Journal of ELECTRICAL ENGINEERING, VOL. 54, NO. 3-4, 2003, 78–82

PARAMETER IDENTIFICATION OF STATIC AND DYNAMIC NONLINEAR SYSTEMS WITH SATURATIONS

Jozef Vörös *

The paper deals with the parameter identification of static and dynamic (Hammerstein-type) systems having multisegment piecewise-linear characteristics with saturations. A new form of static multisegment nonlinearity representation with imposed internal variables allows an iterative estimation of nonlinearity parameters using measured input and output data. To demonstrate the feasibility of the identification method, more illustrative examples are included.

K e y w o r d s: nonlinear systems, saturation, Hammerstein model, identification.

1 INTRODUCTION

Piecewise-linear characteristics allow to describe processes with different gains in different input intervals and they are sometimes used as a general tool to approximate nonlinear functions. Static piecewise-linear characteristics are often encountered in control systems either alone or in cascades with linear dynamic systems (Kalaš *et al*, 1985). Piecewise-linear nonlinearities with saturations can be considered as a special case of general multisegment piecewise-linear characteristics, if the gain is zero at one or more intervals of the input domain.

Recently, a new form of multisegment piecewise-linear nonlinearity description has been proposed and applied in the identification of static and dynamic systems (Vörös, 2002). This form can be used for modeling and identification of systems with saturations and zero-slopes can occur in any subdomain of inputs. The estimation of parameters characterizing the nonlinearity can be performed on the basis of input and output data and requires only minimum a priori information on the domain partitions of unknown characteristics with saturations.

2 MULTISEGMENT PIECEWISE–LINEAR NONLINEARITY

The output $x(t)$ of assumed multisegment piecewise-linear nonlinearity according to Fig. 1 can be written as (Tao, Tian, 1998):

$$x(t)=\begin{cases} m_{R1}u(t)\,, & \text{if } 0\le u(t)\le D_{R1}, \\ m_{R2}[u(t)-D_{R1}]+m_{R1}D_{R1}, & \text{if } u(t)>D_{R1}, \end{cases} \tag{1}$$

$$x(t)=\begin{cases} m_{L1}u(t)\,, & \text{if } D_{L1}\le u(t)<0\,, \\ m_{L2}[u(t)-D_{L1}]+m_{L1}D_{L1}, & \text{if } u(t)<D_{L1}, \end{cases} \tag{2}$$

where $|m_{R1}|<\infty$, $|m_{R2}|<\infty$ are the corresponding segment slopes and $0\le D_{R1}<\infty$ is the constant for the positive inputs, while $|m_{L1}|<\infty$, $|m_{L2}|<\infty$ are the corresponding segment slopes and $-\infty<D_{L1}\le 0$ is the constant for the negative inputs.

According to the approach proposed in (Vörös, 2002) the above characteristic can be described in the following input-output form:

$$\begin{aligned} x(t) = m_{R1}h[-u(t)]u(t) + (m_{R2}-m_{R1})h[D_{R1}-u(t)]u(t) \\ - D_{R1}f_1(t) + m_{L1}h[u(t)]u(t) \\ + (m_{L2}-m_{L1})h[u(t)-D_{L1}]u(t) - D_{L1}f_2(t)\,, \end{aligned} \tag{3}$$

where the internal variables

$$f_1(t)=f_1[u(t)]=(m_{R2}-m_{R1})h[D_{R1}-u(t)]\,, \tag{4}$$

$$f_2(t)=f_2[u(t)]=(m_{L2}-m_{L1})h[u(t)-D_{L1}]\,, \tag{5}$$

are generally unmeasurable and the switching function $h(\cdot)$ defined as follows:

$$h(\alpha)=\begin{cases} 0\,, & \text{if } \alpha>0\,, \\ 1\,, & \text{if } \alpha<0\,, \end{cases} \tag{6}$$

switches between two sets of values, *ie*, $(-\infty,\alpha)$ and (α,∞).

Now, the piecewise-linear characteristic with saturation(s) can be represented by the above description, if the parameters $m_{R2}=0$ and/or $m_{L2}=0$ for positive and/or negative inputs, respectively. As in all these cases the parameters appear in the above equation as the subtrahends in the subtraction terms where the corresponding minuends m_{R1} and/or m_{L1} are always nonzero, no zero parameter will occur in the description.

The proposed description for the multisegment piecewise-linear characteristics (3) can be generalized as follows. After defining the internal variables:

$$f_{1,i}(t)=(m_{R,i}-m_{R,i-1})h[D_{R,i}-u(t)]\,, \tag{7}$$

$$f_{2,j}(t)=(m_{L,j}-m_{L,j-1})h[u(t)-D_{L,j}]\,, \tag{8}$$

* Slovak Technical University, Faculty of Electrical Engineering and Information Technology, Ilkovičova 3, 812 19, Bratislava, Slovakia, e-mail: jvoros@elf.stuba.sk

ISSN 1335-3632

$i = 1, \ldots, n_R$ and $j = 1, \ldots, n_L$, where $|m_{R,i}| < \infty$, $|m_{L,j}| < \infty$ are the segment slopes, $0 \le D_{R,i} < D_{R,i+1} < \infty$ are the constants representing the partition for the positive inputs, while $-\infty < D_{L,j+1} < D_{L,j} \le 0$ are the constants representing the partition for the negative inputs, the output equation for the general multisegment piecewise-linear characteristics can be written as

$$x(t) = \sum_{i=1}^{n_R} \Big\{ (m_{R,i} - m_{R,i-1}) h[D_{R,i-1} - u(t)] u(t) - D_{R,i-1} f_{1,i}(t) \Big\} + \sum_{j=1}^{n_L} \Big\{ (m_{L,j} - m_{L,j-1}) h\big[u(t) - D_{L,j-1}\big] u(t) - D_{L,j-1} f_{2,j}(t) \Big\}, \quad (9)$$

assuming

$$0 \le D_{R,1} < D_{R,2} < \cdots < D_{R,nR} < \infty, \quad (10)$$

$$-\infty < D_{L,nL} < \cdots < D_{L,2} < D_{L,1} \le 0. \quad (11)$$

Also in this case, if the slopes $m_{R,nR}$ and $m_{L,nL}$ are put to zero, then the characteristic is of (end) saturation type. Note, that the saturation can generally appear at any interval of the domain partition.

The estimation of the parameters characterizing the nonlinearity with saturations can be performed using the iterative method with the internal variables' estimations proposed in (Vörös, 2002). It means that an error criterion based on (3) or (9) is repeatedly minimized using the previous estimates of parameters in the switching functions and in the estimates of internal variables $f_1(t)$, $f_2(t)$ or $f_{1,i}(t)$, $f_{2,j}(t)$, resulting from (4)–(5) or (7)–(8).

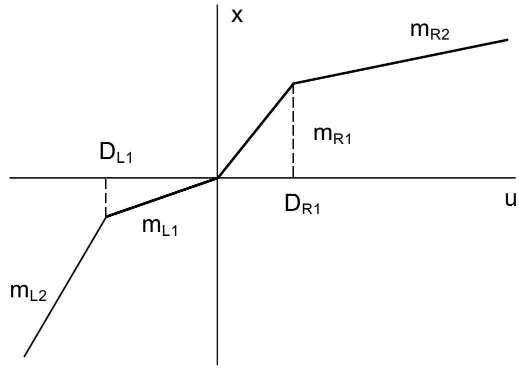

Fig. 1. Piecewise-linear characteristic

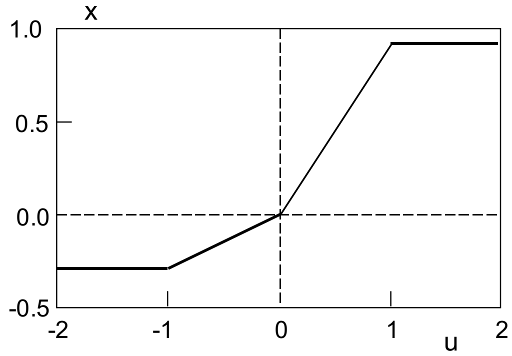

Fig. 2. Example 1 — saturation characteristic

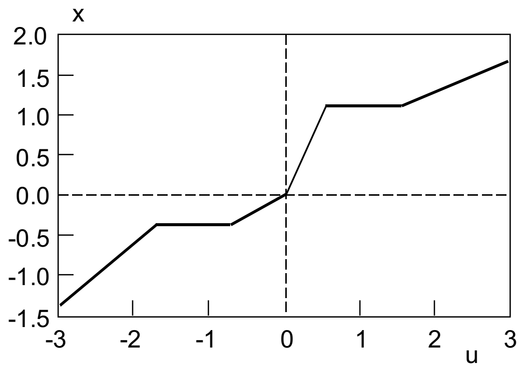

Fig. 3. Example 2 — middle segment saturation characteristic

3 HAMMERSTEIN MODEL WITH SATURATION

The Hammerstein model is given by the cascade connection of a static nonlinearity block followed by a linear dynamic system (Haber, Keviczky, 1999). The nonlinear characteristic with saturations can be described by (3). The difference equation model of the linear block representing the model output part can be given as

$$B(q^{-1}) y(t) = A(q^{-1}) x(t), \quad (12)$$

where $x(t)$ and $y(t)$ are the inputs and outputs, respectively, $A(q^{-1})$ and $B(q^{-1})$ are scalar polynomials in the unit delay operator q^{-1}

$$A(q^{-1}) = a_0 + a_1 q^{-1} + \cdots + a_m q^{-m}, \quad (13)$$

$$B(q^{-1}) = 1 + b_1 q^{-1} + \cdots + b_n q^{-n}. \quad (14)$$

The linear block input is identical with the nonlinear block output. However, a direct substitution of (3) or (9) into (12) would result in a very complex expression, which is nonlinear in both the parameters and data.

To simplify this problem the key term separation principle can be applied in (12). Assuming $a_0 = 1$ (this is always possible in the given model), (12) can be rewritten as follows:

$$y(t) = x(t) + [A(q^{-1}) - 1] x(t) + [1 - B(q^{-1})] y(t), \quad (15)$$

and then (3) can be half-substituted only for the separated $x(t)$ giving

$$y(t) = m_{R1} h[-u(t)] u(t) + (m_{R2} - m_{R1}) h[D_{R1} - u(t)] u(t) - D_{R1} f_1(t) + m_{L1} h[u(t)] u(t) + (m_{L2} - m_{L1}) h[u(t) - D_{L1}] u(t) - D_{L1} f_2(t) + [A(q^{-1}) - 1] x(t) + [1 - B(q^{-1})] y(t). \quad (16)$$

Table 1.

Iter.	m_{R1}	m_{L1}	$m_{R2}-m_{R1}$	$m_{L2}-m_{L1}$	D_{R1}	D_{L1}	$J\,(MSE)$
1	0.9000	0.3000	−0.2639	−0.1041	–	–	0.54484430538554
2	0.9000	0.3000	−0.5219	−0.1798	1.2561	−1.0156	0.02779423276122
3	0.8765	0.2998	−0.8765	−0.2998	1.7245	−1.6689	0.05140331001933
4	0.7475	0.2647	−0.7475	−0.2647	1.0269	−1.0006	0.00749939351759
5	0.9000	0.3000	−0.9000	−0.3000	1.2040	−1.1335	0.00598525896083
6	0.8765	0.2930	−0.8765	−0.2930	1.0000	−1.0000	0.00021710499296
7	0.9000	0.3000	−0.9000	−0.3000	1.0269	−1.0237	0.00015034379850
8	0.9000	0.2998	−0.9000	−0.2998	1.0000	−1.0000	0.00000000860216
True	0.9000	0.3000	−0.9000	−0.3000	1.0000	−1.0000	

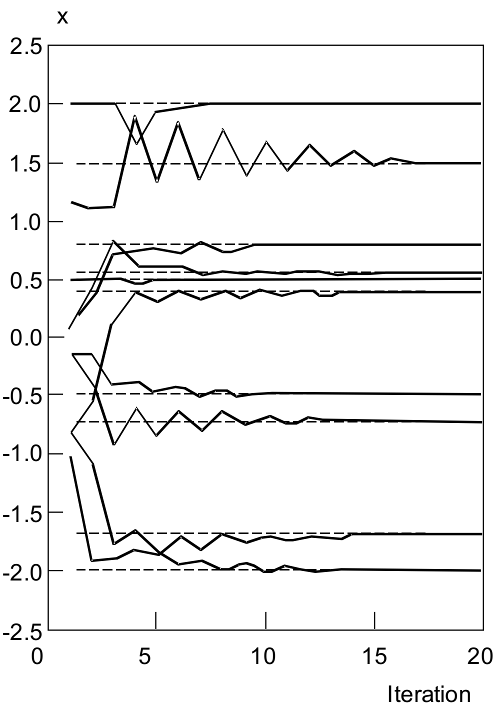

Fig. 4. Example 2 — static model parameter estimation

The equation (16) and those of (3)–(5), defining the internal variables $x(t)$, $f_1(t)$, and $f_2(t)$, represent a special form of the Hammerstein model with saturations. The model has a minimum number of parameters and all of them enter the expressions linearly, except D_{R1} and D_{L1}, which appear both linearly and nonlinearly.

The Hammerstein model parameter estimation can be performed using the iterative method with internal variable estimation (Vörös, 2002). In this case an error criterion based on (16) is repeatedly minimized using the previous estimates of parameters and the estimates of internal variables $x(t)$, $f_1(t)$, $f_2(t)$, resulting from (3)–(5).

In the same way, the general case of multisegment piecewise-linear nonlinearity can be incorporated into the Hammerstein model. After the half-substitution of (9) into (15) the resulting form of model can be used for the identification of more complex dynamic systems with saturations.

4 EXAMPLES

The proposed method for the parameter identification of nonlinear static and dynamic systems having multi-segment piecewise-linear characteristics with saturations was implemented and tested by means of MATLAB. Several systems were simulated and the estimation of the parameters and the internal variables were carried out on the basis of input and output data records using the least squares method, *ie*, the function 'arx' from System Identification Toolbox was used for the parameter estimations.

Static systems with saturations

To illustrate the process of parameter estimation, let us assume the asymmetric saturation characteristic shown in Fig. 2. The system outputs were generated for the parameters $m_{R1}=0.9$, $D_{R1}=1.0$, $m_{L1}=0.3$, $D_{L1}=1.0$, $m_{R2}=0$ and $m_{L2}=0$, using random inputs uniformly distributed in the interval $[-2;2]$.

The initial values of the parameters were chosen zero, except ${}^0D_{R1}=0.1$ and ${}^0D_{L1}=-0.1$, which were used for computing the initial values of internal variables ${}^0f_1(t)$ and ${}^0f_2(t)$ to start up the iterative algorithm. On the basis of 50 samples of inputs and outputs the iterative method with internal variable estimation was applied, *ie*, the mean squares error was repeatedly minimized for the parameters with the internal variables generated by (4)–(5) using the preceding estimates of corresponding parameters. The parameter estimates are included in Table 1, where the true values of parameters are in the bottom row and the values of output error (MSE) are in the last column. Note that in the first iteration D_{R1} and D_{L1} were not estimated.

The next example illustrates the use of general description (9) for the special characteristic shown in Fig. 3 having "middle segment saturations" with the following parameters:

$p_1 = m_{R1} = 2.0$ $\quad p_6 = D_{L1} = -0.7$

$p_2 = m_{L1} = 0.5$ $\quad p_7 = m_{R3} - m_{R2} = 0.4$

$p_3 = m_{R2} - m_{R1} = -2.0$ $\quad p_8 = m_{L3} - m_{L2} = 0.8$

$p_4 = m_{L2} - m_{L1} = -0.5$ $\quad p_9 = D_{R2} = 1.5$

$p_5 = D_{R1} = 0.55$ $\quad p_{10} = D_{L2} = -1.7$

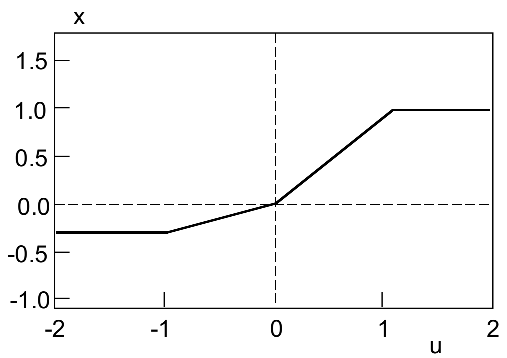

Fig. 5. Example 3 — nonlinear block characteristic

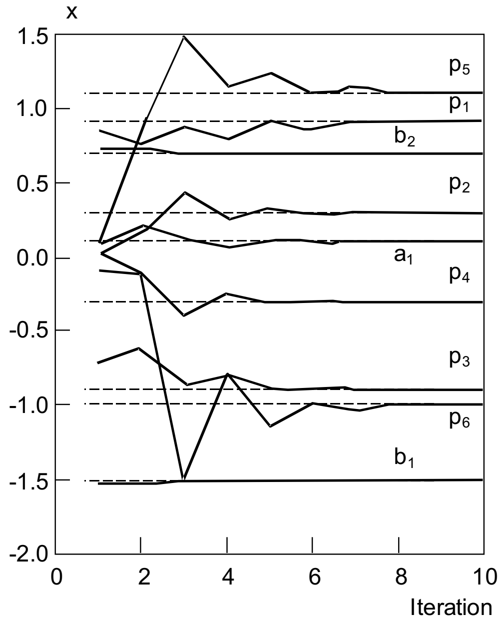

Fig. 6. Example 3 — Hammerstein model parameter estimation

The outputs of this nonlinear system were simulated for uniformly distributed random inputs with $|u(t)| < 4$. The initial values of the parameters were chosen zero, except ${}^0D_{R1} = 0.1$, ${}^0D_{L1} = -0.1$, ${}^0D_{R2} = 1$, ${}^0D_{L2} = -1$, which were used to compute the initial values of internal variables ${}^0f_{1,i}(t)$ and ${}^0f_{2,j}(t)$ to start up the iterative algorithm. To ensure the inequalities (10)–(11) the following limits were chosen: ${}^0D_{R1} < D_{R1} < {}^0D_{R2} < D_{R2} < 2$, ${}^0D_{L1} > D_{L1} > {}^0D_{L2} > D_{L2} > -2$. On the basis of 300 samples of inputs and outputs the iterative method with internal variable estimations was applied. The process of parameter estimation finished after about 18 iterations reaching MSE less than 0.0000001. Its graphical plot in Fig. 4 shows good convergence of the estimates of the parameters to their true values (dashed lines). The parameters appear in the following top-down order: $p_1, p_9, p_8, p_5, p_2, p_7, p_4, p_6, p_{10}, p_3$.

Dynamic systems with saturations

The following examples show the parameter estimation process for the Hammerstein systems with two different multisegment piecewise-linear nonlinearities. In the first example the linear dynamic system was given by the difference equation

$$y(t+1) = x(t) + 0.1x(t-1) + 1.5y(t) - 0.7y(t-1),$$

and the nonlinear block was given by the characteristics shown in Fig. 5, *ie*, the nonlinearity was described by the following parameters:

$$p_1 = m_{R1} = 0.9 \qquad p_4 = m_{L2} - m_{L1} = -0.3$$
$$p_2 = m_{L1} = 0.3 \qquad p_5 = D_{R1} = 1.1$$
$$p_3 = m_{R2} - m_{R1} = -0.9 \quad p_6 = D_{L1} = -1.0$$

The identification was carried out with 300 samples of uniformly distributed random inputs with $|u(t)| < 2$, and the simulated outputs. The mean squares error based on (16) with the estimated internal variables was repeatedly minimized for the model parameters. The initial values of the parameter estimates were chosen zero, except ${}^0D_{R1} = 0.1$ and ${}^0D_{L1} = -0.1$, which were used to evaluate the initial values of internal variables ${}^0f_1(t)$, ${}^0f_2(t)$ and ${}^0x(t)$. The process of parameter estimation of the Hammerstein model is graphically shown in Fig. 6. The parameters' estimates are equal to the true values after about 8 iterations.

The second example illustrates the use of the Hammerstein model with the general description (9). The simulated system consisted of the same linear dynamic system as in the first example and the nonlinear static block with the asymmetric multisegment piecewise-linear characteristic with saturations shown in Fig. 7 was given by the following parameters:

$$p_1 = m_{R1} = 0.9 \qquad p_6 = D_{L1} = -0.5$$
$$p_2 = m_{L1} = 0.2 \qquad p_7 = m_{R3} - m_{R2} = -0.2$$
$$p_3 = m_{R2} - m_{R1} = -0.7 \quad p_8 = m_{L3} - m_{L2} = -1.0$$
$$p_4 = m_{L2} - m_{L1} = 0.8 \qquad p_9 = D_{R2} = 1.6$$
$$p_5 = D_{R1} = 0.6 \qquad p_{10} = D_{L2} = -1.2$$

The identification was carried out with 1000 samples, using random inputs with $|u(t)| < 4$. The initial values of the parameters were chosen zero, except ${}^0D_{R1} = 0.1$, ${}^0D_{L1} = -0.1$, ${}^0D_{R2} = 1$, ${}^0D_{L2} = -1$, which were used to compute the initial values of internal variables ${}^0f_{1,i}(t)$, ${}^0f_{2,j}(t)$ and ${}^0x(t)$ to start up the iterative algorithm. The process of parameter estimation is graphically shown in Fig. 8 (the order of parameters: $p_9, p_1, p_4, b_2, p_5, p_2, a_1, p_7, p_6, p_3, p_8, p_{10}, b_1$). Evidently, the model parameters' estimates are almost equal to the true ones (dashed lines) after about 20 iterations.

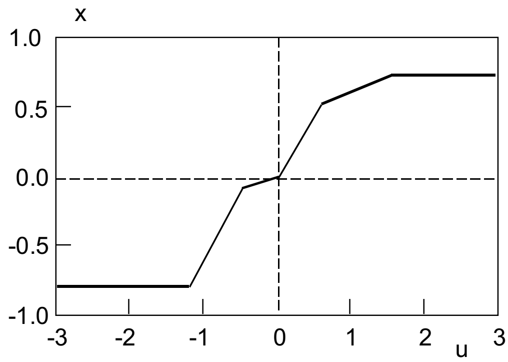

Fig. 7. Example 4 — nonlinear block characteristic

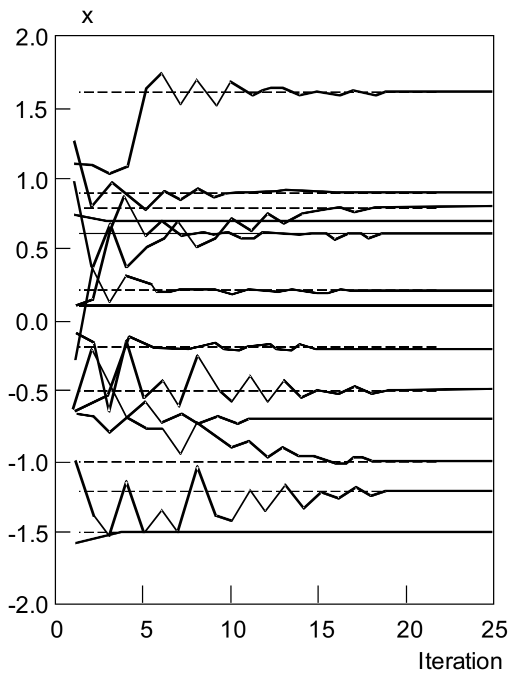

Fig. 8. Example 4 — Hammerstein model parameter estimation

5 CONCLUSION

A new approach to the parameter identification of nonlinear static and dynamic Hammerstein-type systems having piecewise-linear nonlinearities with saturations has been presented. The representation of static characteristics with saturations is considered as a special case of the recently proposed universal form of multisegment piecewise-linear characteristic representation. This form is also used in the nonlinear static block of Hammerstein model.

For the model parameters' estimation an iterative method with internal variables' estimation is applied. A priori knowledge is restricted to the limits for $D_{R,i}$ and $D_{L,j}$, *ie*, the constants determining the partition of nonlinearity domain. The presented examples of identification process demonstrate the feasibility and good convergence properties of the proposed technique for static and dynamic systems with different types of saturation.

Acknowledgement

The author gratefully acknowledges financial support from the Slovak Scientific Grant Agency (VEGA).

References

[1] HABER, R.—KEVICZKY, L.: Nonlinear System Identification — Input-Output Modeling Approach, Kluwer Academic Publishers, Dorchrecht/Boston/London, 1999.

[2] KALAŠ, V.—JURIŠICA, L.—ŽALMAN, M.—ALMÁSSY, S.—SIVIČEK, P.—VARGA, A.—KALAŠ, D.: Nonlinear and Numerical Servosystems, Alfa/SNTL, Bratislava, 1985. (in Slovak)

[3] TAO, G.—TIAN, M.: Discrete-Time Adaptive Control of Systems with Multisegment Piecewise-Linear Nonlinearities, IEEE Trans. Automatic Control **43** No. 5 (1998), 719–723.

[4] VÖRÖS, J.: Modeling and Parameter Identification of Systems with Multisegment Piecewise-Linear Characteristics, IEEE Trans. Automatic Control **47** No. 1 (2002), 184–188.

Received 25 July 2002

Jozef Vörös (Ing, PhD) was born in Hurbanovo on July 9, 1949. He graduated in automatic control from the Faculty of Electrical Engineering of the Slovak Technical University, Bratislava in 1974 and received his PhD degree in control theory from the Institute of Technical Cybernetics of the Slovak Academy of Sciences, Bratislava in 1983. Since 1992 he has been with the Faculty of Electrical Engineering and Information Technology at the Slovak University of Technology, in Bratislava, where he is acting as a senior research scientist in the Department of Automation and Control. His recent research interests include the area of modeling of 2D and 3D objects in robotics using quadtree and octree representations. He is also interested in the analysis of nonlinear systems.