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COMMUNICATIONS

THE PERFORMANCE OF THE FOLLOWER JAMMER WITH A WIDEBAND–SCANNING RECEIVER

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Modern wideband-scanning receivers achieve a scanning speed of tens of gigahertz per second. It allows constructing follower jammers that can jam contemporary frequency hopping systems. The existing articles about the influence of follower jammers on frequency hopping systems are focused on geometrical or power aspects of jamming only. Analyses of the jamming probability (*ie*, what relative part of transmission is jammed) are not published yet. In this article, a mathematical model for computation of this jamming probability is derived. An analysis of the performance of follower jammers is performed, too.

K e y w o r d s: follower jammer, frequency hopping system.

1 INTRODUCTION

The influence of the follower jammer on the frequency hopping system (further FH system only) is solved in this article. We assume a scenario where the follower jammer operates against a single FH system. For example, this scenario can occur in a police action against organized crime.

The radio terminal of any FH system periodically changes its transmission channel. The period of these changes T_h and the parameter $R_h = 1/T_h$ are called the dwell interval and the hopping rate, respectively. The actual transmission channel is selected from a set of F radio channels pseudorandomly. We suppose that all these channels constitute a continuous frequency bandwidth W. For example, the most used FH systems are military systems with $T_h = 2$–$10\,\mathrm{ms}$, $F = 2320$ channels and $W = 58$ MHz [1].

During of a dwell interval, the follower jammer scans the bandwidth W and finds the actual transmission channel of the FH system. When the jammer is successful, it transmits a jamming signal in this channel until the end of the dwell interval. The efficiency of the follower jammer especially depends on the scanning speed of its scanning receiver. The wideband-scanning receiver simultaneously analyzes all channels from the so-called scan window with bandwidth W_s. The performance parameter of the scanning receiver is the scanning speed $R_s = W_s/T_z$, where T_z is the analysis time of the single scan window. For example, contemporary performance receiver ([2], p. 354) has parameters $W_s = 10$ MHz and $R_s = 72$ GHz/s.

The published articles about the influence of follower jammers on FH systems are focused on geometrical [3] or power aspects [4] of jamming. Analyses of the jamming probability (*ie*, what relative part of the transmission is jammed) are not published yet. In this article, a mathematical model for computation of this jamming probability is derived. The analysis of the performance of follower jammers is performed, too.

2 MODEL

Let us suppose that one FH system operates in the bandwidth W only. Next, we suppose that the follower jammer knows the parameters of the FH system and knows the moments of channel changes, too. Exactly at these moments ($t = 0$), the jammer initiates searching of the actual channel. Let t_1 be the moment when the scanning receiver finds the actual transmission channel of the FH system. Let t_2 be the moment when the follower jammer initiates jamming of the found channel. The quantity $\tau_r = (t_2 - t_1)$ is the activation delay. The next relevant quantity is the propagation delay τ_d. This delay is the difference between the propagation time of the signal along the broken line FH transceiver – jammer – FH receiver and the signal along line FH transceiver – FH receiver.

The scanning receiver analyzes k scan windows during a single dwell interval — see equation (4). Let us index these windows in the order of their analyzing by number $j = 1, 2, \dots, k$. When FH terminal transmits in j-th scan window, then this transmission is found at moment $t_0 = j\,T_z$ and the FH receiver is jammed at the moment t_r

$$t_r = t_0 + \tau_r + \tau_d = j\,T_z + (\tau_r + \tau_d) = j\,T_z + T_r\,, \qquad (1)$$

where we denote the time $T_r = (\tau_r + \tau_d)$ as the jamming delay.

It follows from condition $t_r < T_h$ that the follower jammer is able to analyze at most m scan windows during the single dwell interval:

$$m = \left\lfloor \frac{T_h - T_r}{T_z} \right\rfloor = \left\lfloor \frac{T_t}{T_z} \right\rfloor\,, \qquad (2)$$

where $\lfloor x \rfloor$ is the maximum integer equal to or smaller than x and $T_t = (T_h - T_r)$ is the theoretically maximum jammed part of the dwell interval.

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The overall number of scan windows in the FH system bandwidth is

$$n = \left\lceil \frac{W}{W_s} \right\rceil , \tag{3}$$

where $\lceil x \rceil$ is the minimum integer equal to or greater than x.

When $m \geq n$, then the follower jammer is able to analyze all scan windows (*ie*, the entire bandwidth W) during the dwell interval. The jammer can jam all dwell intervals of the FH system in this case. We name this situation as the situation with guaranteed jamming. When $m < n$, then the follower jammer is able to analyze some scan windows only. Then the FH system is jammed only when it transmits in any analyzed window. We name this situation as the situation with occasional jamming.

Let k be the number of scan windows which the follower jammer analyzes in the dwell interval. It is evident that

$$k = \mathrm{Min}\{m, n\} . \tag{4}$$

When $k = m$, then jamming is occasional, when $k = n$, jamming is guaranteed.

The follower jammer analyzes all scan windows randomly with uniform probability $q = 1/n$. The probability $q\,(n-k)$ is the probability that the FH system operates in the scan window which is not analyzed. The jammed period of the dwell interval y is equal to zero in this case. On the contrary, the FH system is found out in the j-th analyzed window, where $j = 1, 2, \ldots, k$. In this case, the dwell interval is jammed for a period y:

$$y = T_h - t_r = T_h - (j\,T_z + T_r) = T_t - j\,T_z . \tag{5}$$

Now, we can express the probability distribution of the jammed period of the dwell interval:

$$p(y) = \begin{cases} q\,(n-k) , & y = 0 , \\ q , & y = T_t - j\,T_z, j = 1, 2, \ldots, k . \end{cases} \tag{6}$$

The average jammed period of the dwell interval Y is given by

$$Y = \sum_y y\,p(y) = \frac{k}{n}\left(T_t - T_z\,\frac{k+1}{2}\right) . \tag{7}$$

We can specify this expression for both possible cases of the jamming by substituting k:

$$Y = \begin{cases} T_t - T_z\,\frac{n+1}{2} , & m \geq n , \\ \frac{m}{n}\left(T_t - T_z\,\frac{m+1}{2}\right) , & m < n , \end{cases} \tag{8}$$

where the upper expression is valid for the situation with guaranteed jamming and the bottom expression is valid for the situation with occasional jamming.

3 ANALYSIS

Let us define the average relative jammed time of the dwell interval h:

$$h = \frac{Y}{T_h} . \tag{9}$$

This parameter is called the jamming probability. We use this probability as an indicator of the following jammer effectiveness. We perform the analysis from the viewpoint of requirements on the scanning receiver. Hence, we neglect factors that do not relate to the receiver performance, *ie*, we neglect the jamming delay ($T_r = 0$). In consequence of this presumption, the theoretically maximum jammed part of the dwell interval T_t is equal to the dwell interval T_h. Then, from (7) and (9) it is valid

$$h = \frac{k}{n}\left(1 - \frac{T_z}{T_h}\,\frac{k+1}{2}\right) . \tag{10}$$

We can derive the dependence of quantity m on the jamming probability h for the case of occasional jamming from this expression. This dependence makes easy the next analysis. We express the ratio T_h/T_z from (10) and substitute $k = m$. Then we substitute the acquired expression into (2) and obtain

$$m = \left\lfloor \frac{m\,(m+1)}{2\,(m - n\,h)} \right\rfloor . \tag{11}$$

The solution of this equation (see Appendix) is the expression:

$$m = \lfloor 2\,n\,h + 1 \rfloor . \tag{12}$$

Let us introduce the limiting value h^* of the jamming probability. This value is the minimum value of the jamming probability for the case of guaranteed jamming. From this definition, it follows that $h = h^*$, when $m = n$. At the same time, the value h must be minimum. Hence, we can write equation (12) in the form $n = 2\,n\,h^* + 1$. From this equation we can obtain the resulting expression for the limiting value h^* of the jamming probability:

$$h^* = \frac{n-1}{2\,n} . \tag{13}$$

The value h^* divides the interval of jamming probability values into two subintervals. The subinterval $h \geq h^*$ defines the case of the guaranteed jamming and the subinterval $h < h^*$ defines the case of the occasional jamming.

Now, we express the quantity T_z from (10) and substitute this expression in the definition of the scanning speed $R_s = W_s/T_z$. We obtain the equation

$$R_s = \frac{W_s}{T_h}\,\frac{k+1}{2}\,\frac{k}{k - n\,h} . \tag{14}$$

We specify this equation for both cases of jamming:

$$R_s = \begin{cases} \frac{W_s}{T_h}\,\frac{n+1}{2}\,\frac{1}{1-h} , & h \geq h^* \\ \frac{W_s}{T_h}\,\frac{m+1}{2}\,\frac{m}{m-n\,h} , & h < h^* , \end{cases} \tag{15}$$

where the quantity m is given by (12).

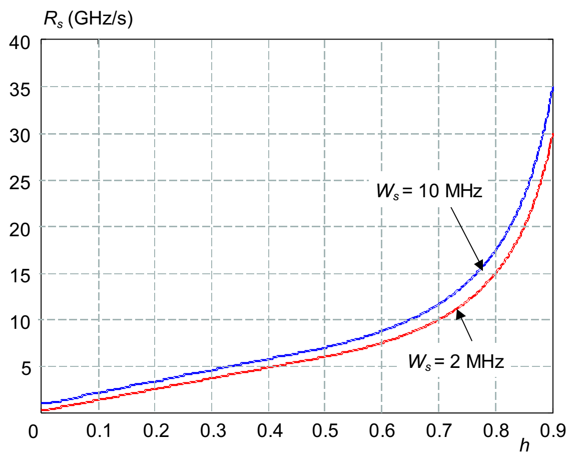

Fig. 1. The dependences of the necessary scanning speed R_s on the jamming probability h. The dependences are valid for the FH system bandwidth $W = 58$ MHz, hopping rate $R_h = 100$ hop/s and for the scan windows $W_s = 2$ and 10 MHz.

Now, we can determine the requirements for performance of the scanning receiver. Let us illustrate it on the following problem. We want to jam two types of FH systems with the bandwidth $W = 58$ MHz. The hopping rate is 100 hop/s for system A) and 330 hop/s for system B). The transmitted voice is coded by the adaptive delta modulation without any error control coding with the date rate 16 kb/s. From [5], we can estimate that the voice is unintelligible when the jamming probability h is equal to 0.4. Let us assume that the scan window of the scanning receiver W_s is equal to 10 MHz. Then we can find that the FH system A) is paralyzed by the follower jammer with the scanning speed $R_s = 5.8$ GHz and the FH system B) is discarded by the follower jammer with the scanning speed $R_s = 19$ GHz.

Now, let us analyze expression (15). The first conclusion is the fact that the necessary scanning speed R_s is directly proportional to the hopping rate $R_h = 1/T_h$ for any given value of the jamming probability h. It means that when the hopping rate R_h of the FH system is increased d times, we must increase the scanning speed d times for the same value of the jamming probability h.

We can derive the next conclusions from Fig. 1. Dependences of the scanning speed R_s on the jamming probability h are displayed here for $W = 58$ MHz, $R_h = 100$ hop/s and $W_s = 2$ or 10 MHz. It is evident that the requirements on the scanning speed R_s increase approximately directly proportional to the jamming probability when $h < h^*$. (Note: The value h^* is 0.483 and 0.417 for $W_s = 2$ and 10 MHz, respectively.) But these requirements increase hyperbolically when $h \geq h^*$, *ie*, for greater values of the jamming probability.

The last conclusion from Fig. 1 is the fact that for two scanning receivers with the same scanning speed, the receiver with a narrower scan window is more efficient. For example, we need the receiver with $W_s = 2$ MHz and $R_s = 10$ GHz/s for the jamming probability $h = 0.7$. But, we need $R_s = 11.7$ GHz/s when the bandwidth of the scan window $W_s = 10$ MHz. The necessary increment of R_s is 17 %.

4 CONCLUSION

This article analyzes a situation when a follower jammer with a wideband-scanning receiver jams a single FH system. The main contribution of the article is a mathematical model for computation of the jamming probability. The analysis of the performance of follower jammers is accomplished, too.

The following facts result from the analysis of the model:

1. The necessary scanning speed R_s of the scanning receiver is directly proportional to the hopping rate R_h of the FH system.
2. For smaller values of the jamming probability h, the necessary scanning speed R_s of the scanning receiver increases approximately directly proportionally. For greater values of quantity h, the requirements on the scanning speed increase hyperbolically.
3. In the case of scanning receivers with the same scanning speed, it is preferable to use the scanning receiver with the most narrow scan window W_s.

APPENDIX

In this Appendix, we prove that expression (12) is the solution to equation (11). It follows from equation (11):

$$m + 1 > \frac{m\,(m+1)}{2\,(m - n\,h)} \geq m\,.$$

We multiply the left and the right inequality by the term $2\,(m-n\,h)/(m+1)$ and $2\,(m-n\,h)/m$, respectively. After some arrangements, we obtain two following inequalities: $m+1 > 2\,n\,h + 1 \geq m$. It follows from these inequalities that $m = \lfloor 2\,n\,h + 1 \rfloor$. This result is the searched solution (12).

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