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NEW METHOD OF SUMMATION FOR MEASUREMENT ERRORS BASED ON PIECE–WISE LINEAR APPROXIMATION OF PROBABILITY DISTRIBUTION

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The article describes a new method for summation of measurement errors. It is based on the piece-wise linear approximation of probability distribution (frequency function). The calculation results are compared with the calculation according to the well known technique and example. The offered method allows to increase the accuracy of determination of the numerical characteristics for resulting error up to 10–11 % and to visualise the resulting distribution low. A software package for IBM PC compatible computers is realised.

Keywords: measurement, probability distribution (frequency function), characteristic function, measuring error, virtual instrument.

1 INTRODUCTION

The problem of summation for errors is one of the main issues of metrology. It arises at measuring instruments creation as well as at evaluation of resulting errors of measurement. The general approach to the determination of probability distribution for resulting error of measurement is reduced to a calculation of summary distribution as a sum of independent random values and application of the probability theory methods. Such a method is most accurate but rather labourious because all characteristics and errors must be taken into account. Such calculation claims additional initial information and must make heuristic decisions during the calculation. Sometimes it causes serious difficulties [1]. So, for example, the change from standard deviation to the entropy or the confidence values is the most difficult operation at the last stage of summation for errors. This operation assumes the frequency function determination as the entropy factor or quantile multiplier of the resulting error is dependent on the probability distribution. However, classical techniques do not permit the exact determination of resulting distribution low. Another so called η-method of summation of an error's components is described in [2]. It is based on the excess definition. However, the excess does not determine unequivocally the type of probability distribution.

The necessity for a more accurate determination of the metrological characteristics and of the resulting distribution law give rise to intelligent precision microprocessor based measuring instruments with complex measuring algorithms, processing and representation of the measured information. The same problem is urgent for virtual measuring instruments. Even if all produced measurement modules and blocks in the virtual measuring instrument or system have certified metrological characteristics (specifications), there is always the problem as how to characterise the metrological properties of the arranged system as a whole, especially if that has been tailored for the user [3]. The measurement result is complete only if accompanied by a statement of its uncertainty. Unfortunately, none of the software products available on the market contains built-in tools for evaluation of error characteristics for the received measurement results.

A uniform technique of automated construction of the frequency function and determination of performance for resulting error of n components which would easily lead to algorithmization and could be realised as computer software does not exist. At the same time the class of probability distributions for errors of measurements is determined in [4], and all of them, except the Rayleigh distribution [5], are met in practice. Hence, the issue of accurate summation of error components for measurement is a topical and timely problem.

2 METHOD OF SUMMATION FOR MEASUREMENT ERRORS

The proposed method is based on the Liapunov characteristic functions (LCF) [5–8]. In comparison with the early offered method of frequency function construction [8] in the new method the piece-wise linear approximation of probability distribution $W(\delta)$ with an automatic change of the discretization step dependent on the given accuracy is used instead of the step approximation. It has allowed to receive an optimum speed/accuracy ratio at the computer realisation of algorithm for this method.

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Such a probability distribution as rectangular, triangular, trapezoidal and some others are approximated without error. The distribution which can not be precisely approximated by straight-line segments is approximated with a beforehand given accuracy. At piece-wise linear approximation, this function can be presented as

$$W(\delta) = \sum_{i=1}^{n} \Big(1\,(\delta_i) - 1\,(\delta_{i+1})\Big)\Big(A_i\delta + W_i\delta_{i+1} - W_{i+1}\delta_i\Big)\,, \tag{1}$$

where A_i is the coefficient which is calculated according to the following equation:

$$A_i = \frac{W_{i+1} - W_i}{\delta_{i+1} - \delta_i}\,; \tag{2}$$

W_i, δ_i are the coordinates of i-th point, where the distribution $W(\delta)$ is approximated; $1\,[\cdot]$ is the unit Heavyside function; n is the number of points which approximate the distribution $W(\delta)$ of the random error δ.

Having calculated the LCF $\theta(j\omega)$ of the distribution $W(\delta)$, we shall receive

$$\theta(j\omega) = M\big(e^{j\omega\delta}\big) = \int_{-\infty}^{+\infty} e^{j\omega\delta} W(\delta)\mathrm{d}\delta = \theta_1(j\omega) + j\theta_2(j\omega)\,, \tag{3}$$

where $\theta_1(j\omega)$, $\theta_2(j\omega)$ are the real and imaginary parts of the required LCF accordingly. We shall determine the θ_1 and θ_2 separately:

$$\begin{aligned}\theta_1 = \int_{-\infty}^{+\infty} W(\delta)\cos\omega\delta\mathrm{d}\delta &= \sum_{i=1}^{n} A_i\Big(\omega^{-2}\cos\omega\delta_{i+1} \\ &+ \delta_{i+1}\omega^{-1}\sin\omega\delta_{i+1} - \omega^{-2}\cos\omega\delta_i - \omega^{-1}\delta_i\sin\omega\delta_i\Big) \\ &+ \Big(W_i\delta_{i+1} - W_{i+1}\delta_i\Big)\,\omega^{-1}\Big(\sin\omega\delta_{i+1} - \sin\omega\delta_i\Big)\,.\end{aligned} \tag{4}$$

After elementary rearrangements of the composed sum and assuming that there are some points with $W = 0$ at the beginning and at the end of frequency function curve, we shall receive:

$$\theta_1(\omega) = -\frac{2}{\omega^2}\sum_{i=1}^{n} A_i \sin\frac{\omega(\delta_i + \delta_{i+1})}{2} + \frac{1}{\omega}\sum_{i=1}^{k} B_i\sin\omega\delta_i\,, \tag{5}$$

where k is the number of the 1$^{\mathrm{st}}$ sort breaks in the frequency function curve $W(\delta)$; B_i is the coefficient, which can be calculated according to the formula

$$B_i = W_{i,1} - W_{i,2}\,, \tag{6}$$

where $W_{i,1}$, $W_{i,2}$ are the left-hand and right-hand borders in the 1$^{\mathrm{st}}$ sort break point δ_i accordingly:

$$W_{i,1} = \lim_{\delta\to\delta_{i+0}} W(\delta) \tag{7}$$

$$W_{i,2} = \lim_{\delta\to\delta_{i-0}} W(\delta) \tag{8}$$

Let us determine the imaginary part of the LCF:

$$\theta_2 = \int_{-\infty}^{+\infty} W(\delta)\sin\omega\delta\mathrm{d}\delta\,. \tag{9}$$

After similar transformations we shall receive:

$$\theta_2(\omega) = \frac{1}{\omega^2}\sum_{i=1}^{n} A_i \sin\frac{\omega(\delta_i - \delta_{i+1})}{2}\cos\frac{\omega(\delta_i - \delta_{i+1})}{2} + \frac{1}{\omega}\sum_{i=1}^{k} B_i\cos\omega\delta_i\,. \tag{10}$$

The real part $\theta_1(j\omega)$ of the characteristic function $\theta(j\omega)$ is even, and the imaginary part $\theta_2(j\omega)$ is an odd function of the frequency ω. Therefore, it is enough to calculate $\theta(j\omega)$ in the positive area ($\omega > 0$), and in the negative area it is possible to calculate the LCF according to the following equation:

$$\theta(j(-\omega) = \theta_1(\omega) - j\theta_2(\omega) \tag{11}$$

For computer representation of the LCF it is also necessary to approximate the last one by a piece-wise line and to store the initial values of the real and imaginary parts. The approximation is executed automatically with a beforehand given accuracy ϵ. Thus the spectrum is limited by the frequency $\omega_{\max}$, at which

$$\begin{aligned}|\theta(j\omega_{\max}| &\le \theta_{\max}\delta_0 \\ |\theta'(j\omega_{\max}| &\le \theta'_{\max}\delta_0\end{aligned} \tag{12}$$

where $\omega_{\max}$ is the maximum value of frequency ω from the range $[0, \omega_{\max}]$ within the limits of which the value of LCF is calculated; δ_0 is the calculation (approximation) accuracy of $\theta(j\omega)$; $|\theta_{max}|$ is the maximum value of the LCF module; $|\theta'_{\max}|$ is the maximum value of the derivative module of the LCF.

If the LCFs are known (the LCF of error's components are in PC's memory point-by-point), it is necessary to multiply them together and to approximate the resulting LCF by a piece-wise line with accuracy δ_0. Hence, it is possible to present the summary LCF $\theta_\Sigma(j\omega)$ as

$$\theta_\sigma(j\omega) = \sum_{i=1}^{n}\Big(1(\omega_i) - 1(\omega_{i+1})\Big)\Big(\frac{\theta_{i+1} - \theta_i}{\omega_{i+1} - \omega_i}\omega + \theta_i\omega_{i+1} - \theta_{i+1}\omega_i\Big)\,. \tag{13}$$

Having executed the return transformation taking into account that the real part of any LCF is an even function of ω, and the imaginary part is an odd function of ω, we shall receive the probability distribution of the resulting random error $W_\Sigma(\delta)$:

$$W_\Sigma(\delta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\theta_1(\omega)\cos\omega\delta\mathrm{d}\omega + \frac{1}{2\pi}\int_{-\infty}^{+\infty}\theta_2(\omega)\cos\omega\delta\mathrm{d}\omega + \frac{j}{2\pi}\int_{-\infty}^{+\infty}\theta_1(\omega)\sin\omega\delta\mathrm{d}\omega + \frac{j}{2\pi}\int_{-\infty}^{+\infty}\theta_2(\omega)\sin\omega\delta\mathrm{d}\omega \quad (14)$$

Taking into account that

$$\int_{-\infty}^{+\infty} A(\omega)\mathrm{d}\omega = 0\,, \quad (15)$$

where $A(\omega)$ is an odd function of the ω

$$\int_{-\infty}^{+\infty} B(\omega)\mathrm{d}\omega = 2\int_{0}^{+\infty} B(\omega)\mathrm{d}\omega$$

and $B(\omega)$ is an even function of the ω, we shall receive

$$W_\sigma(\delta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\theta_1(\omega)\cos\delta\omega\mathrm{d}\omega - \frac{1}{2\pi}\int_{-\infty}^{+\infty}\theta_2(\omega)\sin\omega\delta\mathrm{d}\omega\,. \quad (17)$$

The received equation has a similar structure with the one considered above. Therefore we shall bring the final result of transformation:

$$W_\Sigma(\delta) = \frac{1}{\pi\delta}\sum_{i=1}^{n} C_i\cos\omega_i\delta D_i\sin\omega_i\delta\,, \quad (18)$$

where n is the number of points of $W_\Sigma(\delta)$ approximation; of C_i and D_i are coefficients which are calculated as follows:

$$C_i = \frac{P_i - P_{i-1}}{\omega_i - \omega_{i-1}} - \frac{P_{i+1} - P_i}{\omega_{i+1} - \omega_i} \quad (19)$$

$$D_i = \frac{R_i - R_{i-1}}{\omega_i - \omega_{i-1}} - \frac{R_{i+1} - R_i}{\omega_{i+1} - \omega_i} \quad (20)$$

where P_i, R_i are the real and imaginary parts of the summary LCF of $\theta_2(j\omega)$ in point ω_i. For determination of the numerical characteristics of the distribution on its LCF we shall use the formulas from [5], in which we shall replace the derivatives by the final differences. Then the logarithmic LCF and its four derivatives will have the following form:

$$\theta_{l,i} = \ln(\theta_i)\,, \qquad i = 1,\ldots,5 \quad (21)$$

$$\theta'_{l,i} = \frac{\theta_{l,i+1} - \theta_{l,i}}{\omega_{i+1} - \omega_i}\,, \quad i = 1,\ldots,4 \quad (22)$$

$$\theta''_{l,i} = \frac{\theta'_{l,i+1} - \theta'_{l,i}}{\omega_{i+1} - \omega_i}\,, \quad i = 1,\ldots,3 \quad (23)$$

$$\theta'''_{l,i} = \frac{\theta''_{l,i+1} - \theta''_{l,i}}{\omega_{i+1} - \omega_i}\,, \quad i = 1,\ldots,2 \quad (24)$$

$$\theta^{IV}_{l,i} = \frac{\theta'''_{l,i+1} - \theta'''_{l,i}}{\omega_{i+1} - \omega_i}\,. \quad (25)$$

The equations for numerical characteristics determination of error with allowance for equations (20–24) will have the following form:

$$M(\delta) = i^{-1}\theta'_{l,i}\,, \quad (26)$$

$$D(\delta) = -\theta''_{l,i}\,, \quad (27)$$

$$\sigma = \sqrt{D}\,, \quad (28)$$

$$K_{as} = \theta'''_{l,i}\left[\theta''_{l,i}\right]^{-3/2}\,, \quad (29)$$

$$K_{exc} = \theta^{IV}_{l,i}\left[\theta''_{l,i}\right]^{-2}\,. \quad (30)$$

For realisation of an automatic change for discretization step at calculation of any function (the frequency function or LCF), its value is calculated in three points. The arguments of these points will be distinguished one from the other by the discretization step h. The absolute accuracy of calculations is determined as the module of the difference between the value of function in the central point and in the middle of a straight-line segment which connects its two extreme points of approximation. Then the absolute accuracy of calculations is determined by the formula

$$\Delta E = \left|\frac{(2F_2 - F_1 - F_3)}{h}\right|\,, \quad (31)$$

where F_1, F_2, F_3 are the values of the function in the 1^{st}, 2^{nd} and 3^{rd} points of approximation; h is the discretization step.

The discretization step can be decreased or increased depending on the required calculation accuracy.

3 SOFTWARE PACKAGE

The developed method can be formalised rather easily. On the basis of the offered algorithm a software package for automatic construction of the probability distribution and determination of the performances for the resulting error of n components was developed The friendly interface of the software package allows to enter the accuracy ϵ in %, to choose the distribution low for the components and to input their characteristics in a dialogue mode. The real calculation accuracy will be $\sqrt{n}$ times higher than the given, where n is the number of components of the

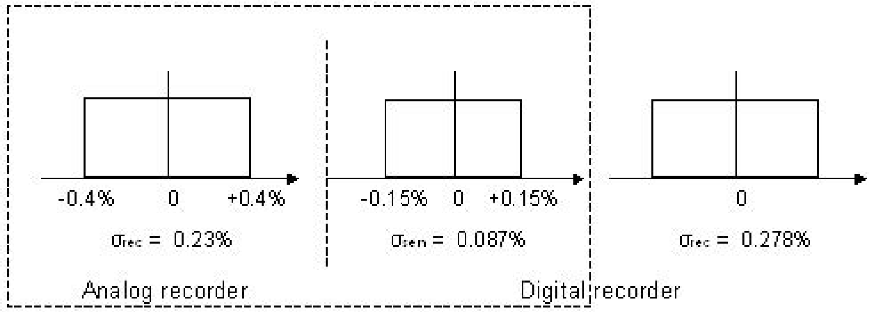

Fig 1

Table 1.

Error, %	Distribution Law for the Component	Resulting Error, % (at $P = 0.98$)	
		Method [1]	Proposed Method ($E = 0.01\,\%$)
$\sigma_{rec} = 0.23$ $\sigma_{sen} = 0.087$	uniform uniform	0.25	0.28
Summary Frequency Function		Not being defined	Trapezium

resulting error. In addition to the Gaussian, triangular, trapezoidal, uniform, anti-modal 1 and anti-modal 2 frequency functions [3], the software package permits to use also the Erlang and linear-broken distribution. The last one takes a special place, as it represents by itself any probability distribution which can be represented by a final sequence of points, connected by straight-line segments. In this case all points of bends are entered consistently, from $\delta_{\min}$ up to $\delta_{\max}$.

The software package gives to the operator the next output information:

- the numerical characteristics of errors for the entered component as well as the resulting error (mean, dispersion, standard deviation, coefficients of asymmetry and excess, confidence interval at any probability, up to $P_{\max} = 0.9999$);
- the frequency function curve;
- amplitude-frequency and phase-frequency characteristics for the LCF.

Numerical characteristics and the received composition curve of probability distribution give more complete and authentic characteristics about metrological performances of measuring instruments and systems. Hence, on the basis of the received information it is possible to establish individual prices for each measuring instrument of the same type, based on its accuracy.

The developed software package consists of three main parts:

- Program for objects generation of the friendly interface;
- Main program, realising the interface functions, processing and check of the entered data;
- Subroutines library.

The library, in its turn, contains the developed subroutines for LCF determination, the LCF return transformation, multiplication of two LCF, graphic construction of the distribution low and a subroutine for determination of the numerical characteristics for random errors on its probability distribution.

4 EXAMPLE AND RESULTS

We shall consider an example of calculation of an error for the measuring channel and compare the results received by the usage of the hand-operated technique, described in [1], and the proposed automated method of summation for errors. The calculations were done for the initial data from the well known (classical) example [1].

Let it be required to calculate a resulting error for the measuring channel consisting of serial units (sensor, amplifier and recorder). An error of zero of the measuring channel with an analog recorder includes four components: the recorder error σ_{rec}, the intrinsic sensor error σ_{sen}, the temperature sensor error σ_t and summary error of two rigidly correlated components: the amplifier zero displacement error at temperature fluctuation and temperature error of the recorder $\sigma_{t(ampl+rec)}$. It is possible to neglect the last two components (for consistency of comparison of the two techniques), because one of them is 9 times and the other in 13.5 times less, than the recorder error. The probability distributions of the components are uniform (Fig. 1).

The comparative results of calculation are adduced in Tab 1.

Table 2.

Error, %	Distribution Law of the Component	Resulting Error, % (at $P = 0.98$)	
		Method [1]	Proposed Method ($E = 0.01\,\%$)
$\sigma_{rec} = 0.278$ $\sigma_{sen} = 0.087$	uniform uniform	0.291	0.327
Summary Frequency Function		Not being defined	Trapezium*

**Numerical characteristics of the resulting probability distribution, received at usage of the developed software package are: $M = 0$; $D = 2.83 \times 10^{-2}$; $SDOM = 0.168$; $K_{as} = 0$; $K_{exc} = -1$; confidence interval $-3.211 \times 10^{-1} \leq X \leq 3.267 \times 10^{-1}$ at $P = 0.98$.*

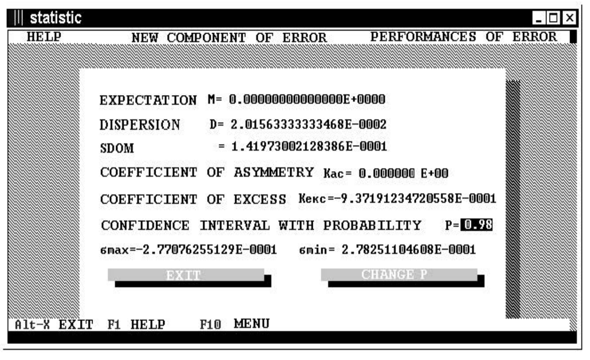

Fig 2

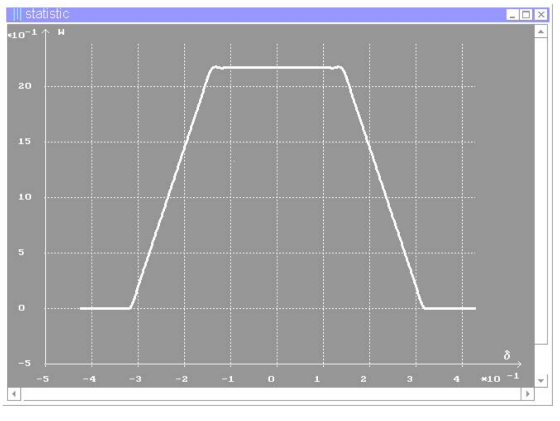

Fig 3

Numerical characteristics of the resulting distribution law, received at the usage of the developed software package, are shown in Fig. 2. The resulting distribution of the two components σ_{reg} and σ_{sen} is shown in Fig. 3.

If the two omitted component were not neglected, the resulting error would be equal to 0.26 %.

The channel error with the digital recorder includes the same error of the sensor and an error of the digital voltmeter (Fig. 1). It is also possible to neglect the temperature sensor error and the amplifier zero displacement error because of temperature fluctuation. It is fair according to the rule to neglect small components at summation of errors. Comparative results of calculation are shown in Tab 2.

The timely performances of program execution of the offered algorithm on IBM PC compatible computer with processor Pentium Pro and clock frequency 200 MHz are adduced in Tab 3. The minimum required hardware: 486DX4 processor with clock frequency 80 MHz.

Table 3.

Operation	Time of Execution sec
Data enter and approximation of frequency function for the first component (distribution law is uniform)	1
Data enter and approximation of frequency function for the second component (distribution law is uniform)	$4 + 11$
Performances calculation for the resulting error	5
Construction of frequency function curve for the resulting error	$1 + 4$

5 CONCLUSIONS

The offered method of summation of errors allows to increase the accuracy of determination of the numerical characteristics for the resulting error up to 10–11 %, completely to automate the calculation, visualisation and registration of the distribution law and, consequently, to increase the labour productivity at individual certification of each measuring instrument.

The received results can be fixed on the basis of computer techniques for the analysis and synthesis of the probability distributions at design of measuring instruments and determination of the accuracy class at certification of measuring instruments and systems. The offered technique can also be useful at the decision of a problem of automatic on-line evaluation of uncertainties of final measurement result in virtual measuring instruments as well as for other similar problems connected with the analysis and research of results of various measurements.

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