DESIGN OF A CORRECTING CIRCUIT FOR AN ELECTROMAGNETIC ACTUATOR

Daniel Mayer — Jan Teplý *

The aim of the paper is the design of a correcting circuit that allows to increase the attraction force at the beginning of the armature travel of an electromagnetic actuator. The equations describing dynamic operational states of the actuator are defined and numerically solved.

Key words: electromagnetic actuator, electromagnet, network synthesis

1 INTRODUCTION

Operating characteristics of an electromagnetic actuator have a rather problematic locus: at the beginning of the shift the attraction force is too small. The shape of the characteristic can be changed with the help of an electronic source but at the cost of higher expenses. At the cost of higher expenses the shift the attraction force is too small. The shape of the characteristic can be changed with the help of an electronic source but at the cost of higher expenses. At the beginning of the armature travel of an electromagnetic actuator the equations for the dynamic state of the actuator are formulated under the assumption of a correcting circuit. That is why the equations for the dynamic state of the actuator are formulated under the assumption of a correcting circuit.

Even though the electromagnetic actuator is among simple electric apparatuses, construction of a mathematical model is not a simple matter, if we want to describe exactly the distribution of the magnetic field (i.e. if we respect the non-linear character of the magnetic circuit and magnetic flux leakage). In our case the aim is not an exact investigation of the dynamic behaviour of the actuator but a comparison of its effect with or without a correcting circuit. That is why the equations for the dynamic state of the actuator are formulated under the common simplifying preconditions.

2 MATHEMATICAL MODEL OF LINEAR ELECTROMAGNETIC SYSTEM

In Fig. 1 the actuator is shown with winding divided into two coils. In Fig. 2 their connection to the correcting circuit, which consists of capacitor C and diode D, is presented.

Presuppose that a) permeability of the magnetic circuit $\mu = \text{const}$, b) magnetic flux leakage does not depend on the armature operating position and can be neglected. The force applied to the actuator armature is defined from the energy of the magnetic field

$$W_m = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + L_{12} i_1 i_2$$

with self-inductances of coils $L_1$, $L_2$ and their mutual inductance $L_{12}$. We obtain them by the solution of the magnetic circuit. There is then

$$L_1 = G_m N_1^2, \quad L_2 = G_m N_2^2, \quad L_{12} = G_m N_1 N_2$$

where $G_m$ is magnetic conductivity. Magnetic reluctance is

$$R_m = \frac{1}{G_m} = R_{mo} + R_{mu}(x)$$

Magnetic conductivity $G_m$ is obtained from reluctance $R_m$, that consists of reluctance of those parts of the magnetic circuit that do not change with the armature travel $R_{mo}$ and reluctance of variable of air gap $R_{mu}(x)$.

$$G_m = \frac{1}{R_m} = \frac{1}{R_{mo} + R_{mu}(x)}$$

From the stated presumptions we obtain $R_{mu}(x) = kx$, where $k$ is a constant. The force of the electromagnet is then

$$f_m = \frac{\partial W_m}{\partial x} = \frac{1}{2} \frac{\partial G_m}{\partial x} (N_1^2 i_1^2 + N_2^2 i_2^2 + 2N_1 N_2 i_1 i_2)$$

After substitution from previous equations to equation (4) the following is obtained

$$f_m = -\frac{k}{2 R_m} (N_1^2 i_1^2 + N_2^2 i_2^2 + 2N_1 N_2 i_1 i_2)$$

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Parameters of particular variants.

<table>
<thead>
<tr>
<th>Variation</th>
<th>Parameters</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>N = 2550, R = 16.5 Ω</td>
<td>without correcting circuit</td>
</tr>
<tr>
<td>2a.</td>
<td>N₁ = 128, R₁ = 0.83 Ω, N₂ = 2422, R₂ = 15.67 Ω</td>
<td>C = 100 µF</td>
</tr>
<tr>
<td>2b.</td>
<td>dtto.</td>
<td>C = 500 µF</td>
</tr>
<tr>
<td>3a.</td>
<td>N₁ = 384, R₁ = 2.48 Ω, N₂ = 2166, R₂ = 14.02 Ω</td>
<td>C = 100 µF</td>
</tr>
<tr>
<td>3b.</td>
<td>dtto.</td>
<td>C = 500 µF</td>
</tr>
</tbody>
</table>

From the Newton law the following equation is valid

\[ M \frac{d^2 x}{dt^2} = f_m + B \frac{dx}{dt} + K(l - x) \]  

(6)

where \( M \) is the mass of armature, \( B \) is the damping constant, \( K \) is the stiffness of the spring and \( l \) is the total displacement of the armature. From Kirchhoff’s laws the equations for the electric circuit are obtained (Fig. 2):

\[ R_{l1}i_1 + u_{L1} + u_C = U_0 \]

\[ -i_1 + i_2 + C \frac{du}{dt} = 0 \]

\[ R_{l2}i_2 + R_{D}i_2 + u_{L2} - u_C = 0 \]

(7)

where the resistance of the diode is

\[ R_D = \begin{cases} 
0 & \text{if } i_2 \geq 0 \\
\infty & \text{if } i_2 < 0
\end{cases} \quad \text{and} \quad u_{L1} = \frac{d\Phi_{c1}}{dt}. \]

As the entire magnetic flux linked with the first coil is

\[ \Phi_{c1} = L_{11}i_1 + L_{12}i_2, \]

the voltage on the inductance of the first coil is

\[ u_{L1} = L_1(x) \frac{d}{dt} \left( \frac{d}{dx} i_1 + i_2 \frac{d}{dx} \frac{d}{dt} + L_2(x) \frac{d}{dt} i_2 + i_2 \frac{d}{dx} \frac{d}{dt} \right). \]

(8)

Similarly, the voltage on the inductance of the second coil is

\[ u_{L2} = L_2(x) \frac{d}{dt} i_2 + i_2 \frac{d}{dx} \frac{d}{dt} + L_{21}(x) \frac{d}{dt} i_1 + i_2 \frac{d}{dx} \frac{d}{dt}. \]

(9)

The actuator is thus described by the system of nonlinear equations (6) and (7). We introduce the velocity of the armature travel of the actuator

\[ v = \frac{dx}{dt}. \]

The unknowns are quantities \( i_1, i_2, u_C, v \) and \( x \). After modification, this system is expressed by the following matrix equation:

\[ \begin{bmatrix} L_1(x) & L_{12}(x) & 0 & 0 & 0 \\
L_{12}(x) & L_2(x) & 0 & 0 & 0 \\
0 & 0 & C & 0 & 0 \\
0 & 0 & 0 & M & 0 \\
0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\
i_2 \\
u_C \\
v \\
x \end{bmatrix} + \]

\[ + \begin{bmatrix} R_{l1}i_1 + \frac{dL_1(x)}{dx} i_1v + \frac{dL_{12}(x)}{dx} i_2v + u_C \\
R_{l2}i_2 + R_{D}i_2 + \frac{dL_2(x)}{dx} i_2v + \frac{dL_{12}(x)}{dx} i_1v - u_C \\
-f_m - Bv - K(l - x) \\
v \end{bmatrix} = [U_0, 0, 0, 0, 0]^T. \]  

\[ (10) \]

3 NUMERICAL EXAMPLE

Judgement of the influence of the correcting circuit on the working characteristics of the actuator is done in comparison with the characteristics of the actuator without the correcting circuit.
Calculation of the characteristics of the actuator is done for an actuator with supply voltage $U_o = 24$ V. Its mechanical parameters are: $M = 10$ kg, $B = 100$ Nsm$^{-1}$, $K = 1000$ Nm$^{-1}$ and travel $l = 4.5$ mm. For the calculation of the magnetic parameters the magnetic circuit is divided into eight parts, Fig. 3, and their reluctance is calculated. The total reluctance of parts independent of displacement of the core (in Fig. 3 these are parts 1 to 7), at permeability of ferromagnetic material $\mu_r = 1000$ is

$$R_{m0} = \sum_{k=1}^{7} R_{mk} = 1.366 \times 10^6 H^{-1}.$$  

Reluctance variation with armature displacement is

$$R_{mv} = 6.755 \times 10^8 x.$$  

Five variants of the calculation were done, see Table 1.

The system of non-linear differential equations (10) is solved numerically with the help of standardly constructed function ODE23s (modified Rosenbrock formula of order 2) in the universal calculating and programming environment MATLAB.

In Fig. 4 the time evolution of the force of electromagnet $f_m$ is evident. The switch-in moment is marked by plus. It is evident that the initial slope is influenced by capacity $C$ of the capacitor of correcting circuit and by the number of turns ($N_1$, ie inductance $L_1$) and resistor $R_1$.

In Fig. 5 there is a plot showing the displacement $x$ of the electromagnet. Also here the accelerating effect of capacity $C$ and inductance $L_1$ is evident.

In Fig. 6 there are the currents $i_1$ and $i_2$ for variant 2b. The correcting circuit increases the initial impulse of current $i_1$ after switch-on. Results is in a big rate of rise of the force of electromagnet. This current impulse cannot be neglected when dimensioning the winding of coil 1. Both currents $i_1$ and $i_2$ get for $t \to \infty$ the same value.

4 CONCLUSIONS

In the presented work a simple way of allowing an increase of the attraction force of electromagnetic actuator, especially shortly after voltage supply, is investigated.
At the calculation neither leakage nor non-linearities of magnetic circuit were respected, which influences the responses of the characteristics of the actuator, but in our case the focus was on comparison of characteristics of actuators with different parameters of the elements of the correcting circuit.

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References


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