ELECTRONICALLY TUNABLE HIGH-ORDER HIGHPASS FILTERS WITH MINIMUM OF COMPONENTS

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A new method for realization of an electronically tunable high-order admittance function that can be used for the synthesis of non-cascade active frequency filters is introduced. Operational transconductance amplifiers (OTA) are employed as active elements. Electronically controlled high-order highpass filters are presented. As an example, two types of fourth-order Butterworth highpass filters are designed, simulated (using PSPICE) and measured.

Keywords: Admittance function, highpass filter, operational transconductance amplifier (OTA)

1 INTRODUCTION

Active filters based on transconductance amplifiers with a single- or balanced-current output (OTA or BOTA) [1]–[6] have recently received considerable attention. The transconductance gain $g_m$ of the OTA and BOTA can be continuously controlled by an external voltage or current source over several decades, which lends electronic tunability to circuit parameters. It may be emphasized that electronic tunability becomes very important when the circuit is in a variety of design specifications and in the integrated form. Many authors often do not make use of this possibility. A lot of transconductance amplifiers have appeared on the market in the past few years, for example the CA3080 (Intersil), LM13600 and LM13700 (National Semiconductor), LT1228 (Linear Technology), NE5517 (Philips Semiconductors), MAX435 or MAX436 (MAXIM) circuits. The interest of technical community in these elements is evidenced by the offer of manufacturers.

It is possible to show that one-port elements with high-order admittance are suitable for non-cascade active frequency filter synthesis. Two possible realizations of such OTA based one-port elements and also their possible utilization for the tunable fourth-order highpass filter synthesis will be shown below. Results of the proposed filters simulation, on the level of SPICE models, and measurement results on function samples will be shown as well.

2 ADMITTANCE FUNCTION REALIZATION

Suppose that we have to design an analog $N$th-order highpass filter using the non-cascade design procedure. An active frequency filter working in the voltage mode can be realized as a voltage divider or in the current mode as a current divider containing high-order admittance elements, as shown in Fig. 1 and Fig. 2. The transfer function in both cases is given by the following relation:

$$T(p) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{I_{\text{out}}}{I_{\text{in}}} = \frac{Y_2(p)}{Y_1(p) + Y^{(n)}(p)},$$

(1)

The $N$th-order admittance function $Y^{(n)}(p)$ can be expressed as

$$Y^{(n)}(p) = Y^0(p) + Y^1(p) + \ldots + Y^n(p),$$

where

$$Y^0(p) = \frac{1}{pE_0} = \frac{1}{R},$$

$$Y^1(p) = \frac{1}{pE_1} = \frac{1}{pL},$$

$$Y^2(p) = \frac{1}{p^2E_2} = \frac{1}{p^2E}, \quad \text{etc.}$$

Note that the one-port element with 2nd-order admittance is known as a FDNR $E$-element [7].

However, the realization of the above individual high-order immittance elements is not economical. A general procedure for the realization of immittance function employing a relatively small number of active elements was published in [8], [9] and [10]. In the following text we show a similar design procedure for a one-port with admittance function $Y^{(n)}(p)$ on the OTA base.

Designing an $n$th-order highpass filter using eq. (1) we have two possible solutions: either $Y_1(p) = pC_0$ and

$$Y^{(n)}(p) = \frac{1}{pE_0} + \frac{1}{pE_1} + \frac{1}{p^2E_2} + \ldots + \frac{1}{p^{N-1}E_{N-1}}$$

(2)

or $Y_1(p) = G_0$ and

$$Y^{(n)}(p) = \frac{1}{pE_1} + \frac{1}{p^2E_2} + \frac{1}{p^3E_3} + \ldots + \frac{1}{p^N E_N}.$$  

(3)

The main problem is now the synthesis of admittance function $Y^{(n)}(p)$.
The input admittance of the network in Fig. 3a can be written as
\[ Y_{in} = \frac{I_{in}}{V_{in}} = \sum_{i=1}^{n} \frac{I_{i}}{V_{in}} = g_{m1} + \frac{g_{m1}g_{m2}}{pC_{1}} + \frac{g_{m1}g_{m2}g_{m3}}{p^{2}C_{1}C_{2}} + \cdots + \frac{g_{m1}g_{m2}g_{m3} \cdots g_{m(n+1)}}{p^{n}C_{1}C_{2} \cdots C_{n}} \] (4)
and the input admittance of the network in Fig. 3b can be written as
\[ Y_{in}^{(n)}(p) = \frac{I_{in}}{V_{in}} = g_{m(n+1)} + \frac{g_{mn}g_{m(n+1)}}{pC_{n}} + \frac{g_{m(n-1)}g_{m(n+1)}}{p^{2}C_{(n-1)}C_{n}} + \cdots + \frac{g_{m1}g_{m2}g_{m3} \cdots g_{mn}g_{m(n+1)}}{p^{n}C_{1}C_{2} \cdots C_{n}} \] (5)

A disadvantage of the circuit in Fig. 3a is the using of ungrounded passive elements when realizing in the integrated form. The circuit in Fig. 3b, which uses grounded passive elements, does not have this disadvantage and is thus more suitable for monolithic integration.

### 3 ELECTRONICALLY TUNABLE FOURTH-ORDER HIGHPASS FILTER REALIZATION

When realizing the parallel arranged admittance in Fig. 1 by one of the networks in Fig. 3 and choosing \( Y_{1}(p) = pC_{0} \) and assuming \( g_{m1} = g_{m2} = g_{m3} = g_{m4} = g_{m} \) (we consider only the fourth-order highpass filter for illustration), then according to (1) the following formula will be valid for the voltage transfer function:
\[ T(p) = \frac{(p^{4}K_{4})}{(p^{4}K_{4} + g_{m}p^{3}K_{3} + g_{m}^{2}p^{2}K_{2} + g_{m}^{3}pK_{1} + g_{m}^{4})} \] (6)
where we have denoted \( K_{4} = C_{0}C_{1}C_{2}C_{3} \), \( K_{3} = C_{1}C_{2}C_{3} \), \( K_{2} = C_{2}C_{3} \) and \( K_{1} = C_{3} \).

We have obtained fourth-order highpass filters whose cut-off frequency can be set by a simultaneous change in the transconductance gains of all OTA elements by means of control current \( I_{SET} \) (see Fig. 4).

The voltage transfer function of a highpass filter in normalized form \( (s = p/\omega_{3dBc}) \) is given by
\[ T(s) = \frac{s^{4}}{s^{4} + s^{3}c_{41} + s^{2}c_{42} + sc_{43} + c_{44}} \] (7)
where

\[ c_{41} = \frac{g_m}{\omega_{3dBe} C_0}, \quad c_{42} = \frac{g_m^2}{\omega_{3dBe}^2 C_0 C_1}, \]
\[ c_{43} = \frac{g_m^3}{\omega_{3dBe}^3 C_0 C_1 C_2}, \quad c_{44} = \frac{g_m^4}{\omega_{3dBe}^4 C_0 C_1 C_2 C_3}. \]

(8)

3.1 Simulation results

As an example, let us design two highpass filters from Fig. 4 with the Butterworth approximation of transfer function with cut-off frequency in the range from 100 Hz to 100 kHz. The transfer function coefficients taken from the charts for non-cascade design are: \( c_{41} = 2.6131, c_{42} = 3.4142, c_{43} = 2.6131, c_{44} = 1.0000 \). Considering LT1228 circuits [11] instead of OTA elements, it is possible to set the transconductance gain in an interval \( g_m \in (0.01; 10) \) mS. The capacitances of capacitors can be computed from relation (8). We obtain: \( C_0 = 6.09 \) nF, \( C_1 = 12.18 \) nF, \( C_2 = 20.79 \) nF, \( C_3 = 41.59 \) nF.

3.2 Experimental results

LT1228 circuits have also been used for experimental verification. A symmetrical supply voltage of ±5 V has been used for these circuits. Magnitude frequency responses of the tunable highpass filters from Fig. 4 measured by the HP3589A circuit analyzer are shown in Fig. 6. These responses are quite good over a wide frequency range.

In the last LT1228 circuit the buffer which forms its part was used in order to separate the load. The buffers of the other LT1228 circuits were not made use of.

Since all OTAs have to be controlled by identical DC current, an improved multi-output Wilson current mirror with unipolar transistors in Fig. 7 [12] has been proposed. Then the filter can be controlled by a single central current source (eg DAC with current output). Transistor Q2 is supposed to have a fourfold channel width compared with the other transistors. The resistance of resistor R2 is therefore four times less than the resistance of resistor R1. This type of multi-output current mirror exhibits an identical unity transfer to all outputs. This feature cannot be gained with a bipolar multi-output current mirror.

The transconductance temperature coefficient has a value of ~3300 ppm/°C according to the datasheet of LT1228 circuit [11]. This temperature influence on the setting of the transconductance value can be easily compensated using, for example, an LT1004-2.5 reference diode in the source of the control current, as described in the datasheet of LT1228 [11].

As it is necessary to change simultaneously several \( g_m \) transconductances of OTA, when the cut-off frequency of
that the accuracy of all $g_m$ settings is $\pm 1\%$ for each setting of $g_m$, see Fig. 8.

### 3.3 Sensitivity analysis

A detailed sensitivity analysis of the designed filters (Fig. 4 a,b) was made. As the voltage transfers of both filters have the same form, the sensitivity characteristics are also identical.

In the sensitivity computation we will start from the primary voltage transfer of the filters from Fig. 4

$$T(s) = \frac{s^4 C_0 C_1 C_2 C_3}{D}, \quad (10)$$

where

$$D = s^4 C_0 C_1 C_2 C_3 + s^3 C_1 C_2 C_3 g_{m1} + s^2 C_2 C_3 g_{m1} g_{m2} + s C_3 g_{m1} g_{m2} g_{m3} + g_{m1} g_{m2} g_{m3} g_{m4}. \quad (11)$$

The partial general sensitivity functions [13], [14] of the filter transfer function to the individual parameters of passive and active components (capacitors and transconductances $g_m$ of OTA) are

$$S_{C_0}^{T(s)} = -S_{g_{m1}}^{T(s)} = \frac{1}{D} \left( s^3 C_1 C_2 C_3 g_{m1} + s^2 C_2 C_3 g_{m1} g_{m2} + s C_3 g_{m1} g_{m2} g_{m3} + g_{m1} g_{m2} g_{m3} g_{m4} \right)$$

$$= \frac{s^3 c_{41} + s^2 c_{42} + s c_{43} + c_{44}}{s^4 + s^3 c_{41} + s^2 c_{42} + s c_{43} + c_{44}}, \quad (12)$$

$$S_{C_1}^{T(s)} = -S_{g_{m2}}^{T(s)} = \frac{1}{D} \left( s^2 C_2 C_3 g_{m1} g_{m2} + s C_3 g_{m1} g_{m2} g_{m3} + g_{m1} g_{m2} g_{m3} g_{m4} \right)$$

$$= \frac{s^2 c_{42} + s c_{43} + c_{44}}{s^4 + s^3 c_{41} + s^2 c_{42} + s c_{43} + c_{44}}. \quad (13)$$
The worst-case multiparameter relative sensitivity of the transfer magnitude of the filter synthesized with the help of cascade synthesis from two second-order high-pass filters with multifeedback [15] was also made. Figure 10a again for standardized frequency. The resulting cut-off frequency the sensitivity has its maximum. The sensitivity of the filter transfer is mostly concentrated in the cut-off frequency area. In the stop-band area the values of the sensitivities are equal to plus or minus one. But in this area an adequate attenuation of the input signal is sufficient in the realization.

For an easy determination of the worst case of the resulting transfer magnitude influence, it is suitable to know the so-called worst-case multiparameter sensitivity of the transfer magnitude. This sensitivity expresses the maximum influence on the resulting frequency magnitude response, which is caused by the changes in the values of all filter parameters (capacitors and transconductances), as is evident from the definition equation

$$WS_{T}^{(s)} = \sum_{i=1}^{N} |R_{S_{T}^{(s)}_{q_{i}}}|,$$  \hspace{1cm} (16)

where \( q = (q_1, q_1, \ldots, q_N) \) is the vector of the filter parameters and \( N \) is the total number of parameters. The worst-case multiparameter sensitivity of the transfer magnitude for the filter from Fig. 4 is shown in graphs in Fig. 10a, again for standardized frequency. The resulting curve confirms the above facts that the sensitivity in the pass-band is equal to zero and that in the area of the cut-off frequency the sensitivity has its maximum.

To allow comparison with the cascade synthesis, the sensitivity analysis of the fourth-order high-pass filter designed with the help of cascade synthesis from two second-order high-pass filters with multifeedback [15] was also made. Figure 10b gives the worst-case multiparameter sensitivity of the transfer magnitude of the filter. From the comparison of the curves it follows that the sensitivity of the cascade filter is high in the pass-band area, in contrast to non-cascade synthesis with sensitivity equal to zero. The slightly higher sensitivity in the area of the cut-off frequency of the non-cascade synthesis usually does not matter.
The limits of the resulting frequency magnitude response are easy to determine from the worst-case multiparameter sensitivity. The frequency magnitude response of the designed filters including maximum limits for all filter parameters is given in Fig. 11. Figure 12 gives for comparison also the maximum limits for 5% tolerances of all filter parameters of the cascade high-pass filter. The graphs only confirm the results that the sensitivity of the non-cascade variant is lower in particular in the pass-band area.

4 CONCLUSIONS

A method of realizing electronically tunable high-order admittance elements suitable for non-cascade synthesis of active frequency filters has been demonstrated. Two such one-port networks using modern OTA elements have been presented. One of them employs grounded passive elements. The network with grounded passive elements is more suitable for monolithic integration. It has been shown that by placing any of the one-port networks mentioned above in the frequency-dependent voltage divider we obtain an electronically tunable highpass filter in the voltage mode. We obtain an electronically tunable highpass filter in the current mode if it is appropriately located in the frequency-dependent current divider. The proposed structures are suitable only for all-pole filters (Butterworth, Chebychev). The electronically tunable highpass filters have been created, analysed for sensitivity and measured to verify theoretical results. The simulated and measured results are in excellent agreement with theoretical predictions. From the viewpoint of the sensitivity it can be said that the designed solution of the high-pass filter is less sensitive than the cascade solution, in particular in the pass-band area, where the sensitivity is equal to zero.

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References


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