

# THEORETICAL AND PRACTICAL LIMITS OF TRANSMISSION CHANNELS USED BY THE ADSL TECHNOLOGY IN THE METALLIC LINE'S ENVIRONMENT IN THE ACCESS NETWORK IN SLOVAKIA

Rastislav Róka \*

For successful expansion of new multimedia services through xDSL technologies in the access network in Slovakia, it is necessary exactly to know miscellaneous features of real environments and particular transmission characteristics of homogeneous symmetric lines. To evaluate the transmission channel limits, it is also unavoidable to know environmental noise characteristics because of their negative influences on the transmitted information signals. This article comprehensively discusses two qualitative parameters of metallic homogeneous lines — the channel capacity and the cut-off rate. Also, the bandwidth efficiency is presented as a function of the signal-to-noise ratio for both parameters.

**K e y w o r d s:** channel capacity, cutoff rate, bandwidth efficiency

## 1 INTRODUCTION

In the future, an integrated communication network should be able to transport signals of many various kinds of services and applications, including voice, video, data and multimedia services, for a large amount of subscribers. To accomplish this objective, the utilization of already existing metallic parts of the access network appears as a good starting point. As a result of the effort to utilize unloaded twisted-pair telephone loops to transport high-speed signals, a family of xDSL technologies arises. In present days, many various high-speed digital communication xDSL systems are proposed to transport digital information signals through subscriber loop interfaces [1].

For successful expansion of new multimedia services through xDSL technologies in the access network primarily created from homogeneous symmetric lines, it is necessary exactly to know miscellaneous features of real environments and particular transmission characteristics of these lines. On this basis, it is possible to specify if there is a potential to utilize the existing subscriber lines for the transport of high-speed information signals. The transmission capacity of the channel belongs to basic parameters that assign the quality of the metallic line. These basic parameters of the internal environment for homogeneous symmetric lines are given primarily by material and geometric characteristics of wires. In our analysis, primary constants of the line — resistance, inductivity, capacity and conductivity — are considered for given core diameters accomplishing conditions of the standards ETSI ETR 328 [2] and ANSI T1.413 [3].

Except for qualitative features of the subscriber line, it is also unavoidable to know environmental noise charac-

teristics because of their negative influences on the transmitted information signals. Noise resources of the real environment can be classified as performance limiting noises or capacity limiting noises [4]. The performance limiting noises (impulses, radiofrequency interference) are intermittent in nature, geographically variable and unpredictable, and therefore are usually accounted for in planning rules by using a safety margin. The capacity limiting noises (thermal noise, crosstalks) are usually slowly changing, their noise levels are often predictable and relatively easy to take into account in analyses. Just from this reason, we focused on the best known noise types — AWGN, NEXT and FEXT — in our analysis.

The goal of our analysis is to quantify qualitative parameters of metallic homogeneous lines, to assign the extent and the importance of negative environmental influences on the transmitted information signals and to specify numerically the upper limits in the term of the bandwidth efficiency for both parameters the channel capacity and the cutoff rate. The analytical output should be the qualification of the correlation between these parameters in dependence on various values for the signal-to-noise ratio.

## 2 THEORETICAL PART

If the information rate  $R$  from the source is lower than the channel capacity  $C$  ( $R < C$ ), then is theoretically possible to attain the reliable, errorless signal transport through the given channel using appropriate coding techniques. On the other side, if the information rate is higher than the channel capacity ( $R > C$ ), the errorless transport is impossible irrespective of an amount of the signal

\* Department of Telecommunications, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology, Ilkovičova 3, 812 19 Bratislava, Slovakia, rrroka@ktl.elf.stuba.sk

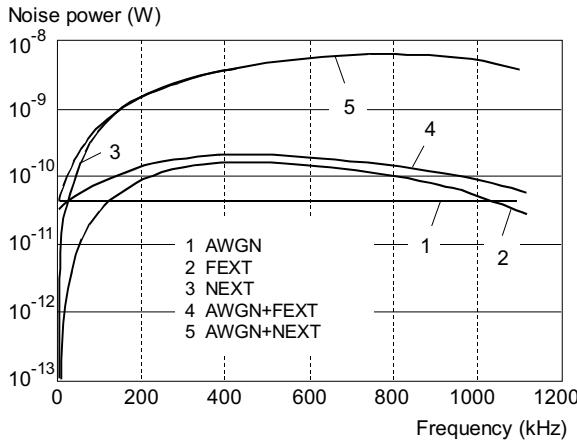


Fig. 1. Frequency characteristics of noise powers for various noise types

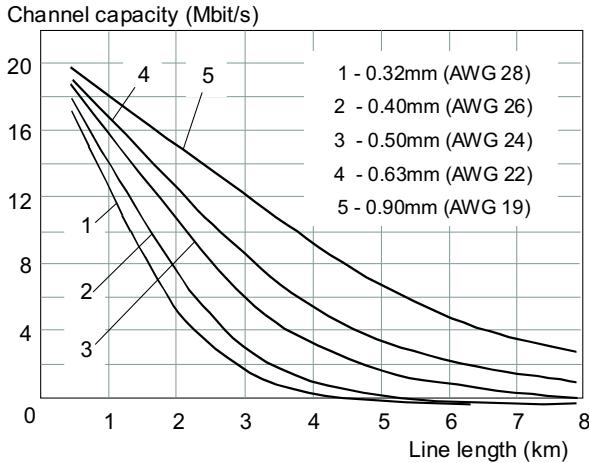


Fig. 2. Channel capacity's characteristics versus line length for various core diameters of wires

processing realized in the transceivers [5]. A memoryless communication channel's capacity is usually calculated as the maximum throughput, measured in information units per second [bit/s], assuming a certain fixed symbol rate, measured in symbols per second [symbol/s]. The capacity of digital channels is generally dedicated by the basic formula for the capacity of continuous flat channels with the coloured Gaussian noise:

$$\begin{aligned} C_{CHANNEL} &= \int_0^W \log_2(1 + SNR(f)) df \\ &= \int_0^W \log_2 \left( 1 + \frac{|H_C(f)|^2 PSD_{S1}(f)}{|H_X(f)|^2 PSD_{S2}(f)} \right) df \quad [\text{bits/s}] \end{aligned} \quad (1)$$

where  $C_{CHANNEL}$  is the channel capacity,  $SNR$  is the signal-to-noise ratio in the real environment,  $W$  is the upper limit of the frequency range of the channel bandwidth,  $H_C$  is the transmission function of the channel path in a complex form,  $PSD_{S1}$  is the power spectral density of the information signal,  $H_X$  is the transmission function of the crosstalk path in a complex form and  $PSD_{S2}$  is the power spectral density of the crosstalk signal.

Any physical system limits the available frequency bandwidth because its transfer characteristic must eventually roll off at sufficiently high frequencies. This departure from a flat spectral shape translates into intersymbol interferences in the time domain at high enough signalling rates using those frequencies [6]. Except the bandwidth limitation, several additive noise components impair a transmission on the local loop, including the electronic noise (generally white), the crosstalk noise coupled from adjacent loops in the cable bundle between the customer premises and the central office (usually non-white) and the residual echo noise remaining after any echo cancellation.

The simplest case of the noise is the white (thermal) noise AWGN with the fixed standard level  $\sigma_1^2 = -140 \text{ dBm/Hz}$ . We used the power spectral density for the increased noise AWGN  $\sigma_{22} = N_0/2 = -110 \text{ dBm/Hz}$ . The crosstalk is a case of the noise that arises from the electromagnetic coupling of signals in various transmission paths where both pairs — the first pair (disturbing) and the second pair (disturbed) of lines — are situated in the same telephone cable bundle. At the information signal transport, we distinguish two types of crosstalks [7]:

– the near-end crosstalk NEXT — the NEXT transmission path can be modelled using its transmission function  $H_{NEXT}$  in the form

$$|H_{NEXT}(f)|^2 = K_{NEXT} \cdot f^{3/2} \quad (2)$$

where  $K_{NEXT} = 0.882 \times 10^{-14} \cdot N_d^{0.6}$  and  $N_d$  is the number of disturbing pairs (disturbers).

– the far-end crosstalk FEXT — the FEXT transmission path can be modelled using its transmission function  $H_{FEXT}$  in the form

$$|H_{FEXT}(f)|^2 = K_{FEXT} \cdot l \cdot 3280 \cdot f^2 \cdot |H_C(f)|^2 \quad (3)$$

where  $K_{FEXT} = 3.083 \times 10^{-20}$  and  $l$  is the line length in km.

On the basis of the mentioned relations (2) and (3) and by means of acquired findings about particular noise types, we executed analytical computations and a simulation of various environmental noise influences. Calculated results are represented in Fig. 1.

Figure 1 represents frequency characteristics of noise powers of the white noise, both types of the crosstalk and their combinations for the line with a copper (Cu) core diameter  $\phi 0.4 \text{ mm}$ , the line length of 1 km and input signal power 10 mW. It is clearly seen that the largest noise power is contained in the NEXT crosstalk and, in the comparison with it, noise powers AWGN and FEXT are neglected. From calculations and simulations, which we performed, it is possible to draw a conclusion that dependencies for other core diameters are very similar dependencies. From our analysis also results that a noise power of the FEXT crosstalk for longer line lengths is expressively decreased.

In the next part of our analysis, we researched the influence of the core diameter of wires on the parameter of the channel capacity and we focused on numerical representations of this dependency. In Fig. 2, graphical dependencies of the transmission channel capacity as a function of line lengths are represented in the environment of the AWGN, with the input signal power 10 mW and with the power spectral density of the downstream signal ADSL  $PSD_{ADSLds}$ . It is clearly seen that the wire with core diameter  $\phi = 0.9$  mm has a better upper limit than other wires, primarily for long line lengths.

Except influences of the white noise and crosstalks, digital subscriber lines transmitting information signals at very high-speed rates are also affected by impulse noises. This influence of impulse noises on the performance of xDSL digital systems in the environment of metallic lines is a content of another of our studies that calls for a construction of other mathematical techniques and simulation methods.

If the environment is characterized with only a fixed level of the AWGN, then there exists an optimal transmitting spectral density that is band-limited [8]. The channel capacity  $C_{NEXT}$  in the environment with only the NEXT will still not be affected by the shape of the frequency spectrum  $PSD_S(f)$ , but only by the frequency range of  $PSD_S(f)$ . At the assumption  $PSD_{S1}(f) = PSD_{S2}(f)$  it is clear that the power spectral density  $PSD_S(f)$  of the signal with unlimited bandwidth maximizes the value of  $C_{NEXT}$ .

$$C_{NEXT} = \int_0^W \log_2 \left( 1 + \frac{|H_C(f)|^2}{|H_{NEXT}(f)|^2} \right) df \quad [\text{bits/s}] \quad (4)$$

In the case of a channel in the environment with only FEXT, it is possible to draw an equivalent conclusion for the channel capacity  $C_{FEXT}$  — the channel capacity is affected only by the frequency range of  $PSD_S(f)$ . Of course, the value of capacity  $C_{FEXT}$  is much less than the value of capacity  $C_{NEXT}$ , because the NEXT crosstalk unlike the FEXT crosstalk signal does not propagate through the whole line length and therefore has a much more harmful impact on the transmitted information signal, whereby the upper limit of  $C_{NEXT}$  is expressively decreased.

In the case that there exists a specific combination of the above-mentioned noises and crosstalks (in addition to the NEXT interference, the AWGN noise is also presented), then, *eg*, the channel capacity  $C_{NEXT-AWGN}$  is determined for the special type of signals under the condition that the transmitted power  $P$  is constrained, *ie* [8],

$$C_{NEXT-AWGN} \sup \int_0^W \log_2 \left( 1 + \frac{|H_C(f)|^2 PSD_S(f)}{|H_{NEXT}(f)|^2 PSD_S(f) + \frac{N_0}{2}} \right) df \quad [\text{bits/s}] \quad (5)$$

where the sup operation is carried out over all  $PSD_S(f)$  satisfying the average power constraint

$$2 \int_0^\infty PSD_S(f) df \leq P. \quad (6)$$

While the channel capacity  $C_{CHANNEL}$  determines the limits of the error-free transmission at the infinite circuit complexity in transceivers, the cutoff rate  $R_0^*$  is regarded as a practical transmission limit for the moderate coding complexity [6]. We can calculate the cutoff rate  $R_0^*$  given the critical rate  $R_{cr}$  and the error exponent at that rate  $E_{cr}$  ( $R_{cr}$ ).

$$R_0^* = R_{cr} + E_r(R_{cr}). \quad (7)$$

Here particular components in the environment with the combination of NEXT and AWGN influences are given by the following formulas

$$R_{cr} = \sum_{n=1}^N \frac{W}{N} \log_2 \left( \frac{B_{cr} H_n^2}{w_n + K_n P_n} \right) \quad [\text{bits/s}] \quad (8)$$

$$E_r(R_{cr}) = \frac{WP}{4B_{cr}} - \sum_{n=1}^N \frac{W}{N} \log_2 \left( 2 - \frac{w_n + K_n P_n}{B_{cr} H_n^2} \right) \quad [\text{bits/s}] \quad (9)$$

where  $N$  is the number of parallel subchannels,  $W$  is the upper margin of the frequency range of the channel bandwidth,  $H_n^2$  equals the power transfer gain of subchannel  $n$ ,  $P$  is the total input signal power,  $P_n$  equals the input signal power assigned to subchannel  $n$ ,  $w_n$  equals the two-sided additive Gaussian noise in subchannel  $n$ ,  $K_n = K_{NEXT} \cdot f_n^{1.5}$  is the NEXT noise in subchannel  $n$ ,  $B_{cr}$  is a variable that is determined iteratively.

Each term in the sum of formulas (8) and (9) represents the contribution of the single-sided subchannel, which is included into the total sum only if the formula (10) is valid

$$B_{cr} \leq \frac{H_n^2}{w_n + K_n P_n} \quad (10)$$

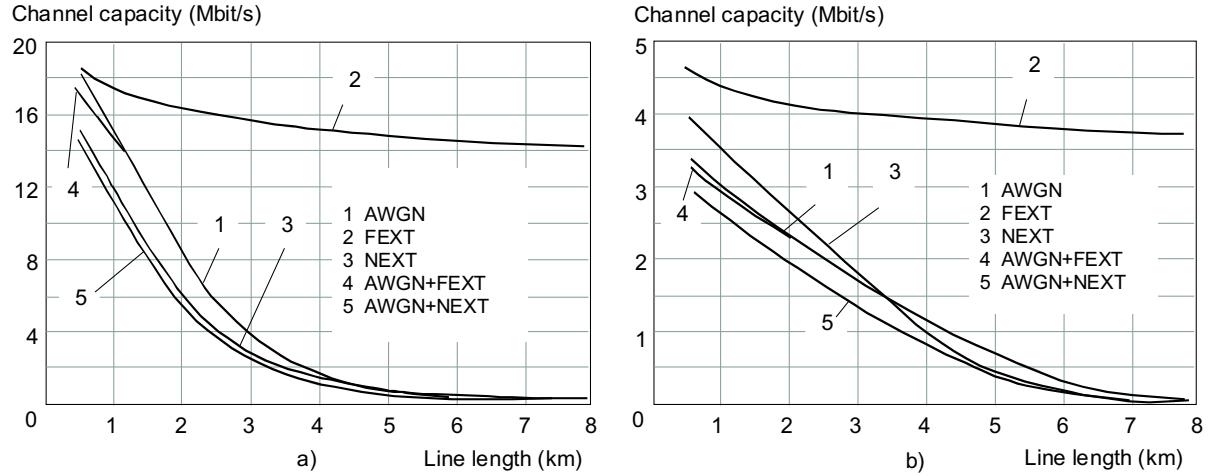
We must determine one variable iteratively. In our case, it is the value of  $B_{cr}$ , which must satisfy the following power constraint

$$P = \sum_{n=1}^N P_n \quad (11)$$

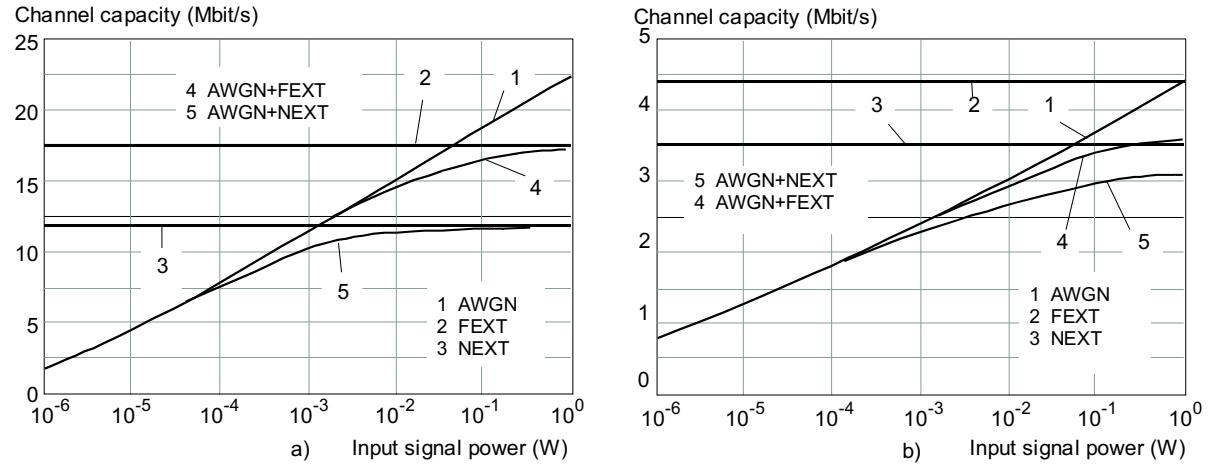
where the input signal power assigned to subchannel  $n$  is given as

$$P_n = \frac{4B_{cr}(B_{cr} H_n^2 - w_n - K_n P_n)}{2B_{cr} H_n^2 - w_n - K_n P_n}. \quad (12)$$

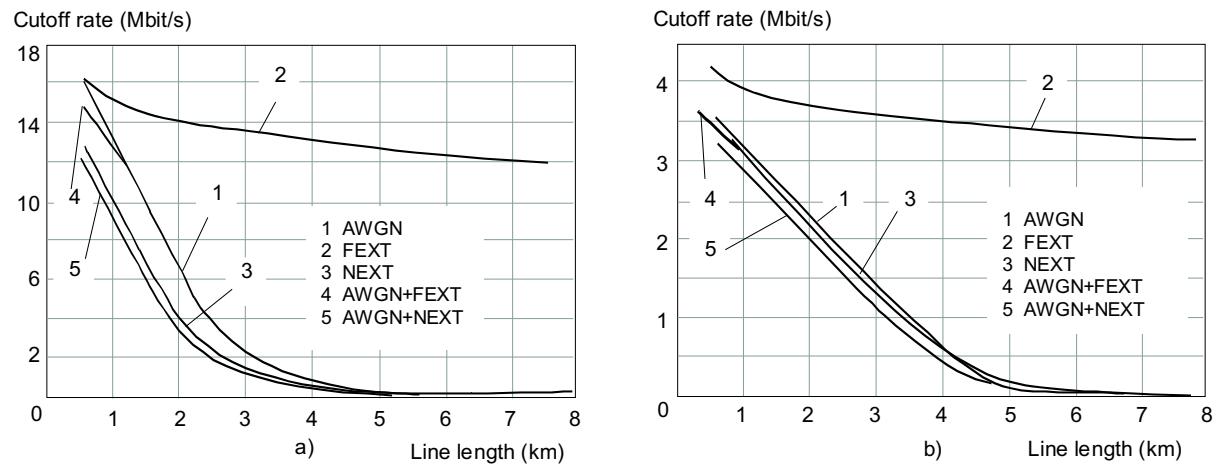
The computing approach of the  $R_0^*$  calculation is following: we guess a value for  $B_{cr}$  and the corresponding power level for each subchannel is determined by rearranging (12). Values  $P_n$  are determined for all subchannels with nonzero input signal power level and then formula (11) is checked for the equality. As necessary, a value  $B_{cr}$  is iteratively adjusted.



**Fig. 3.** Channel capacity characteristics versus line length for downstream (a) and upstream (b) ADSL signals for various environmental types



**Fig. 4.** Channel capacity characteristics versus input signal power for downstream (a) and upstream (b) ADSL signals for various environmental types

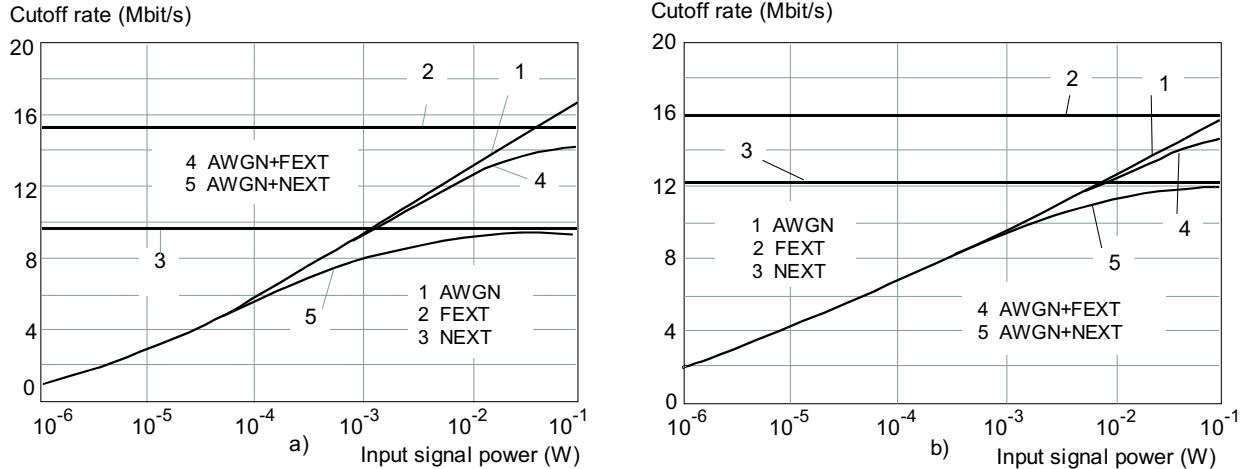


**Fig. 5.** Cutoff rate characteristics versus line length for downstream (a) and upstream (b) ADSL signals for various environmental types

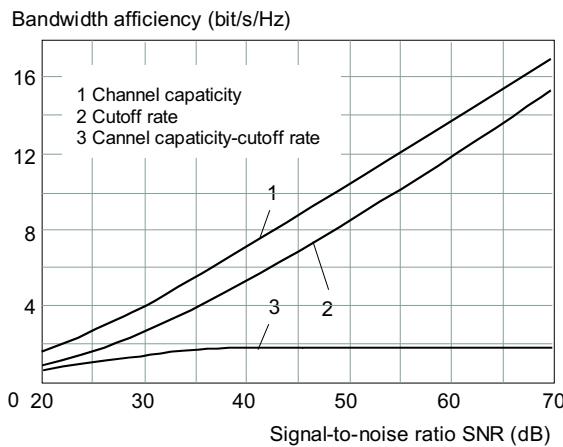
### 3 EXPERIMENTAL PART

On the basis of mathematical formulas and findings mentioned in part 2, we performed the analysis of theoret-

ical and practical limits of transmission channels used by the ADSL technology in the metallic line's environment in the access network in Slovakia. Analytical calculations and simulations were executed for various characteristics



**Fig. 6.** Cutoff rate characteristics versus input signal power for downstream and upstream ADSL signals for various environmental types



**Fig. 7.** Characteristics of the bandwidth efficiency versus the signal-to-noise ratio

of the metallic line's environment and information signals transmitted by the ADSL technology. Results of our analysis are presented in Figs. 3–7.

Graphical results from our analysis of the transmission channel capacity as a function of line lengths for various environmental types for the line with core diameter  $\phi = 0.4$  mm and input signal power 10 mW and for downstream and upstream ADSL signals are presented in Figs. 3a,b. It is emerged from these graphs that the channel capacity for the downstream signal is higher by one order than the channel capacity for the upstream signal. It is possible to see that the values of the channel capacity with increasing line length are largely falling for dominant noises AWGN and NEXT.

In Fig. 4a,b, there are presented graphical results from the analysis of the transmission channel capacity as a function of input signal powers for various environmental types for the line with core diameter  $\phi = 0.4$  mm and line length 1 km and for downstream and upstream ADSL signals. It is emerged from these graphs that the channel capacities in the environment with only the NEXT

or the FEXT crosstalk are independent of the input signal power. For low values of the input signal power, the main limiting environmental factor is the AWGN noise and, for high values of the input signal power, the NEXT and FEXT crosstalks become the dominant environmental influences.

Graphical results from the analysis of the transmission cutoff rate as functions of line lengths and input signal powers are presented in Figs. 5a,b and Figs. 6a,b. It is obvious that the characteristic dependences of both qualitative parameters — the channel capacity and the cutoff rate — are very close.

To compare the channel capacity and the cutoff rate, and to use them in our next analysis, we used a parameter called bandwidth efficiency. Figure 7 illustrates the characteristic of the bandwidth efficiency as a function of the signal-to-noise ratio. This dependence represents the correlation between theoretical and practical limits of qualitative parameters for homogeneous symmetric lines, specifically for a line with core diameter  $\phi = 0.4$  mm, line length 1 km and in the AWGN environment. At low SNR values, the difference between  $C$  and  $R_0^*$  is increased since the white noise dominates the performance. At high SNR values (approximately from the 40 dB value), the difference is invariant and has a value  $\approx 1.85$  bit/s/Hz.

#### 4 DISCUSSION

In this article, the channel capacity and cutoff rate formulas are derived for transmission channels corrupted by various types of noise influences and their combinations. Using these formulas, we calculated the maximum throughputs (theoretical and practical) attainable on metallic homogeneous lines.

The most important negative influence of the real environment is the NEXT crosstalk, which has a noise power more than 2 orders higher than the AWGN and the FEXT. The FEXT noise power is strongly decreasing with an increasing length, but at short lengths of metallic lines using xDSL technologies it is necessary to take this

influence into account, above all for asymmetric applications at very high downstream rates.

For a prospective utilization of metallic lines by means of xDSL technologies, for instance in combination with optical networks, it is necessary to consider the utilization of new metallic homogeneous lines with a larger core diameter ( $\phi$  0.9 mm) or replacement of old subscriber shunts by new ones.

From graphical characteristics of the channel capacity and cutoff rate, it follows that actual metallic lines are not effectively utilized for information signal transport at very high-speed rates and there still exist possibilities to reach an increase in transmission rates closer to their practical limits for both downstream and upstream directions of the transport.

At low values of the SNR ratio, the main environmental negative influence is the AWGN, because this type of the noise is always present on the line, and the NEXT and FEXT noise powers are low due to low input signal powers. In this case, the difference between bandwidth efficiencies expressed in bit/s/Hz for the channel capacity  $C$  and cutoff rate  $R_0^*$  is increasing. However, at high values of input signal powers, the value of the SNR ratio is also increased and the NEXT crosstalk becomes a dominant influence. In this case, parts of the curves for the bandwidth efficiency for  $C$  and  $R_0^*$  are nearly parallel and the difference between them becomes unvaried.

## 5 CONCLUSION

From the analysis of the channel capacity and the cut-off rate in various types of environmental influences, it follows that the present bit rates on metallic subscriber shunts in the access network in Slovakia are deeply below their theoretical limits. This conclusion indicates that an effort for the proposal of new modulation techniques utilized by xDSL modems has to be more intensive for solving the "bottleneck" problem in the access network consisting from metallic homogeneous symmetric lines. Effective utilization of metallic lines for broadband signal transports allows fast and cheap expansion of high-speed data and multimedia services for a large amount of subscribers.

## 6 APPENDIX — ABBREVIATIONS

ADSL	— Asymmetric Digital Subscriber Line
FEXT	— Far End Crosstalk
ANSI	— American National Standard Institute
NEXT	— Near End Crosstalk
AWGN	— Additive White Gaussian Noise
PSD	— Power Spectral Density
ETSI	— European Telecommunications Standards Institute
SNR	— Signal-to-Noise Ratio
xDSL	— "x" Digital Subscriber Line

## REFERENCES

- [1] CUCHRAN, J.—RÓKA, R.: The Analysis of the Feasibilities of the Utilizing of xDSL Technologies in the Access Networks of Slovak Telecom, Final Report of the Project ZoD 1163, Bratislava, February 1999. (in Slovak)
- [2] ETSI — Technical Report ETR 328: Transmission and Multiplexing (TM) — ADSL, November 1996.
- [3] ANSI — Standard T1.413: Network and Customer Installation Interfaces — ADSL, August 1995.
- [4] COOK, J. et al.: The Noise and Crosstalk Environment for ADSL and VDSL Systems, IEEE Communications Magazine, May 1999, pp. 73–78.
- [5] PROAKIS, J. et al.: Communication Systems Engineering, Prentice-Hall, New Jersey, 1994.
- [6] ASLANIS, J. et al.: Achievable Information Rates on Digital Subscriber Loops: Limiting Information Rates with Crosstalk Noise, IEEE Transactions on Communications, Vol. 40, No. 2, February 1992, pp. 361–372.
- [7] WERNER, J.-J.: The HDSL Environment, IEEE Journal on Selected Areas in Communications, Vol. 9, No. 6, August 1991, pp. 785–800.
- [8] KALET, I. et al.: On the Capacity of a Twisted-Wire Pair: Gaussian Model, IEEE Transactions on Communications, Vol. 38, No. 3, March 1990, pp. 379–383.

Received 21 May 2001

**Rastislav Róka** was born in Šaľa, Slovakia on January 27, 1972. He received his MSc degree in telecommunications from the Slovak University of Technology, Bratislava, in 1995. Since 1997, he has worked as a senior lecturer at the Department of Telecommunications, FEI STU, Bratislava. At present, his research activity has been focused on the high-speed signal transport through metallic access networks by means of xDSL technologies using various modulation and coding techniques.