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DUAL VERSION OF MODIFIED PSEUDO–WIGNER DISTRIBUTION

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In the field of quadratic TFRs, the Cohen class of TFRs has got a very important position. It follows from the fact that a number of quadratic TFRs can be obtained by suitable selection of the Cohen TFR kernel. The TFRs are defined using corresponding variables, such as window functions in spectrograms defined in a time or frequency domain. Distributions described by variables defined in the frequency domain are usually known as dual versions to distributions originally described by variables described in the time domain. In this paper, a dual version to the modified pseudo-Wigner distribution will be introduced. Based on the synthetic multicomponent signal and measured EKG signal analysis, it will be shown that the dual modified pseudo-Wigner distribution can provide a good time-frequency resolution and good cross-terms reduction.

K e y w o r d s: Cohen class of TFRs, Wigner distribution, spectrogram, modified pseudo-Wigner distribution

1 INTRODUCTION

Time-frequency representations (TFRs) of signals have been intensively studied in the last decade. This process has been stimulated by hundreds of their applications, especially in the field of nonstationary signal processing. At the present time, a number of linear and quadratic TFRs are available. In the field of quadratic TFRs, the Cohen class of TFRs has possessed a very important position. Wigner distribution, pseudo-Wigner distribution, spectrogram, Page distribution and Rihaczek distribution can be given as examples of TFRs belonging to the Cohen class of TFRs. It can be shown that these TFRs can be obtained by a suitable selection of the Cohen TFR kernel [1, 2, 4].

It is well-know that the TFRs are defined using corresponding variables such as window functions in spectrogram, described in time or frequency domains. A distribution described by variables defined in the frequency domain is usually referred to as a dual version to a distribution originally described by variables described in the time domain. As an example of a dual TFR, the Wigner distribution defined in the frequency domain to the Wigner distribution defined in the time domain can be given.

In this paper, a dual version of the pseudo-Wigner distribution and a dual version of the modified pseudo-Wigner distribution will be introduced. In order to follow this intention of ours, firstly, the Cohen class of TFRs will be presented in the next section. Here, it will be shown how the Wigner distribution (WD), pseudo-WD and the spectrogram can be derived from the Cohen TFR. In section 3, the modified pseudo-Wigner distribution (MPWD) originally introduced by L. Stankovic in [3] will be described. Then, by using phase-modified short time Fourier transform and a window function defined in the frequency domain, the dual modified pseudo-Wigner distribution (DMPWD) will be introduced. The properties of the MPWD and DMPWD will be illustrated on an analysis of a synthetic multicomponent signal and a measured EKG signal. The results of these analyses will show that DMPWD can provide a good time-frequency resolution and good cross-terms reduction. At the end, the conclusion remarks will be presented.

2 COHEN CLASS OF TFRS

A lot of different TFRs (*eg* WD, spectrogram, Rihazcek distribution, etc.) have been derived nowadays. L. Cohen has shown in [4] that a number of quadratic TFRs can be derived from the Cohen TFR defined by the equation

$$CD(t,\omega)=\frac{1}{4\pi^2}\iiint e^{j\theta t-j\tau\omega-j\theta u}\,\phi(\theta,\tau)x\big(u+\frac{\tau}{2}\big)x^*\big(u-\frac{\tau}{2}\big)\mathrm{d}u\mathrm{d}\tau\mathrm{d}\theta\,, \quad (1)$$

where $\phi(\theta,\tau)$ is a transformation kernel, $x(t)$ is analysed signal, $*$ represents complex conjugation operation and variables t, f, τ, θ represent time, frequency, lag (time lag) and doppler (frequency lag) respectively. The set of TFRs which can be obtained from the Cohen TFR is known as the Cohen Class of TFRs.

The WD belongs to the most important quadratic TFRs. It is a special case of the Cohen TFR for the transformation kernel $\phi(\theta,\tau)$ equal one. With regard to this fact and taking into account equation (1) it follows

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that the WD of the signal $x(t)$ can be defined in the time domain as [1, 2, 4]

$$WD(t,\omega) = \int_{-\infty}^{\infty} x\big(t+\frac{\tau}{2}\big)x^*\big(t-\frac{\tau}{2}\big)e^{-j\omega\tau}\mathrm{d}\tau\,, \quad (2)$$

where the function

$$q(t,\tau) = x\big(t+\frac{\tau}{2}\big)x^*\big(t-\frac{\tau}{2}\big) \quad (3)$$

is known as the symmetric time auto-correlation function.

The WD definition in the frequency domain can be derived expressing the signal $x(t)$ in terms of its spectrum $X(\omega)$ and substituting it into the equation (2) [1, 2, 4]

$$WD(t,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X\big(\omega+\frac{\theta}{2}\big)X^*\big(\omega-\frac{\theta}{2}\big)e^{j\theta t}\mathrm{d}\theta\,, \quad (4)$$

where the function

$$Q(\omega,\theta) = X\big(\omega+\frac{\theta}{2}\big)X^*\big(\omega-\frac{\theta}{2}\big) \quad (5)$$

is known as the symmetric frequency auto-correlation function. The WD definitions in the time (2) and frequency (4) domains are exactly the same in the case that $X(\omega)$ represents the Fourier transform of the signal $x(t)$.

The WD represents the signal in the two-dimensional domain of the time and frequency. The distribution representing the signal in the two-dimensional domain of the lag and doppler is referred to as the ambiguity function (AF) and defined as

$$AF(\theta,\tau) = \int_{-\infty}^{\infty} x\big(t+\frac{\tau}{2}\big)x^*\big(t-\frac{\tau}{2}\big)e^{-jt\theta}\mathrm{d}t\,. \quad (6)$$

It can be easily shown (*eg* [4]) that the AF and the WD are related by two-dimensional Fourier transform

$$WD(t,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} AF(\tau,\theta)\,e^{-jt\theta}e^{j\omega\tau}\mathrm{d}\theta\mathrm{d}\tau\,. \quad (7)$$

Other relations between WD and AF, the symmetric time and frequency auto-correlation function is illustrated in Fig. 1.

Inspecting equations (1) and (6) it can be shown that other quadratic TFRs belonging to the Cohen class can be also obtained from the WD by means of the formula

$$CD(t,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C(\tau,\theta)AF(\tau,\theta)\,e^{-jt\theta}e^{j\omega\tau}\mathrm{d}\theta\mathrm{d}\tau\,, \quad (8)$$

where $C(\tau,\theta)$ represents the kernel function. The equation mentioned above can be understood as two-dimensional filtering of the WD in the "frequency" domain, where the AF is the image of the WD in the "frequency" domain and the kernel function $C(\tau,\theta)$ represents the transfer function of the two-dimensional filter. With regard to this fact the WD has an excellent time-frequency resolution because $C(\tau,\theta)$ is equal to 1 and there is no low-pass filtering. The WD features also extensive cross-terms which have an oscillatory character and which are not removed or reduced by low-pass filtering. The cross-terms make the interpretation of the WD more difficult. In order to reduce these disturbing cross-terms it is possible to derive a TFR belonging to the Cohen class of TFRs by means of equation (8), where the kernel function $C(\tau,\theta)$ possesses a low-pass character. The main disadvantage of this method is the reduction of cross-terms at the expense of a good time-frequency resolution.

A spectrogram defined as a square of the short time Fourier transform (STFT) [1, 2]

$$SPEC(t,\omega) = \big|STFT(t,\omega)\big|^2 = \Big|\int_{-\infty}^{\infty} x(\tau)w^*(\tau-t)\,e^{-j\omega t}\mathrm{d}\tau\Big|^2 \quad (9)$$

can be considered as an example of the Cohen TFR, where the kernel function $C(\tau,\theta)$ has a low-pass character in both dimensions of the time-frequency plane. More exactly, the kernel function $C(\tau,\theta)$ is equal to the AF of the window function $w(t)$. With regard to these facts the spectrogram has a poor time-frequency resolution but good reduction of the disturbing cross-terms.

Pseudo-WD (PWD) described *eg* in [1] is another Cohen class TFR given by

$$PWD(t,\omega) = \int_{-\infty}^{\infty} x\big(t+\frac{\tau}{2}\big)x^*\big(t-\frac{\tau}{2}\big)w\big(\frac{\tau}{2}\big)w^*\big(\frac{\tau}{2}\big)e^{-j\omega\tau}\mathrm{d}\tau\,. \quad (10)$$

By means of equation (1) it can be shown that in the case of PWD the kernel function $C(\tau,\theta)$ is equal to the square of the window function $w(t/2)$. Then, the PWD can be understood as a WD filtered only in the direction of the frequency axis and so possessing the same resolution as the WD in the direction of the time axis and worse resolution than the WD in the direction of the frequency axis. The main advantage of the PWD over the WD is that the PWD reduces cross-terms in the direction of the frequency axis.

In the next section it will be shown that there exists a distribution which combines the properties of the PWD and spectrogram. In certain cases this so-called modified PWD can provide a good time-frequency resolution and good reduction of cross-terms, both at once.

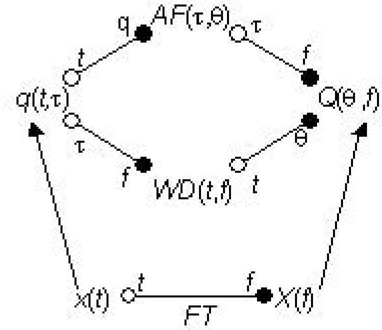

Fig 1 Scheme of the relations between the WD, the AF, the symmetric time and frequency auto-correlation function

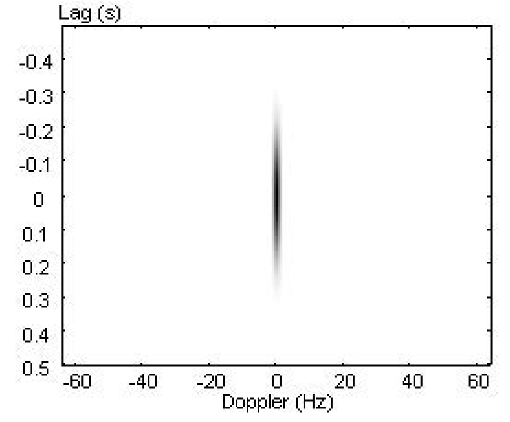

Fig 2 Kernel function of spectrogram using Blackman window function

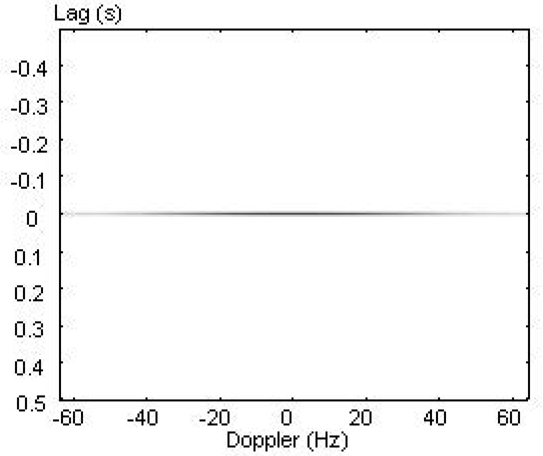

Fig 3 Kernel function of PWD using Blackman window function

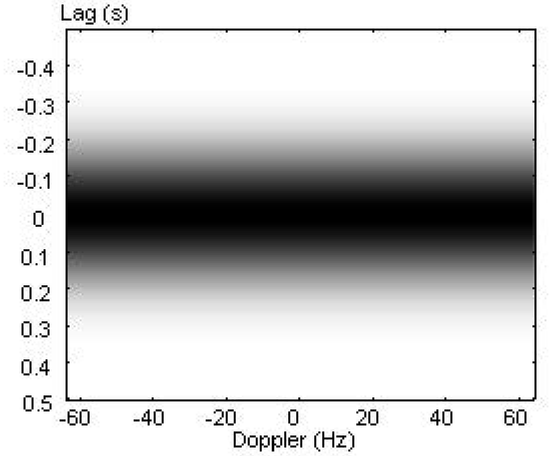

Fig 4 Kernel function of spectrogram defined in frequency domain using Blackman window function

3 MODIFIED PSEUDO–WIGNER DISTRIBUTION

Let us consider the signal $x(t)$. The MPWD of the signal $x(t)$ is defined by the next formula [3]

$$MPWD(t,\omega) = \frac{1}{\pi}\int_{\theta} P(2\theta) STFT(t,\omega+\theta) STFT^{*}(t,\omega-\theta)\mathrm{d}\theta\,, \quad (11)$$

where $P(\theta)$ is the window function and the STFT is defined in the time domain using the window function $w(t)$ as

$$STFT(t,\omega) = \int_{\tau} x(\tau) w^{*}(\tau - t)\, e^{-j\tau\omega}\mathrm{d}\tau\,. \quad (12)$$

There are two special cases gained by an appropriate choice of the window function $P(\theta)$. The first one is setting up the window function $P(\theta)$ equal to $2\pi\delta(\theta)$, where $\delta(\theta)$ is a Dirac pulse. Then the spectrogram is obtained from the MPWD. An example of the kernel function $C(\tau,\theta)$ used to obtain the spectrogram from the WD according to equation (8) is illustrated in Fig. 2. The Blackman window function $w(t)$ was applied in this example. It is easy to see the low-pass character of the kernel function $C(\tau,\theta)$, which causes extensive reduction of cross-terms in both directions (time and frequency) of the spectrogram.

Setting up window the function $P(\theta)$ equal to 1 for all θ represents the second important case of the window function $P(\theta)$ selections. Then the PWD is obtained from the MPWD. An example of the kernel function $C(\tau,\theta))$ used to obtain the PWD from the WD is illustrated in the Fig. 3. The Blackman window function $w(t)$ was used in this example. It is easy to see that the low pass character of the kernel function $C(\tau,\theta))$ is maintained only in the lag direction, which causes reduction of the cross-terms only in the frequency direction of the PWD and preserving good resolution in the time direction of the PWD.

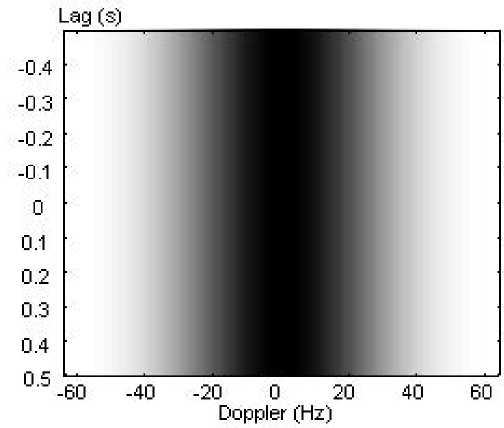

Fig 5 Kernel function of DPWD using Blackman window function

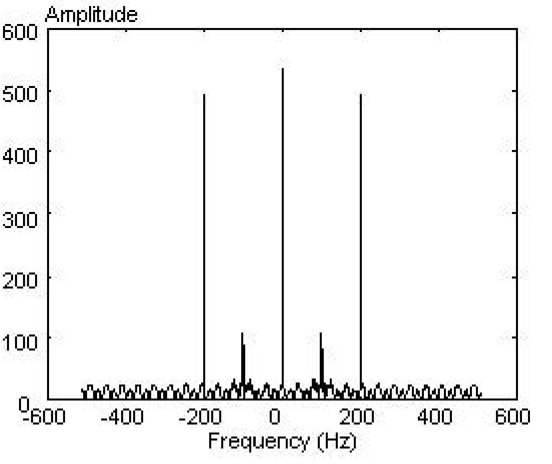

Fig 6 Synthetic signal $x(t)$

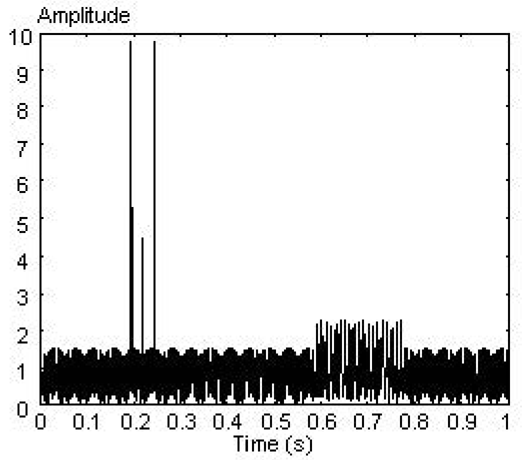

Fig 7 Spectrum of signal $x(t)$

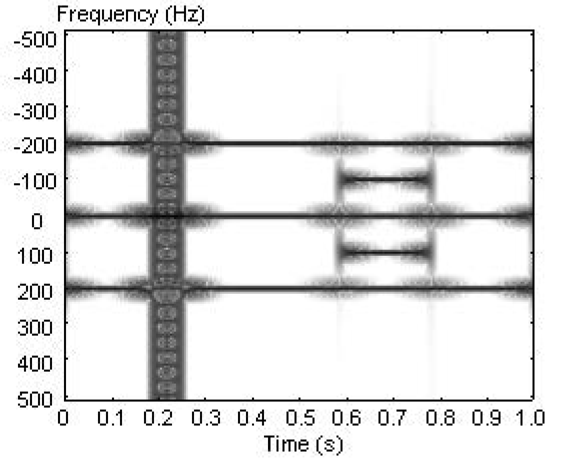

Fig 8 MPWD (longer window function) of signal $x(t)$

The MPWD can be considered as a modification of the PWD but is qualitatively and numerically quite different from the PWD. The above mentioned extreme cases of choosing window function $P(\theta)$ illustrate the ability of the MPWD to be a transform between the PWD and the spectrogram and to combine their good properties by choosing an appropriate window function $P(\theta)$. In the case of $P(\theta)$ selection it holds that the wider the window function $P(\theta)$, the better the time-frequency resolution and the worse the cross-terms reduction.

In certain cases cross-terms can be completely removed and the time-frequency resolution of the PWD will be also preserved. Let us consider, *eg*, a signal created by a sum of frequency modulated signals. The cross-terms arising from each pair of signals will appear only if the distance between instantaneous frequencies of the frequency modulated signals is less than the length of the window function $P(2\theta)$ extended by the auto-term width [3].

In the next section a TFR dual to the PWD will be introduced. Then, it will be shown that there is a possibility to derive a dual form of the MPWD, which will combine the properties of the spectrogram and of the dual version of the PWD in a similar way as the MPWD combines the properties of the spectrogram and the PWD.

4 DUAL VERSION OF MODIFIED PSEUDO–WIGNER DISTRIBUTION

In the previous section it was shown that there is a TFR combining the properties of the spectrogram and of the PWD. The same idea can be followed deriving a dual TFR combining the properties of the spectrogram and a dual distribution to the PWD. In the next paragraph a description of the spectrogram defined in the frequency domain will be presented. Then, TFR dual to the PWD will be introduced. This section will be concluded by the definition of the dual MPWD and the description of its basic properties.

Let us consider signal $x(t)$ and its spectrum $X(\omega)$. The STFT of the signal $x(t)$ defined in the frequency domain is [5]

$$STFT(t,\omega) = \frac{e^{-j\omega t}}{2\pi} \int_{\theta} X(\theta)\, W^*(\theta-\omega)\, e^{j\theta t} \mathrm{d}\theta\,, \quad (13)$$

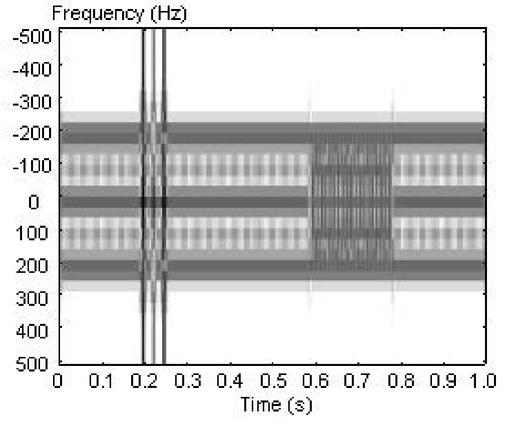

Fig 9 MPWD (shorter window function) of signal $x(t)$

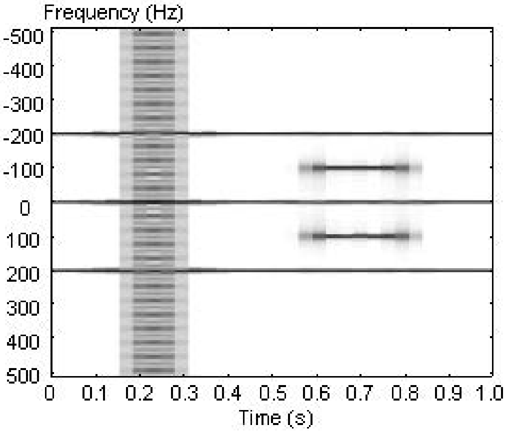

Fig 10 DMPWD (longer window function) of signal $x(t)$

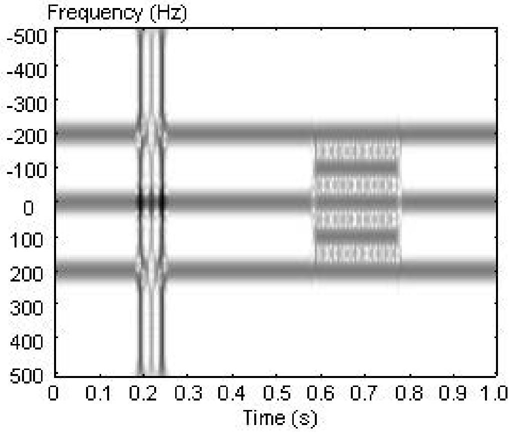

Fig 11 DMPWD (shorter window function) of signal $x(t)$

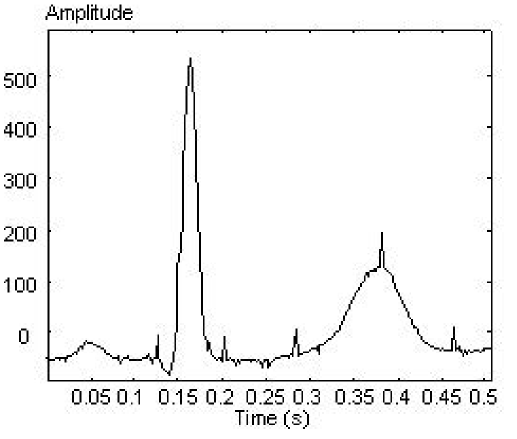

Fig 12 Measured EKG signal

where $W(\omega)$ is the window function defined in the frequency domain.

The definition of STFT in the frequency domain by equation (13) is equal to the definition of STFT in the time domain by equation (12). The properties of the spectrogram defined in the frequency domain (13) are the same as the properties of the spectrogram defined in the time domain by (9). It means that the spectrogram has a bad time-frequency resolution and good cross-terms reduction. As it was mentioned before, the spectrogram defined in the frequency domain can be understood as two-dimensional filtering of the WD by means of equation (8), where the kernel function $C(\tau,\theta)$ is equal to the AF of the window function defined in the frequency domain $W(\omega)$. An example of the spectrogram kernel function $C(\tau,\theta)$ is illustrated in Fig. 4. The Blackman window function $W(\omega)$ was used in this example. It is easy to see the low pass character of the kernel function $C(\tau,\theta)$, which causes extensive reduction of cross-terms in both directions (time and frequency) of the spectrogram defined in the frequency domain.

The PWD is defined by equation (10). It was mentioned before that the PWD can be understood as a WD filtered only in the direction of the frequency axis. With regard to this fact it possesses the same resolution as the WD in the direction of the time axis and worse resolution than the WD in the direction of the frequency axis.

Following a similar approach another TFR can be derived from the WD by means of low-pass filtering in the direction of the time axis. This TFR will be referred to as dual PWD and defined as

$$DPWD(t,f) =$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} W\big(\frac{\theta}{2}\big)W^*\big(\frac{\theta}{2}\big)X\big(\omega+\frac{\theta}{2}\big)X^*\big(\omega-\frac{\theta}{2}\big)e^{j\theta\, t}\mathrm{d}\theta\,, \quad (14)$$

where $W(\omega)$ is the window function defined in the frequency domain. The DPWD possesses dual properties with regard to the PWD. It means it has the same resolution as the WD in the direction of the frequency axis and worse resolution than the WD in the direction of the time axis. The main advantage of the DPWD over the WD is that the DPWD reduces cross-terms in the direction of the time axis. The DPWD can be understood

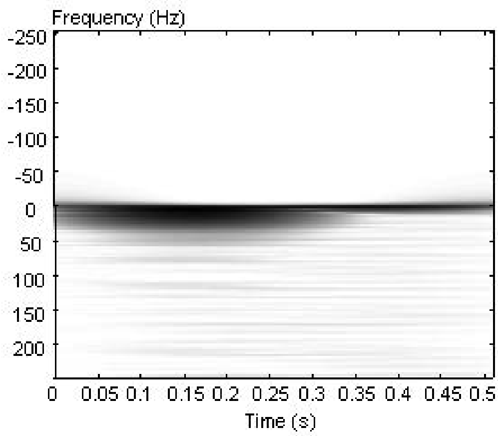

Fig 13 Spectrum of measured EKG signal

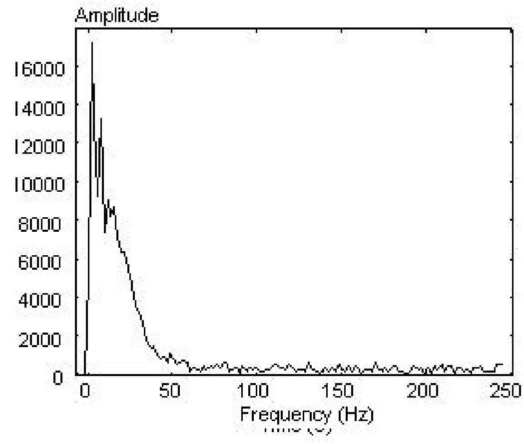

Fig 14 Spectrogram of signal $x(t)$ defined in time domain

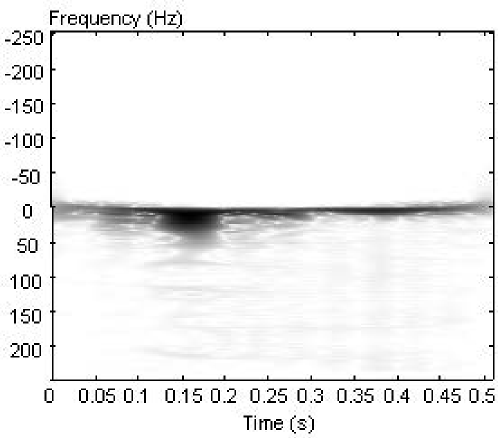

Fig 15 Spectrogram of signal $x(t)$ defined in frequency domain

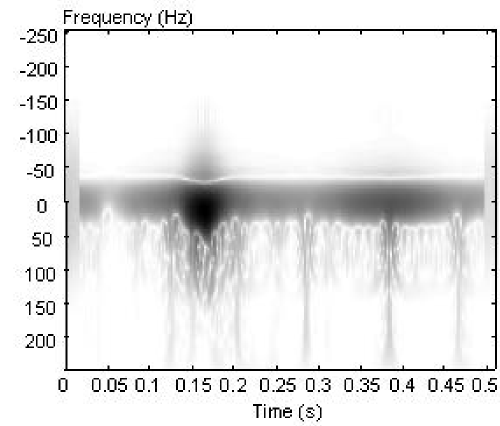

Fig 16 MPWD of signal $x(t)$

as two-dimensional filtering of the WD, where the kernel function $C(\tau,\theta)$ is equal to the square of the window function defined in the frequency domain $W(\omega)$. An example of the DPWD kernel function $C(\tau,\omega)$ is illustrated in the Fig. 5. The Blackman window function defined in the frequency domain $W(\omega)$ was used in this example. It is easy to see the low-pass character of the kernel function $C(\tau,\theta)$, which is only in the doppler direction, which causes reduction of the cross-terms only in the time direction of the DPWD and preserving good resolution in the frequency direction of the DPWD. The duality of the PWD and DPWD can be also presented comparing kernel functions $C(\tau,\theta)$ of the PWD and the DPWD displayed in Figs. 2 and 3, respectively.

The DMPWD can be derived according to the derivation of the MPWD. The DMPWD of the signal $x(t)$ is defined by the formula

$$DMPWD(t,\omega) = \frac{1}{\pi}\int_{\tau} P(2\tau) PMSTFT(t+\tau,\omega) PMSTFT^*(t-\tau,\omega)\mathrm{d}\tau\,, \tag{15}$$

where $P(\tau)$ is the window function and PMSTFT is the phase-modified STFT defined in frequency domain using the window function $W(\omega)$ as

$$PMSTFT(t,\omega) = \int_{\theta} X(\theta)\, W^*(\theta-\omega)\, e^{j\theta t}\mathrm{d}\theta\,. \tag{16}$$

There are two special cases gained by an appropriate choice of the window function $P(\tau)$. The first one is setting up the window function $P(\tau)$ equal $2\pi\delta(\tau)$. Then the spectrogram defined in the frequency domain is obtained from the DMPWD. The second case is setting up the window function $P(\tau)$ equal 1 for all τ. Then the DPWD defined by equation (14) is obtained from the DMPWD.

These two special cases illustrate the ability of the DMPWD to be a transform between the DPWD and the spectrogram and to combine their good properties choosing an appropriate window function $P(\tau)$. The wider the window function $P(\tau)$, the better the time-frequency resolution and the worse the cross-terms reduction.

In certain cases cross-terms can be completely removed and the time-frequency resolution of the DPWD will be preserved. Let us consider *eg* a signal created by a sum of short-time signals. Then cross-terms will appear only if the distance between group delays of the short-time signals is less than the length of the window function $P(2\tau)$ extended by the auto-term width.

Fig 17 DMPWD of signal $x(t)$

5 EXPERIMENTAL RESULTS

In order to illustrate the basic properties of the DMPWD and to compare them with those of the MPWD some experimental results will be presented in this section. First, a synthetic signal will be analyzed. Then the performance of the DMPWD and the MPWD will be illustrated based on the measured EKG signal.

5.1 Synthetic Signal Analysis

Let us consider a multi-component signal

$$\begin{aligned} x(t) = 0.5 + 9.8\delta(t-0.194) + 9.8\delta(t-0.244) \\ + 4.5\delta(t-0.219) + 0.45\sin(400\pi t) \\ + 0.1\,\mathrm{rect}\Big(\frac{t-0.68}{0.2}\Big)\sin(200\pi t)\,, \end{aligned} \tag{17}$$

where $\mathrm{rect}(t/T)$ represents a rectangular window function of time length T. From the signal definition it is obvious that it contains DC component, three shifted Dirac impulses, one sinusoidal signal lasting for the whole time and one sinusoidal signal lasting only in a certain time window. The signal $x(t)$ and its spectrum are illustrated in Figs. 6 and 7, respectively. The goal of this experiment is to distinguish all Dirac pulses and sinusoidal components in the time-frequency plain and to suppress cross-terms as much as possible.

First, we will try to analyze the signal $x(t)$ by means of the MPWD. The result is presented in Fig. 8. The Harris window function $w(t)$ of time length 250 ms was used for the computation of STFT by (12), which is a part of the MPWD definition by (11). The Harris window function $P(2\theta)$ was used for the computation of the MPWD and had a frequency length of 88 Hz. The frequency length of the window function $P(2\theta)$ was chosen in such a way as not to give rise to cross-terms between the pairs of sinusoidal components, which are distinguished 100 Hz from each other. From the presented result (Fig. 8) it is clear that this TFR is not able to distinguish three Dirac pulses. The bad time resolution of the MPWD is given by the relatively long window function $w(t)$, which determines the frequency resolution of the MPWD. In the case of a shorter window function $w(t)$ (Fig. 9), *eg* 50 ms, which assures good reduction of the cross-terms between Dirac pulses, the frequency resolution is so poor that it is not possible to distinguish sinusoidal signal components.

Improved results can be obtained by means of the DMPWD. The result is presented in Fig. 10. The Harris window function $W(\omega)$ of the frequency length 256 Hz was used for the computation of the PMSTFT by (16), which is a part of the DMPWD definition by (15). The Harris window function $P(2\tau)$ was used for the computation of the DMPWD and had a time length of 32 ms. The time length of the window function $P(2\tau)$ was chosen in such a way as not to give rise to extensive cross-terms between the pairs of sinusoidal components, which are distinguished 25 ms from each other. From the presented result (Fig. 10) it is clear that this TFR is able to distinguish all signal components in the time-frequency plain although the frequency resolution is not excellent and the cross-terms are present between sinusoidal components. Further improvement is possible by shortening the window function $W(\omega)$ but only at the expense of a worse frequency resolution (Fig. 11).

5.2 EKG Signal Analysis

The second example presents MPWD and DMPWD analyzes of the measured EKG signal. The EKG signal was acquired using 500 Hz sampling frequency. The signal used for the analysis is displayed in Fig. 12 and its spectrum is shown in Fig. 13. From the time waveform of the EKG signal (Fig. 12) it is apparent that the signal contains also impulse noise.

In Fig. 14 it is possible to see the spectrogram of the EKG signal defined in the time domain by equation (9). Here, the Blackman window function of 0.5 s time length was used. This spectrogram features a very bad time resolution whereas frequency resolution is very good. This is caused by a relatively long window function defined in the time domain. Because of these facts impulse noise present in the EKG signal is entirely undistinguishable.

The spectrogram of the EKG signal defined in the frequency domain is displayed in the Fig. 14. The Blackman window function of 500 Hz frequency length was used. Comparing Fig. 14 and Fig. 15 it is easy to see that the spectrogram defined in the frequency domain possesses very good time resolution whereas frequency resolution is very bad. This is caused by a relatively long window function defined in the frequency domain. The presence of impulse noise is visible, which is assured by very good time resolution.

In order to obtain improved time resolution, comparable with that of the spectrogram defined in the time domain, it is possible to use the MPWD. The MPWD of the EKG signal is illustrated in Fig. 16. The difference between the time resolution of the MPWD (Fig. 16) and the spectrogram defined in the time domain (Fig. 14) is apparent but in spite of this fact impulse noise present in the analyzed signal is almost invisible. The time resolution can be further improved using a longer window function $P(\tau)$, but only at the expense of increased cross-terms.

In the case of the DMPWD it is possible to receive an improved frequency resolution, comparable with that of the spectrogram defined in the frequency domain. The DMPWD of the EKG signal is illustrated in Fig. 17. The difference between the frequency resolution of DMPWD (Fig. 17) and the spectrogram defined in the frequency domain (Fig. 15) is apparent. The frequency resolution can be further improved using a longer window function $P(\tau)$ but only at the expense of increased cross-terms.

6 CONCLUSIONS

Some well-known TFRs of Cohen class such as the WD, PWD the spectrogram and their dual versions were shortly described in this paper. Then, the DMPWD was defined according to the definition of the MPWD. The basic properties of MPWD and DMPWD were illustrated based on analyses of a multicomponent synthetic signal and EKG signal disturbed by an impulse noise. The results of the experiments have shown that the DMPWD combines good properties of the WD and the spectrogram. It means it is possible to get a good time-frequency resolution and good cross-terms reduction. The DMPWD definition is very similar to that of the DPWD but the interpretation is qualitatively and numerically quite different from the interpretation of the DPWD. The DMPWD can be considered as the first iteration step in the recursive definition of dual L-Wigner distribution [6]. For the future work it could be interesting to investigate the existence of a formula connecting the MPWD and its dual version defined in this paper.

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