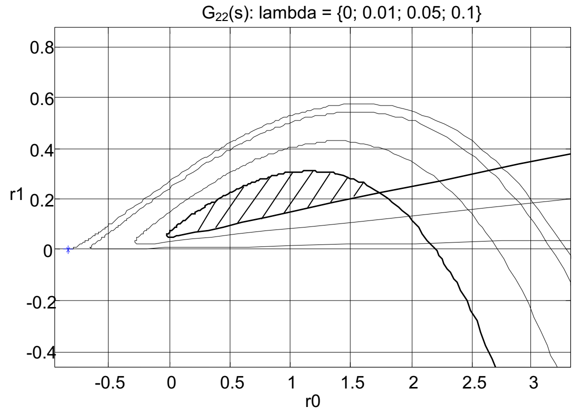

Fig. 4. Neymark D-plots for the 2nd subsystem

Design procedure:

1. Determine the partition of the controlled system into subsystems (diagonal part) and interactions (off-diagonal part)
   $G(s) = G_d(s) + G_m(s)$.
2. Find the characteristic functions of $G_m(s)$: $\lambda_\ell(s)$, $\ell = 1,\dots,m$.
3. Choose any j and specify $p(s,\alpha) = -\lambda_j(s,\alpha)$, $j \in \{1,\dots,m\}$.

The core of the design procedure consists in the following two steps (4, 5).

4. Check whether stability conditions imposed on the corresponding equivalent characteristic functions
   $q_\ell^{eq}(s,\alpha) = p_j(s,\alpha) + \lambda_\ell(s)$, $\ell = 1,\dots,m$, $j \in \{1,\dots,m\}$
   in *Thm* 2 applied to $G_m(s)$ with n_m unstable poles are satisfied (Fig. 2 — Example in the next Section).
   If not, choose the next j and go to 3. If there is no more j applicable, this procedure does not provide a stabilizing controller.
   If yes, go to 5.
5. Design local controllers $R_i(s)$ for all m equivalent subsystems $G_{ii}^{eq}(s,\alpha) = G_{ii}(s,\alpha) - p_j(s,\alpha)$, $i = 1,\dots,m$; j fixed using a suitable frequency-domain technique (*eg* the Neymark D-partition method)

The proposed procedure is illustrated on an example in the next Section.

6 EXAMPLE

Consider a MIMO system with two inputs and two outputs given by the transfer function matrix

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

with

$$G_{11}(s) = \frac{0.01675s^2 - 0.1018s + 0.438}{s^3 + 2.213s^2 + 2.073s + 0.6106},$$

$$G_{12}(s) = \frac{0.01555s^2 - 0.0375s - 0.1106}{s^3 + 2.554s^2 + 1.783s + .5433},$$

$$G_{21}(s) = \frac{0.01325s^2 - 0.03415s + 1.018}{s^3 + 3.927s^2 + 5.815s + 3.547},$$

$$G_{22}(s) = \frac{0.01575s^2 - 0.1252s + 0.442}{s^3 + 3.514s^2 + 2.01s + 0.3872}.$$

Equivalent characteristic loci of both subsystems for the decay rate $\alpha = 0.1$ are in Fig. 2.

Local PI controllers for equivalent subsystems have been designed applying the Neymark D-partition to

$$R_i^{-1}(s) + [G_{ii}(s,\alpha) - p(s,\alpha)] = 0, \quad i = 1,2$$

for the decay rate $\alpha = 0.1$ (Fig. 3, 4).

The resulting PI controllers are

$$R_1(s) = \frac{1.2786s + 0.35525}{s},$$

$$R_2(s) = \frac{0.93272s + 0.19157}{s}$$

The closed-loop eigenvalues show that the prescribed decay rate has been achieved (0.1549).

$$\lambda_i = \{-0.1549;\ -0.1825;\ -0.2286 \pm 0.4133j;\ -0.3464 \pm 0.6648j;\ -0.4129 \pm 0.3888j;\ -1.079 \pm 0.9291j;\ -1.2811;\ -1.7029;\ -1.8115;\ -2.9768\}$$

7 CONCLUSION

In this paper a novel frequency-domain approach to the decentralized controller design has been proposed. Its main advantage is that the plant interactions are included in the design of local controllers through their characteristic functions that are modified to achieve required performance (in terms of a specified decay rate) for the global system. Theoretical results are supported with results obtained by solving several examples, one of which is included.

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