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INTRODUCING THE WEIGHT CONCEPT TO VECTOR DIRECTIONAL FILTERS

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In color images and satellite images, each image point can be represented as a vector of R, G, B components. The filtering of these vector-valued image signals is constrained by the requirement for the preservation of all desired image features, including color information. On that account, vector techniques such as vector median-type filters based on a reduced ordering scheme according to the absolute or Euclidean distance and vector directional filters based on the vector angle distance were developed.

This paper is focused on the design of a new structure of vector directional filters; the novelty of the proposed method is in incorporating the weight concept to directional processing. Thus, a new weighted vector directional filter (WVDF) is developed and it can provide excellent estimates according to subjective and objective criteria, whereas the computational complexity is still equivalent with the basic vector directional filter (BVDF). In addition, the proposed WVDF includes BVDF as a special case.

K e y w o r d s: directional filters, vector filters, weighted directional filters, angle distance

1 INTRODUCTION

Color information is one of the most important features that must be preserved in the case of color image processing. Under this constraint, should noise be suppressed whereas color chromaticity remains saved, digital vector filters are preferred [2, 3, 5–7].

The well-known vector median filter [1, 4, 5] is probably the most frequently used vector filter for noise reduction in color images. This filter belongs to a class of vector filters based on a reduced ordering scheme [3, 5] where the input RGB vectors are ordered according to mutual distances of their magnitude. If the performance of vector median-type filters is inefficient, then a marked improvement can be achieved by weighted structures [2–5]. However, on the ground of the inefficient noise reduction of vector directional filters [3, 5–8], these filters are extended to produce a set of vector samples with approximately equivalent directions in a vector space. This extended filter, simply called a general vector directional filter (GVDF), is connected to a cascade with an additional filter that performs the filtering operation according to magnitude processing. Thus, the computational complexity is rapidly increased and some problems may occur with the choice of an appropriate filter for magnitude processing.

On that account, in this paper a new class of vector directional filters are presented. These weighted vector directional filters (WVDF) represent a new filter family for both theoretical and practical purposes. WVDF can be designed to perform excellent filtering operation in the sense of objective and subjective criteria. In addition, no price is paid for this improved performance since the computational complexity is similar to that of the basic vector directional filter (BVDF) that is the simplest vector directional filter.

2 VECTOR DIRECTIONAL FILTERS

Unlike filters based on magnitude processing, in the case of vector directional filters the output are the vector directions, while the vector magnitudes are used only to evaluate them.

Vector directions indicate the color chromaticity and, thus, vector directional filters perform optimal filtering operation in the sense of sample direction preservation.

Let $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N$ be an input set determined by a filter window with a window size N, where each multivariate sample $\mathbf{x}_i$ (for $i = 1, 2, \ldots, N$) represents an m-dimensional vector. In the case of color images $m = 3$ since vectors consist of three intensities from R, G, B channels. The most important problem related to vector filters is the ordering of multivariate samples, where an ordered input set is given by

$$\mathbf{x}^{(1)} \le \mathbf{x}^{(2)} \le \cdots \le \mathbf{x}^{(r)} \le \cdots \le \mathbf{x}^{(N)} . \qquad (1)$$

The ordering process (1) can be performed [3, 5] according to magnitude and angle distances of input samples. In the case of directional processing the angle distances are defined as

$$\alpha_i = \sum_{j=1}^{N} A(\mathbf{x}_i, \mathbf{x}_j) \quad \text{for } i = 1, 2, \ldots, N \qquad (2)$$

where $A(\mathbf{x}_i, \mathbf{x}_j)$ represents the angle between two m-dimensional vectors $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{im})$ and $\mathbf{x}_j =$

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$(x_{j1}, x_{j2}, \dots, x_{jm})$ given by [3, 5, 6]

$$A(\mathbf{x}_i, \mathbf{x}_j) = \cos^{-1} \frac{\mathbf{x}_i \mathbf{x}_j^\top}{|\mathbf{x}_i| \cdot |\mathbf{x}_j|} = \cos^{-1} \frac{x_{i1}x_{j1} + x_{i2}x_{j2} + \dots + x_{im}x_{jm}}{\sqrt{x_{i1}^2 + x_{i2}^2 + \dots + x_{im}^2}\sqrt{x_{j1}^2 + x_{j2}^2 + \dots + x_{jm}^2}} \tag{3}$$

From (2) and (3) it is clear that for $i = 1, 2 \dots, N$ the angle distance α_i is associated with the input vector-valued sample $\mathbf{x}_i$. The ordered set of angle distances $\alpha_1, \alpha_2, \dots, \alpha_N$ can be expressed as

$$\alpha_{(1)} \le \alpha_{(2)} \le \dots \le \alpha_{(r)} \le \dots \alpha_{(N)} \tag{4}$$

In the case of BVDF filter, the filter output is defined as [3, 5–8]

$$\boldsymbol{y}_{BVDF} = \mathbf{x}^{(1)} \tag{5}$$

ie as a sample associated with minimal angle distance $\alpha_{(1)}$. Clearly, BVDF outputs the sample from the input set that minimizes the sum of angles with other vectors.

However, on the ground of the inefficient noise suppression in medium and highly corrupted images, BVDF was extended to GVDF that outputs a set of r samples with the minimal sum of angles. Mathematically, this operation can be expressed by [3, 5–8]

$$\boldsymbol{y}_{GVDF} = \{\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(r)}\}. \tag{6}$$

However, in the second filter stage the magnitude processing must be performed. Thus, the computational complexity of directional processing is increased. Another problem is related to the choice of the optimal method for magnitude processing. The new method for directional processing should exclude the use of an additional filter and provide only one sample on the filter output. In addition, this output sample should estimate the original sample with high precision. These requirements lead to a combination of weighted angle distances and the choice of one multivariate sample associated with the minimal sum of angles as well as in (5).

3 WEIGHTED ANGLE CONCEPT

Let $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N$ be an input set determined by a filter window and N represents a window size. Let us assume that $w_1, w_2, \dots, w_N$ is a set of integer weights such that each weight w_j for $j = 1, 2, \dots, N$ is associated with m-dimensional input sample $\mathbf{x}_j$.

Then the weighted angle distance β_i associated with the input sample $\mathbf{x}_i$ is given by

$$\beta_i = \sum_{j=1}^{N} w_j A(\mathbf{x}_i, \mathbf{x}_j) \quad \text{for } i = i, 2, \dots, N \tag{7}$$

where $A(\mathbf{x}_i, \mathbf{x}_j)$ is the angle between two m-dimensional vectors $\mathbf{x}_i$ and $\mathbf{x}_j$ defined by (3).

The output of the weighted vector directional filter (WVDF) can be expressed as

$$\boldsymbol{y}_{WVDF} = \mathbf{x}^{(1)} \tag{8}$$

where $\mathbf{x}^{(1)}$ (1) is an ordered multivariate input sample associated with the minimal weighted angle distance $\beta_{(1)}$ according to

$$\beta_{(1)} \le \beta_{(2)} \le \dots \le \beta_{(r)} \le \dots \le \beta_{(N)}. \tag{9}$$

Including of weights to account for angle distances brings new flexibility to the design of filters based on directional processing.

Besides practical purposes related with the markedly improved performance of noise suppression, the signification of WVDF lies in theoretical analysis of this filter class. Clearly, the class of WVDF is very large and it includes many vector directional filters as special cases.

Let $w_j = 1$ for all $j = 1, 2, \dots, N$. Then definition (7) is identical with (2) and WVDF (8) performs the same operation as a BVDF (5).

If the set of integer weights $w_1, w_2, \dots, w_n$ is given by

$$w_j = \begin{cases} N - 2k + 2 & \text{for } j = (N+1)/2 \\ 1 & \text{otherwise} \end{cases} \tag{10}$$

where k is a parameter constrained to have a value $1 \le k \le (N+1)/2$, then the WVDF is equivalent with the center-weighted vector directional filter (CWVDF).

If (10) is valid and the central weight $w_{(N+1)/2}$ has the maximum possible value ($w_{(N+1)/2} = N$), then the WVDF performs no filtering operation, *ie* WVDF is equivalent with an identity filter and central input sample $\mathbf{x}_{(N+1)/2}$ is passed to the filter output. On the other hand, if (10) is valid and the central weight $w_{(N+1)/2} = 1$, then WVDF is equivalent with BVDF again.

From the above examples it is evident that WVDF can perform various filtering operations in the dependence on the set of weights. In addition, weighted angle distances can be incorporated in GVDF too, however, this is not required since the performance of WVDF can be really excellent and thus additional processing is not necessary.

4 CONCLUSION

In this paper a new class of weighted vector directional filters was introduced. To make an estimate, the importance of input multivariate samples is expressed by a set of integer weights taking into account the angle distances. The mentioned weighted angle distances make this filter class very flexible and attractive. Thus, weighted vector directional filters can provide the best estimate without increasing the computational complexity. In addition, the class of weighted vector directional filters includes a basic vector directional filter, identity filter and center-weighted vector directional filter as special cases. The mentioned fact is very important from the theoretical point of view of directional processing.

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