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DISCRETE–TIME VARIABLE STRUCTURE CONTROLLER SYNTHESIS FOR THIRD ORDER OBJECTS WITH FINITE ZERO USING DELTA TRANSFORM

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Analyzed in this paper is the variable structure system (VSS) to control the third order object with finite zero. First, a discrete mathematical model of the system over the canonical space, using the delta transform, was given for the discussed VSS system. Then, decomposition of the canonical space to subspaces with and without control by introducing the output variable delta transform was carried out. Finally, based on the quasi-sliding mode existence conditions, relations to provide this mode over the canonical subspace with control are derived. One illustrative example is used to demonstrate the discrete-time variable structure (DTVS) controller synthesis procedure using the delta transform.

K e y w o r d s: discrete-time variable structure system, delta operator, canonical subspace, quasi-sliding mode

1 INTRODUCTION

The discrete-time variable structure systems (DTVSS) are in a studying phase. Dote and Hoft [1] are the first to define the reaching and sliding mode existence conditions on the switching hyperplane $g(kT) = 0$ into the form: $[g(k+1)T) - g(kT)]g(kT) < 0$. Milosavljević [2] demonstrates that this condition is necessary, but not also sufficient, for the quasi-sliding mode existence. Szarpturk *et al* [3] define a necessary and sufficient condition in the following form: $\big|g\big((k+1)T\big)\big| < |g(kT)|$ and Furuta [4] in the form:

$$\Delta v(kT) < 0 \Leftrightarrow v\big((k+1)T\big) - v(kT) < 0\,; \qquad v(kT) = \frac{1}{2}[g(kT)]^2\,. \tag{1}$$

Mention should be made here that Gao *et al* [5] deviate from the usual procedure to first define the condition of reaching and sliding mode existence and then to determine the control. First, they define the “reaching law” in a discrete form:

$$g\big((k+1)T\big) - g(kT) = -qT - \xi T\,\mathrm{sgn}\big(g(kT)\big)\,, \qquad q>0\,,\ \xi>0\,,\ 1-qT>0\,,$$

which provides all the basic features of the sliding mode and then define the control. Koshkouei and Zinober [6] proposed discrete-time sliding mode controllers with a lattice-wise hyperplane, where sufficient conditions for the existence of the sliding mode are given by:

$$g^+\big((k+1)T\big) < g^+(kT)\,, \quad g^-\big((k+1)T\big) > g^-(kT)\,.$$

Misawa [7] suggested to use a boundary layer and proposed a discrete-time sliding mode control, which ensures the attractiveness and invariance of the boundary layer. Some DTVS controllers were designed with discretizing continuous-time variable structure controllers [8] or with observer [9]. Furuta and Pan [10] proposed a PR-sliding sector for a lazy control and a chattering free controller. Finally, in [11] and [12], they defined the invariant PR-sliding sector and proposed the DTVS controller with the invariant sliding sector.

There were no attempts in the stated papers in the field of DTVSS to analyze the possibilities of implementing sliding modes (quasi-sliding modes) for objects with finite zeros. The problem of implementing sliding modes in analogue VSS, for such objects, has been discussed in several papers, the results of which have been summed up in monographs [13, 14]. It was pointed out there that due to the differentiable features of the object the sliding mode could not be implemented by the control suffering from break, which is characteristic of VSS. Because of that, two methods of implementing VSS with sliding operating modes were proposed:

(i) the control break signal to be passed through the first-order low-pass filters cascade [14, 15],

(ii) to decompose the starting system to subsystems with and without control by introducing the output variable differentials so that the sliding mode can be organized in the subspace with control [13].

This paper will make an attempt to define a new and simple DTVS controller synthesis. The first part of the paper provides a system mathematical model in the controllable canonical subspace, using the delta transform for finding the object discrete transfer function. The second part of the paper determines conditions of the quasi-sliding mode existence on the chosen switching hyperplane using a Lyapunov function. Then, a method is proposed to improve the DTVSS synthesis and the DTVS controller robustness to the object parameters uncertainties (matching conditions). At the end of the paper, an illustrative example to control the third order object with finite zero is given.

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2 PROBLEM STATEMENT

Let us have a continuous-time controlled object controllable, observable and minimum phase. It may be described by the state-space model in normal controllable canonical form by triplet $(\mathbf{A}, \boldsymbol{b}, \boldsymbol{d})$.

$$\dot{\boldsymbol{x}}(t) = \mathbf{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t)\,, \quad x \in \Re^3\,, \in \Re\,,$$
$$y(t) = \boldsymbol{d}\boldsymbol{x}(t)\,, \quad y \in \Re\,.$$

The equivalent discrete-time state-space model may be obtained by different methods. We will choose the delta transform for the following reasons:

(i) its simplicity,
(ii) unchanged form of the mathematical model in continuous-time and discrete-time,
(iii) robustness of the VSS algorithms to the object parameters uncertainties,
(iv) easier DTVS controller synthesis.

Let sampling time (T) be chosen so that the discrete-time model of the given object will be minimum phase. Its delta model is:

$$\delta\mathbf{x}(kT) = \mathbf{A}_\delta(T)\mathbf{x}(kT) + \boldsymbol{b}_\delta(T)u(kT)\,, \; \mathbf{x} \in \Re^3\,, \; u \in \Re\,,$$
$$y(kT) = \boldsymbol{d}_\delta(T)\mathbf{x}(kT)\,, \quad y \in \Re\,. \tag{2}$$

δ transform is the Euler approximation of difference. Here $\delta x(kT)$ stands for first difference:

$$\delta x(kT) = \frac{x\big((k{+})T\big) - x(kT)}{T}\,, \; T \neq 0\,.$$

. Since the pair $(\mathbf{A}, \boldsymbol{b})$ is controllable and $(\mathbf{A}_\delta(T), \boldsymbol{b}_\delta(T)$ are analytic functions of T, the pair $\big(\mathbf{A}_\delta(T), \boldsymbol{b}_\delta(T)\big)$ is controllable for almost all choices of T. Matrices $\mathbf{A}$ and $\mathbf{A}_\delta(T)$ are square 3×3 matrices in companion form with elements a_i and $a_i(T)$, $i = 0, 1, 2$ respectively, where

$$a_0(T) = 1 + \sum_{i=0}^{2}(-1)^i a_i T^{3-i},$$
$$a_1(T) = 3 + \sum_{i=0}^{2}(-1)^i a_i\, T^{3-i}, \;\; a_2(T) = -3 + a_2 T\,.$$

Vectors $\boldsymbol{b}$ and $\boldsymbol{b}_\delta(T)$ are same ($\boldsymbol{b} = \boldsymbol{b}_\delta(T) = [0 \;\; 0 \;\; 1]^\top$). Vector $\boldsymbol{d}$ is a three-dimensional vector with two nonsingular elements. Elements of three-dimensional vector $\boldsymbol{d}_\delta(T)$ are obtained by the following relations:

$$d_0(T) = \sum_{i=0}^{1}(-1)^i d_i T^{3-i}, \;\; d_1(T) = d_1 T^2\,.$$

The difference $p = n - m = 3 - 1 = 2$ will be called the system relative degree. Using the method given in [13] for the continuous-time systems, successively finding p times the first difference of the output variable $y(kT)$, the canonical subspace in the following form:

$$\begin{aligned}
\delta y^{(1)}(kT) &= \boldsymbol{d}_\delta(T)[\mathbf{A}_\delta(T) - \delta\mathbf{I}]\boldsymbol{x}(kT)\,,\\
\delta y^{(0)}(kT) &= y(kT)\,,\\
\delta y^{(2)}(kT) &= \boldsymbol{d}_\delta(T)[\mathbf{A}_\delta(T) - \delta\mathbf{I}]^2\boldsymbol{x}(kT)\\
&\quad + \boldsymbol{d}_\delta(T)[\mathbf{A}_\delta(T) - \delta\mathbf{I}]\boldsymbol{b}_\delta(T)u(kT)
\end{aligned} \tag{3}$$

will be composed. For simplicity, in the future explanation $\boldsymbol{d}_\delta(T)$, $\boldsymbol{b}_\delta(T)$ and $\mathbf{A}_\delta(T)$ will be $\boldsymbol{d}_\delta$, $\boldsymbol{b}_\delta$ and $\mathbf{A}_\delta$, respectively.

The aim of the control system synthesis is to select control $u(kT)$ so that for any arbitrary initial condition the stable discrete-time sliding mode on the hyperplane

$$g(kT) = \boldsymbol{c}\boldsymbol{y}(kT)\,, \tag{4}$$

in the two-dimensional subspace will occur, where

$$\boldsymbol{c} = [c \;\; 1]\,, \quad \boldsymbol{y}(kT) = \Big[y(kT) \;\; \delta y^{(1)}(kT)\Big]\,.$$

3 SYNTHESIS OF THE DTVS CONTROLLER

The system type regulator ($r = \text{const}$) is analyzed in this paper. Let the sliding hyperplane over the canonical subspace be given by relation (4). To choose the sliding hyperplane parameter c, let us determine the equivalent control $u_{eq}(kT)$ from the condition [4] for the system to remain on the sliding hyperplane $g\big((k+1)T\big) = g(kT)$ for each k, that is, from the condition that: $g(kT) = 0$, $\delta g(kT) = 0$. Finding the first difference $\delta g(kT)$ of the expression (4), then substituting the relation (3) and rearranging:

$$\begin{aligned}
\delta g(kT) &= \big[c\boldsymbol{d}_\delta(\mathbf{A}_\delta - \delta\mathbf{I}) + \boldsymbol{d}_\delta(\mathbf{A}_\delta - \delta\mathbf{I})^2\big]\,\mathbf{x}(kT)\\
&\quad + \boldsymbol{d}_\delta(\mathbf{A}_\delta - \delta\mathbf{I})\boldsymbol{b}_\delta u(kT)
\end{aligned} \tag{5}$$

is obtained. The expression in square brackets of relation (5) is a 3-dimensional vector which will be denoted $\mathbf{g}_{eq}$. Let $h = \boldsymbol{d}_\delta(\mathbf{A}_\delta - \delta\mathbf{I})\boldsymbol{b}_\delta$, then relation (5) becomes: $\delta g(kT) = \mathbf{g}_{eq}\mathbf{x}(kT) + hu(kT)$. The equivalent control is $u_{eq}(kT) = -h^{-1}\mathbf{g}_{eq}\mathbf{x}(kT)$.

Now, the discrete-time system model (2) with the equivalent control assumes the following form:

$$\begin{aligned}
\delta\mathbf{x}(kT) &= \mathbf{A}_\delta\mathbf{x}(kT) + \boldsymbol{b}_\delta u_{eq}(kT)\\
&= \big(\mathbf{A}_\delta - \boldsymbol{b}_\delta h^{-1}\mathbf{g}_{eq}\big)\mathbf{x}(kT) = \mathbf{A}_{\delta\, eq}\mathbf{x}(kT)\,.
\end{aligned}$$

Parameter c of the sliding hyperplane (4) is determined in such a way that the system

$$\begin{aligned}
\mathbf{x}(kT) &= A_{\delta\, eq}\mathbf{x}(kT)\,,\\
0 &= \overline{\boldsymbol{c}}\boldsymbol{x}(kT)\,,\\
\overline{\mathbf{c}} &= [\overline{c}_1 \;\; \overline{c}_2 \;\; \overline{c}_3] = \boldsymbol{c}\Big[\boldsymbol{d}_\delta^\top \;\; \big(\boldsymbol{d}_\delta(\mathbf{A}_\delta - \delta\mathbf{I})\big)^\top\Big]^\top,
\end{aligned}$$

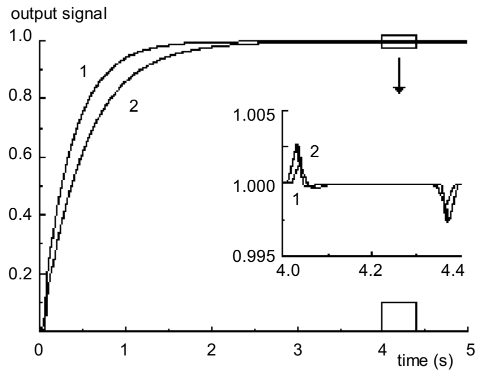

Fig. 1. Step responses of the system for parameters rated values of the object and for the case without (1) or with (2) the proportional controller

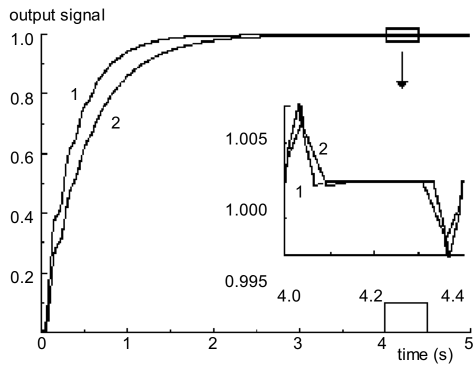

Fig. 2. Step responses of the system for parameters borderline values of the object and for the case without (1) or with (2) the proportional controller

is stable over the complete canonical space (over the subspaces both with and without control), that is, that the roots of the system characteristics equation $\det(\delta \mathbf{I} - \mathbf{A}_{\delta\, eq}) = 0$ are modulo ≤ 1.

To determine control $u(kT)$, the same idea as that in [15] is used. Starting from relations (3), first the expression for $g\big((k+1)T\big)$ is found in the following form:

$$g\big((k+1)T\big) = [c\boldsymbol{d}_\delta \mathbf{A}_\delta + \boldsymbol{d}_\delta(\mathbf{A}_\delta - \delta\mathbf{I})\mathbf{A}_\delta]\,\mathbf{x}(kT) + \boldsymbol{d}_\delta(\mathbf{A}_\delta - \delta\mathbf{I})\boldsymbol{b}_\delta u(kT)\,. \quad (6)$$

The expression in square brackets of relation (6) is a three-dimensional vector of types which will be denoted $-\boldsymbol{g}$. Then, relation (6) becomes:

$$g\big((k+1)T\big) = -\mathbf{g}\mathbf{x}(kT) + hu(kT)\,. \quad (7)$$

To fulfil the condition $g\big((k+1)T\big) = 0$, it is necessary that:

$$u(kT) = h^{-1}\mathbf{g}\mathbf{x}(kT)\,. \quad (8)$$

. Bearing this in mind, the switching part should also be introduced into the expression (8) for $u(kT)$ to provide convergence of the system state trajectories towards the sliding hyperplane.

To determine the control switching part $u(kT)$, the necessary and sufficient condition of the reaching and sliding mode existence, defined by Szarpturk *et al* [3] or Furuta [4], can be used. Starting from condition (1), Theorem 1 is reached:

THEOREM 1. *System (3) is stable if the control*

$$u(kT) = h^{-1}\mathbf{g}\mathbf{x}(kT) + h^{-1}\mathbf{f}\mathbf{x}(kT) + h^{-1}\beta g(kT)\,, \quad (9)$$

where: β is a constant, $|\beta| < 1$, $\beta \neq 0$, $\boldsymbol{f} = [f_1 \;\; f_2 \;\; f_3]$ — commutation coefficients such that:

$$|f_i^+| = |f_i^-| = f_i = \begin{cases} 0 & |g(kT)| \leq \gamma(kT)\,, \\ -f\,\mathrm{sgn}\big(\beta g(kT)x_i(kT)\big) & |g(kT)| > \gamma(kT)\,, \end{cases} \quad i = 1,2,3\,, \quad (10)$$

$$\gamma(kT) = \frac{f}{2|\beta|}\sum_{i=1}^{3}|x_i(kT)|\,, \qquad 0 < f < 2|\beta| \max_{1\leq i\leq 3}\Big(\boldsymbol{c}\left[\boldsymbol{d}_\delta^\top \;\; \big(\boldsymbol{d}_\delta(\mathbf{A}_\delta - \delta\mathbf{I})\big)^\top\right]^\top\Big)\,. \quad (11)$$

P r o o f . Based on relation (9) it can be concluded that the sliding area: $\Phi = \{\mathbf{x}(kT) \mid |g(kT)| \leq \gamma(kT)\}$ has been defined and that the canonical subspace of the state is divided into two areas:

1) The area outside the sliding area Φ, where $|g(kT)| > \gamma(kT)$.

Based on relation (9), expression (7) obtains the following form:

$$g\big((k+1)T\big) = \beta g(kT) + \mathbf{f}\mathbf{x}(kT)\,. \quad (12)$$

Multiplying both sides of relation (12) by $\beta g(kT)$:

$$\beta g(kT) g\big((k+1)T\big) = \beta^2[g(kT)]^2 + \beta g(kT)\mathbf{f}\mathbf{x}(kT) \quad (13)$$

is obtained. Taking into account relations (10) and (11), relation (13) becomes:

$$\beta g(kT)g\big((k+1)T\big) - \beta^2[g(kT)]^2 = -2\beta^2\gamma(kT)|g(kT)|\,.$$

If $\gamma(kT) < |g(kT)|$, the $\beta g(kT)g\big((k+1)T\big) - \beta^2 g(kT)^2 > -2\beta^2[g(kT)]^2$, that is $|g\big(k+1)T\big)| < |\beta||g(kT)|$.

2) The area within the sliding area Φ, where $|g(kT)| < \gamma(kT)$.

Based on relations (9) and (10), expression (7) assumes the following form: $g\big((k+1)T\big) = \beta g(kT)$.

If, for both areas over the canonical subspace of the state, a discrete Lyapunov function in the form $v(kT) = [g(kT)]^2$ is chosen, then $v\big((k+1)T\big) \leq \beta^2 v(kT)$ for $|\beta| < 1$. Based on this relation, it can be concluded that $g(kT)$ decreases, that is $\lim\limits_{k\to\infty} g(kT) = 0$. The vector components $\mathbf{x}(kT)$ are stable. Thus, system (3) is stable both inside and outside the sliding area Φ.

4 DTVS CONTROLLER SYNTHESIS IMPROVEMENT METHOD

Applying the given algorithm, the process of establishing the assigned value is slowly implemented. To speed up the process, a combination of parallel action of the conventional proportional controller and the given DTVS controller is recommended as a possible variant. To determine control $u(kT)$, the same idea (relation (7)) as in the previous section is used. Starting from the necessary and sufficient condition (1) of the reaching and sliding mode existence, Theorem 2 is reached:

THEOREM 2. *System (3) with proportional controller is stable if the control is*

$$u(kT) = h^{-1}\mathbf{g}\mathbf{x}(kT) + h^{-1}\mathbf{f}\mathbf{x}(kT) + h^{-1}\beta g(kT) + h^{-1}p_{reg}e(kT)\,, \quad (14)$$

where: p_{reg} *is a proportional gain* $p_{reg} > 0$,
$e(kT)$ *— error signal* $e(kT) = r - y(kT) = r - \boldsymbol{d}_\delta\mathbf{x}(kT)$,
$\boldsymbol{f} = [f_1 \;\; f_2 \;\; f_3]$ *— commutation coefficients such that:*

$$|f_i^+| = |f_i^-| = f_i = \begin{cases} 0 & |g(kT)| \le \gamma(kT)\,, \\ -f\,\mathrm{sgn}\big(\beta g(kT)x_i(kT)\big) & |g(kT)| > \gamma(kT)\,, \end{cases} \quad i = 1,2,3\,,$$

$$\gamma(kT) = \frac{(f+\overline{d})}{2|\beta|}\sum_{i=1}^{3}|x_i(kT)|\,, \;\; \overline{\boldsymbol{d}} = -p_{reg}\boldsymbol{d}_\delta = [d_1 \;\; d_2 \;\; d_3]\,,$$

$$|d_i| < \overline{d}\,, \;\; i = 1,2,3\,, \quad \overline{d} < f < 2|\beta| \max_{1\le i\le 3} \overline{c}_i - \overline{d}\,.$$

The proof of Theorem 2 is the same as that of Theorem 1, so that there is no need to repeat it.

5 DTVS CONTROLLER ROBUSTNESS

Let the system be given in the perturbed condition in the following form:

$$\delta\mathbf{x}(kT) = \mathbf{A}_\delta\mathbf{x}(kT) + \Delta\mathbf{A}_\delta\mathbf{x}(kT) + \boldsymbol{b}_\delta u(kT) + \boldsymbol{f}(kT)\,, \quad (15)$$

where: $\mathbf{A}_\delta$ and $\boldsymbol{b}_\delta$ — the matrices obtained using the delta transform, $\Delta\mathbf{A}_\delta$ — the system parameter variations, and $f(kT)$ — the external disturbance.

The matching conditions are assumed by [16]:

$\Delta\mathbf{A}_\delta = \boldsymbol{b}_\delta\widetilde{\mathbf{A}}$, $\widetilde{\mathbf{A}}$ — a row vector,

$\boldsymbol{f}(kT) = \boldsymbol{b}_\delta\widetilde{\boldsymbol{f}}(kT)$, $\tilde{f}$ — a scalar.

Then, relation (15) becomes:

$$\delta\mathbf{x}(kT) = \mathbf{A}_\delta\mathbf{x}(kT) + \boldsymbol{b}_\delta\big(u(kT) + \tilde{A}\mathbf{x}(kT) + \tilde{f}(kT)\big)\,. \quad (16)$$

Based on relation (16) it can be concluded that the control and disturbance vectors are co-linear, that is the invariance property applies to the parametric and external disturbances. Now, the control (9) becomes:

$$u(kT) = h^{-1}\mathbf{g}\mathbf{x}(kT) + h^{-1}\mathbf{f}\mathbf{x}(kT) + h^{-1}\beta g(kT) - h^{-1}\mathbf{A}\mathbf{x}(kT) - h^{-1}\tilde{f}(kT)\,. \quad (17)$$

In relation (17), $\tilde{\mathbf{A}}$ and $\tilde{\boldsymbol{f}}$ are unknown, so that the control in this form cannot be implemented and the invariance property does not apply any more. Let the values $\tilde{\mathbf{A}}$, $\tilde{\boldsymbol{f}}$ be replaced with $\mathbf{A}_c$ and f_c, respectively, for which the reaching and existence condition of the sliding mode is fulfilled, the control (4) is

$$u(kT) = h^{-1}\mathbf{g}\mathbf{x}(kT) + h^{-1}\mathbf{f}\mathbf{x}(kT) + h^{-1}\beta g(kT) - h^{-1}\mathbf{A}_c\mathbf{x}(kT) - h^{-1}f_c(kT)\,. \quad (18)$$

To determine $\mathbf{A}_c$ and f_c, the expression for $g\big((k+1)T\big)$ is found in the following form:

$$g\big((k+1)T\big) = (-\mathbf{g} + \Delta\mathbf{g})\mathbf{x}(kT) + hu(kT) + \boldsymbol{e}\boldsymbol{f}(kT) \quad (19)$$

where: $\Delta\boldsymbol{g} = \mathrm{d}_{\delta 1}(\mathbf{A}_\delta + \Delta\mathbf{A}_\delta) + \boldsymbol{g}$, $\boldsymbol{e} = \boldsymbol{d}_{\delta 1} + \boldsymbol{d}_{\delta 2}$,
$\boldsymbol{d}_{\delta 1} = c\boldsymbol{d}_\delta + \boldsymbol{d}_\delta(\mathbf{A}_\delta + \Delta\mathbf{A}_\delta - \delta\mathbf{I})$, $\boldsymbol{d}_{\delta 2} = c\boldsymbol{d}_\delta$.
Using relations (18), expression (19) becomes

$$g\big((k+1)T\big) = \mathbf{f}\mathbf{x}(kT) + \beta g(kT) + \Delta\boldsymbol{g}\mathbf{x}(kT) - \mathbf{A}_c\mathbf{x}(kT) + \boldsymbol{e}\,\boldsymbol{f}(kT) - f_c(kT)\,. \quad (20)$$

To simplify expression (20), the following new parameter uncertainties and external disturbances:

$$\overline{\mathbf{A}}(\boldsymbol{x}(kT) = \Delta\boldsymbol{g}\,\boldsymbol{x}(kT)\,, \quad \overline{\mathbf{A}}_c\big(\boldsymbol{x}(kT)\big) = \mathbf{A}_c\boldsymbol{x}(kT)\,,$$
$$\overline{f}(kT) = \boldsymbol{e}\,\boldsymbol{f}(kT)\,, \quad \overline{f}_c(kT) = f_c(kT)\,.$$

are defined. Let the boundaries $\overline{\mathbf{A}}$ and $\overline{f}$ be known and respectively given:

$$\overline{\mathbf{A}}_{\min} < \overline{\mathbf{A}} < \overline{\mathbf{A}}_{\max}\,, \quad \overline{f}_{\min} < \overline{f} < \overline{f}_{\max}\,.$$

The choice of $\mathbf{A}_c$ and f_c is done to ensure that the sign of the incremental $g(kT)$ of (20) is opposite to the sign of $g(kT)$. Therefore a practical choice is
for $g(kT) > 0$
$\overline{\mathbf{A}}_c = \overline{\mathbf{A}}_{\max}$, $\overline{f}_c = \overline{f}_{\max}$ or $\mathbf{A}_c = \overline{\mathbf{A}}_{\max}$, $\overline{f}_c = \overline{f}_{\max}$,
for $g(kT) < 0$
$\overline{\mathbf{A}}_c = \overline{\mathbf{A}}_{\min}$, $\overline{f}_c = \overline{f}_{\min}$ or $\mathbf{A}_c = \overline{\mathbf{A}}_{\min}$, $\overline{f}_c = \overline{f}_{\min}$.
Now, the control (18) can be fully implemented because all the parameters are known. The previous analysis of the system robustness has shown that the ideal sliding mode is invariant to parametric and external disturbances, while the real sliding mode does not possess that property of invariance because system (15) with control (18) is always dependent on the disturbances.

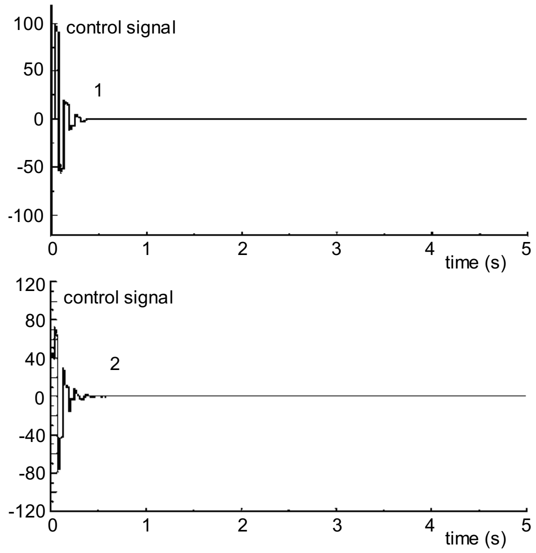

Fig. 3. Control signals of the system for parameters rated values of the object and for the case without (1) or with (2) the proportional controller

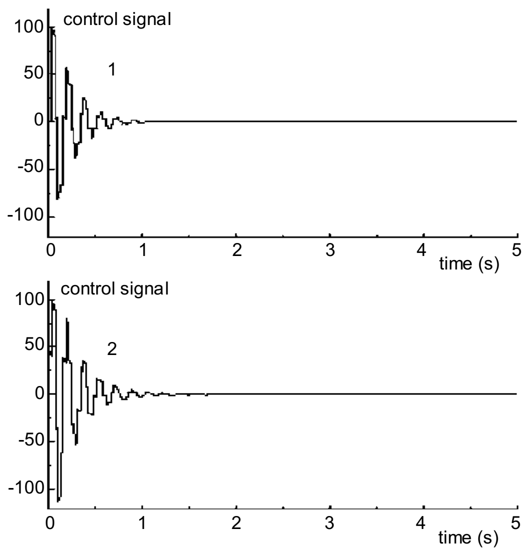

Fig. 4. Control signals of the system for parameters borderline values of the object and for the case without (1) or with (2) the proportional controller

6 ILLUSTRATIVE EXAMPLE

To verify the relations obtained for the DTVS controller synthesis, regulation of a third order object with finite zero, the parameters of which have optional values $5 \le a_0 = a_2 \le 7$, $10 \le a_1 \le 12$, $0.5 \le d_0 \le 1.5$, $19.5 \le d_1 \le 20.5$ has been simulated on a PC. The discretization period $T = 30$ ms is chosen so that:

- the Theorem of sampling will be fulfilled
- the discretization period will be less of minimum object time constants ($T < T_1 = 1$ s, $T < T_2 = 0.5$ s, $T < T_3 = 0.33$ s)
- the discrete-time model of the object will be minimum phase.

Applying the Euler method for the selected discretization period, the DTVSS model with the parameters: $-0.86 \le a_0(T) \le -0.79$, $2.58 \le a_1(T) \le 2.71$, $-2.85 \le a_2(T) \le -2.79$, $-8.235 \cdot 10^{-4} \le d_0(T) \le 1.035 \cdot 10^{-4}$, $4.5 \cdot 10^{-4} \le d_1(T) \le 13.5 \cdot 10^{-4}$ is obtained. Based on (3), the system model over the two- dimensional subspace is

$$\begin{aligned} \delta y^{(1)}(kT) &= [3.6 \quad -12.6 \quad 9] \; 10^{-4}\mathbf{x}(kT) \\ \delta y^{(2)}(kT) &= [3.867642 \quad -7.6491 \quad 3.78] \; 10^{-4}\mathbf{x}(kT) \\ &\qquad + 9 \cdot 10^{-4} u(kT)\,. \end{aligned}$$

The control $u(kT)$ according to Theorem 1 in the form (9), where:

$$\begin{aligned} &h = 9 \cdot 10^{-4}, \quad f = 72 \cdot 10^{-6}, \quad \beta = 0.045\,, \\ &g = [-7.467642 \quad 27.4491 \quad -30.78] \cdot 10^{-4} \end{aligned} \tag{21}$$

is determined.

As a possible improvement variant, a combination of parallel action of the conventional proportional controller and the given DTVS controller is recommended. The proportional gain $p_{reg} = 0.04$ is determined in such a way that the system is stable over the complete canonical space, that is the roots of the system characteristics equation are modulo ≤ 1. The control $u(kT)$ according to Theorem 2 has the form (14), where: $f = 0.4 \cdot 10^{-4}$ and $\boldsymbol{g}$, h and β are as those in relation (21). The simulation results are shown in the form of step responses (Fig. 1), controls (Fig. 3) and sliding hyperplanes (Fig. 5) of the system for the case DTVSS without (settling time is 3.49 s) and with the proportional controller (settling time is 2.069 s). Based on the results obtained, it can be concluded that the process of establishing the assigned value is sped up.

For the parameters borderline values of the object, the simulation results are shown in the form of step responses (Fig. 2), controls (Fig. 4) and sliding hyperplanes (Fig. 6) of the system for the case of DTVSS without (settling time is 3.544 s) and with the proportional controller (settling time is 2.071 s). Based on the results obtained it can be concluded that the system is still stable and that the changes in step responses of the system are only reduced to a slower establishment of the balanced state. Also, it can be said that DTVSS with the proportional controller is more robust because its output variable changes for the borderline values are less with reference to the rated parameter values of the object.

The impact of the external disturbance has been considered supposing that the external disturbance of the constant intensity 0.1 over the interval from 4.0 s to 4.4 s affects the object. The results obtained show that the external disturbance effects are relatively quickly eliminated. Also, it can be said that DTVSS with the proportional regulator is more robust because the changes on the output variable diagram are lesser.

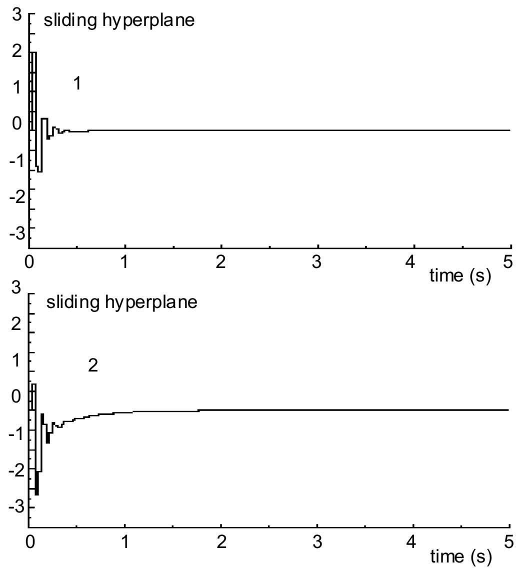

Fig. 5. Sliding hyperplanes of the system for parameters rated values of the object and for the case without (1) or with (2) the proportional controller

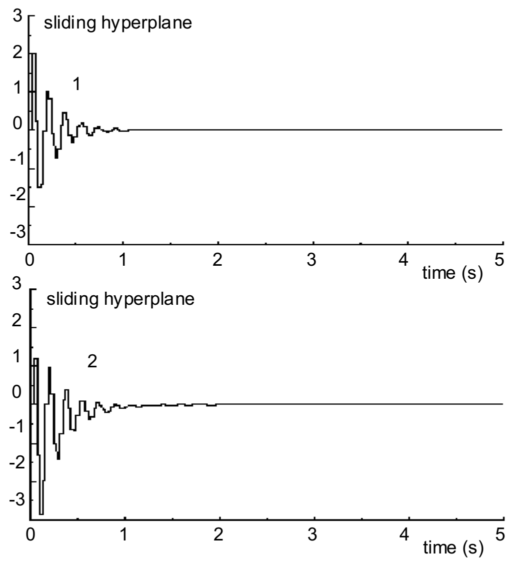

Fig. 6. Sliding hyperplanes of the system for parameters borderline values of the object and for the case without (1) or with (2) the proportional controller

7 CONCLUSION

The paper analyzes the conditions of generation of the quasi-sliding (zigzag) mode movement in DTVSS in the case of a 3^{rd} order object. In an illustrative example, the DTVSS synthesis procedure is shown with and without the classical proportional controller, that is, reaching and sliding mode existence relations have been derived. Based on a check-up, it has been concluded that the proposed control method is possible and that it yields good results (zero error in the steady state, stability robustness) as well as that the DTVSS characteristics in combinations with the conventional proportional controller are better than those of the traditional DTVSS. Thus, it means that parallel combination of the variable structure algorithms and the proportional type classical algorithm is possible.

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