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COMMUNICATIONS

PARAMETER IDENTIFICATION OF HAMMERSTEIN SYSTEMS WITH ASYMMETRIC DEAD–ZONES

Jozef Vörös *

The paper deals with the parameter identification of Hammerstein systems having piecewise-linear nonlinearities with asymmetric dead-zones. A new form of nonlinearity representation provides a special form of Hammerstein model. Parameter estimation is carried out iteratively with measured input and output data records and estimated internal variables. To demonstrate the feasibility of the identification method, illustrative examples are included.

K e y w o r d s: nonlinear systems, Hammerstein model, identification, dead-zone

1 INTRODUCTION

Control systems often have dead-zone characteristics, which are usually poorly known, although they can significantly influence the system performance (Kalaš *et al*, [4]). The dead-zones can be incorporated into block- oriented dynamic models as input blocks giving the Hammerstein model, or as output blocks for the Wiener models (Haber, Keviczky, [3]). Such dynamic models were considered for the adaptive control of nonlinear systems (Tian, Tao, [7]), (Tao, Kokotovic, [6]). However, parameter identification of Hammerstein systems with dead-zones was presented only in (Gu, Bao, Lang, [2]), (Vörös, [8]) and (Bai, [1]). In these cases, the discontinuous nonlinearities with asymmetric piecewise-linear characteristics but with symmetric dead-zones (and preloads) were studied.

Characteristics with asymmetric dead-zones, which have different dead-zone intervals for positive and negative inputs, respectively, can be considered and modeled as a special case of multisegment piecewise-linear characteristics. The recently proposed approach to modeling and parameter identification of multisegment piecewise-linear characteristics can deal with asymmetric dead-zones both in static and in dynamic cases (Vörös, [9]).

This paper deals with a new approach to the parameter identification of nonlinear dynamic systems that can be represented by the Hammerstein model with asymmetric dead-zones. First, a multisegment piecewise-linear nonlinearity representation is described, which is appropriate for these characteristics. Then a special form of Hammerstein model is presented enabling the parameter estimation algorithm to be (quasi)linear, as all the model parameters to be estimated appear linearly (at least once) in the model description. Finally, simulation results for Hammerstein systems having multisegment piecewise-linear nonlinearities with dead-zones are included.

2 PIECEWISE–LINEAR NONLINEARITY WITH DEAD–ZONES

Piecewise-linear nonlinearities with asymmetric dead-zones can be considered as a special case of general multisegment piecewise-linear characteristics. The output $x(t)$ of assumed nonlinearity according to Fig. 1 can be written as [7]:

$$x(t) = \begin{cases} m_{R1}u(t) & \text{if } 0 \le u(t) \le D_{R1} \\ m_{R2}\,[u(t) - D_{R1}] + m_{R1}D_{R1} & \text{if } u(t) > D_{R1} \end{cases} \tag{2.1}$$

$$x(t) = \begin{cases} m_{L1}u(t) & \text{if } D_{L1} \le u(t) < 0 \\ m_{L2}\,[u(t) - D_{L1}] + m_{L1}D_{L1} & \text{if } u(t) < D_{L1} \end{cases} \tag{2.2}$$

where $|m_{R1}| < \infty$, $|m_{R2}| < \infty$ are the corresponding segment slopes and $0 \le D_{R1} < \infty$ is the constant for the positive inputs, $|m_{L1}| < \infty$, $|m_{L2}| < \infty$ are the corresponding segment slopes and $-\infty < D_{L1} \le 0$ is the constant for the negative inputs.

According to the approach proposed in [9] the above (2+2)-segment piecewise-linear characteristic can be described in the following compact form:

$$\begin{aligned} x(t) = m_{R1}h[-u(t)]u(t) + (m_{R2} - m_{R1})h[D_{R1} - u(t)]u(t) \\ - D_{R1}f_1(t) + m_{L1}h[u(t)]u(t) \\ + (m_{L2} - m_{L1})h[u(t) - D_{L1}]u(t) - D_{L1}f_2(t) \end{aligned} \tag{2.3}$$

where the internal variables

$$f_1(t) = f_1[u(t)] = (m_{R2} - m_{R1})h[D_{R1} - u(t)] \tag{2.4}$$

$$f_2(t) = f_2[u(t)] = (m_{L2} - m_{L1})h[u(t) - D_{L1}] \tag{2.5}$$

are unmeasurable and the switching function,

$$h[u(t) - D] = \begin{cases} 0 & \text{if } u(t) - D > 0 \\ 1 & \text{if } u(t) - D < 0 \end{cases} \tag{2.6}$$

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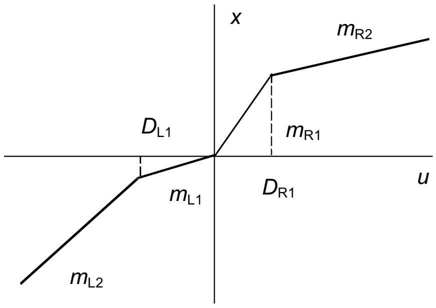

Fig. 1. Piecewise-linear characteristic

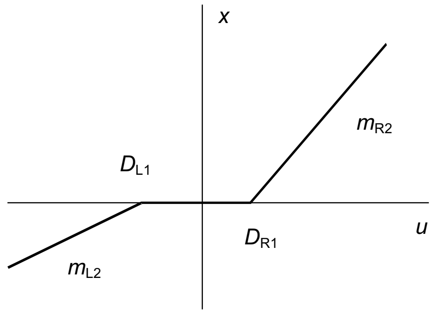

Fig. 2. Piecewise-linear characteristic with dead-zones

switches between two sets of input values $u(t)$.

Now, the piecewise-linear characteristics with asymmetric dead-zones can be represented by the above model having the parameters $m_{R1} = 0$ and $m_{L1} = 0$.

The input/output relation for the characteristic in Fig. 2 can be simplified as follows:

$$x(t) = m_{R2}h[D_{R1} - u(t)]u(t) - D_{R1}f_1(t) + m_{L2}h[u(t) - D_{L1}]u(t) - D_{L1}f_2(t) \quad (2.7)$$

where the internal variables are

$$f_1(t) = m_{R2}h[D_{R1} - u(t)]\,, \quad (2.8)$$

$$f_2(t) = m_{L2}h[u(t) - D_{L1}]\,. \quad (2.9)$$

The constants D_{R1} and D_{L1} determine the dead-zones for positive and negative inputs, respectively.

The description for the (2+2)-segment piecewise-linear characteristic (2.7) can be generalized as follows. After defining the internal variables:

$$f_{1,i}(t) = (m_{R,i+1} - m_{R,i})h[D_{R,i} - u(t)]\,, \quad (2.10)$$

$$f_{2,j}(t) = (m_{L,j+1} - m_{L,j})h[u(t) - D_{L,j}]\,, \quad (2.11)$$

$i = 2, \dots, n_R$ and $j = 2, \dots, n_L$, where $|m_{R,i}| < \infty$, $|m_{L,j}| < \infty$ are the segment slopes, $0 \le D_{R,i} < D_{R,i+1} < \infty$ are the constants representing the partition for the positive inputs, while $-\infty < D_{L,j+1} < D_{L,j} \le 0$ are the constants representing the partition for the negative inputs, the output equation for the general multisegment piecewise-linear characteristics with dead-zones can be written in the following simplified form

$$\begin{aligned} x(t) &= m_{R2}h[D_{R1} - u(t)]u(t) - D_{R1}f_1(t) \\ &+ \sum_{i=2}^{n_R}\big\{(m_{R,i+1} - m_{R,i})h[D_{R,i} - u(t)]u(t) - D_{R,i}f_{1,i}(t)\big\} \\ &+ m_{L2}h[u(t) - D_{L1}]u(t) - D_{L1}f_2(t) + \sum_{j=2}^{n_L}\Big\{\big(m_{L,j+1} \\ &- m_{L,j}\big)h[u(t) - D_{L,j}f_{2,j}(t)\Big\} \end{aligned} \quad (2.12)$$

where

$$0 \le D_{R,1} < D_{R,2} < \dots < D_{R,n_R} < \infty\,, \quad (2.13)$$

$$-\infty < D_{L,n_L} < \dots < D_{L,2} < D_{L,1} \le 0\,, \quad (2.14)$$

and again the constants D_{R1} and D_{L1} determine the dead-zones for the positive and negative inputs, respectively.

3 HAMMERSTEIN MODEL WITH DEAD–ZONES

The Hammerstein model is given by the cascade connection of a static nonlinearity block followed by a linear dynamic system. The difference equation model of its linear block representing the model output part can be given as

$$y(t) = A(q^{-1})x(t) + [1 - B(q^{-1})]y(t)\,, \quad (3.1)$$

where $x(t)$ and $y(t)$ are the inputs and outputs, respectively, $A(q^{-1})$ and $B(q^{-1})$ are scalar polynomials in the unit delay operator q^{-1}

$$A(q^{-1}) = a_0 + a_1q^{-1} + \dots + a_mq^{-m}\,, \quad (3.2)$$

$$B(q^{-1}) = 1 + b_1q^{-1} + \dots + b_nq^{-n}\,, \quad (3.3)$$

and we can put $a_0 = 1$ (it is always possible to fix one parameter in the Hammerstein model). If the nonlinear block is described by (2.7), then the equation

$$\begin{aligned} y(t) &= m_{R2}h\,[D_{R1} - u(t)]\,u(t) - D_{R1}f_1(t) \\ &+ m_{L2}h\,[u(t) - D_{L1}]\,u(t) - D_{L1}f_2(t) \\ &+ \big[A(q^{-1}) - 1\big]\,x(t) + \big[1 - B(q^{-1})\big]\,y(t) \end{aligned} \quad (3.4)$$

and the equations (2.8), (2.9) and (2.7) defining the internal variables $f_1(t)$, $f_2(t)$, and $x(t)$, represent a special form of Hammerstein model with multisegment piecewise-linear nonlinearity including dead-zones. The model has a minimum number of parameters and all of them enter the expressions linearly, except D_{R1} and D_{L1}, which appear both linearly and nonlinearly.

As the internal variables in (3.4) are unmeasurable, an iterative algorithm with internal variables' estimations

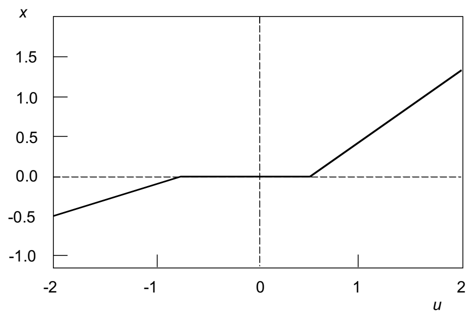

Fig. 3. Example 1 — characteristic with asymmetric dead-zones

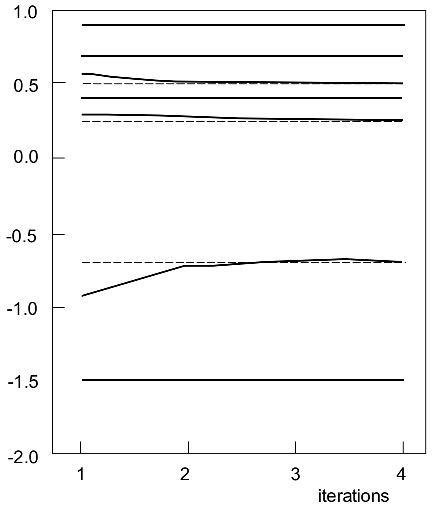

Fig. 4. Example 1 — Hammerstein model parameter estimates

can be considered for the model parameter estimation, similarly as in [9]. The identification algorithm based on the derived special form of Hammerstein model enables the direct estimation of all the model parameters using input/output records and internal variables' estimates.

In the same way, the general case of multisegment piecewise-linear nonlinearity with dead-zones given by (2.12) can be incorporated into the Hammerstein model. The resulting form will be linear in all the parameters to be estimated, except D_{Ri} and D_{Lj}, which again appear both linearly and nonlinearly.

Note that the identification could be performed using the full form of multisegment piecewise-linear nonlinearity description (Vörös, [9]). However, omitting the needless terms leads to the faster convergence of parameter estimates. -

4 ILLUSTRATIVE EXAMPLES

The proposed method for the parameter identification of nonlinear dynamic systems having piecewise-linear nonlinearities with asymmetric dead-zones using the Hammerstein model was implemented and tested by means of MATLAB packages. To illustrate the feasibility of the proposed identification method, the following examples show the parameter estimation process for the Hammerstein systems having piecewise-linear nonlinearities with asymmetric dead-zones.

In the first example the linear dynamic system was given by the difference equation

$$y(t+1) = x(t) + 0.25x(t-1) + 1.5y(t) - 0.7y(t-1),$$

and the nonlinear block was given by the characteristic shown in Fig. 3. This nonlinearity was characterized by the following parameters:

$$m_{R2} = 0.9, \; m_{L2} = 0.2, \; D_{R1} = 0.5, \; D_{L1} = -0.7.$$

The least squares method has been applied for the repeated estimations of both linear and nonlinear block parameters. The initial values of the parameters were chosen zero, except ${}^1D_{R1} = {}^1D_{L1} = 0.1$, to start up the iterative algorithm. The identification was carried out with 200 samples, using a random input with $|u(t)| < 2$. The process of parameter estimation of the Hammerstein system is graphically shown in Fig. 4 (the order of parameters: m_{R2}, b_2, D_{R1}, m_{L2}, a_1, D_{L1}, b_1). The parameters estimates converge to the real values (dotted lines) after 4 iterations.

The next example illustrates the use of general description (2.12) where the linear dynamic system was given by the relation:

$$y(t+1) = x(t) + 0.1x(t-1) + 1.5y(t) - 0.7y(t-1),$$

and the (3+3)-segment piecewise-linear characteristic with dead-zones in Fig. 5 was given by the following set of parameters:

$$p_1 = m_{R2} = 1.3, \; p_2 = m_{L2} = 1.2, \; p_3 = D_{R1} = 0.3,$$
$$p_4 = D_{L1} = -0.4, \; p_5 = m_{R3} - m_{R2} = -0.8,$$
$$p_6 = m_{L3} - m_{L2} = -0.6, \; p_7 = D_{R2} = 1.5,$$
$$p_8 = D_{L2} = -1.7.$$

Also in this case the least squares method has been applied for the simultaneous estimation of both linear and nonlinear block parameters under the same conditions as in the first example. The identification was carried out with 600 samples, using a random input with $|u(t)| < 3$. The results of iterative process in Fig. 6 (the order of parameters: p_7, p_1, p_2, b_2, p_3, a_1, p_4, p_6, p_5, b_1, p_8) show good convergence of the parameters' estimates to the true values after about 10 iterations.

The estimation process for the same Hammerstein system was also tested with additive output noise and a Monte Carlo simulation study of 20 runs was performed. As the iterative processes may stop after distinct numbers of steps and it is impossible to ensure the same conditions for the evaluation in the Monte Carlo run, the number of iterations was limited to 20. The normally distributed random noises with zero mean and the signal-to-noise ratio SNR = 100 were added to the simulated outputs (SNR — the square root of the ratio of output and noise variances). The results for 1200 samples of input/output data are in Table 1, where the estimated parameters are given by the mean values and the standard deviations.

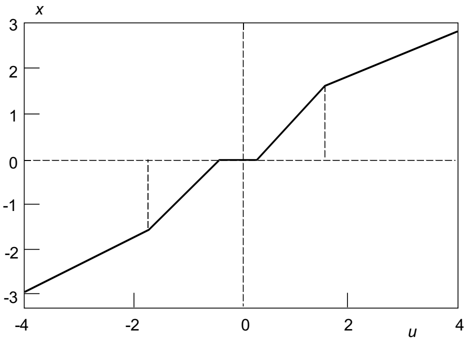

Fig. 5. Example 2 — characteristics with asymmetric dead-zones

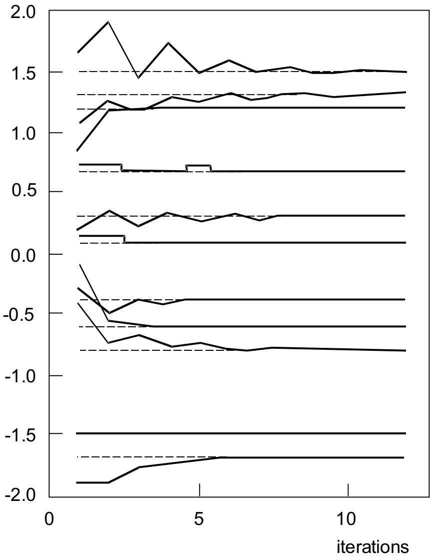

Fig. 6. Example 2 — Hammerstein model parameter estimates

Table 1.

	True	Mean	STD
p_1	1.3000	1.2957	0.0160
p_2	1.2000	1.2064	0.0122
p_3	0.3000	0.2973	0.0087
p_4	−0.4000	−0.4026	0.0091
p_5	−0.8000	−0.7973	0.0220
p_6	−0.6000	−0.6090	0.0138
p_7	1.5000	1.5049	0.0206
p_8	−1.7000	−1.6948	0.0231
a_1	0.1000	0.1022	0.0032
b_1	−1.5000	−1.4980	0.0010
b_2	0.7000	0.6981	0.0009

5 CONCLUSIONS

A new approach to parameter identification of nonlinear dynamic systems having multisegment piecewise-linear nonlinearities with asymmetric dead-zones using the Hammerstein model has been presented. A new and universal form of multisegment nonlinearity representation has led to a special form of Hammerstein model with the minimum number of parameters. All the model parameters to be estimated appear explicitly in the model description and they can be estimated simultaneously.

The presented examples of identification process demonstrate the feasibility and good convergence properties of the proposed approach. As the model parameters are identical with those of linear and nonlinear blocks, respectively, the model can be used in control algorithms.

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