EDDY CURRENT LOSSES IN TRANSFORMER LOW VOLTAGE FOIL COILS

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The paper deals with eddy current losses in transformer low voltage foil coils, where the measured value of the load loss is usually much higher than the one computed by standard procedures. The work brings results explaining analytically the source of the discussed loss and formulas, and a procedure allowing to get results of sufficient preciseness.

K e y w o r d s: transformer, foil coil, load loss (losses), eddy current loss, added loss factor

1 INTRODUCTION

The eddy current loss in a foil coil has not been explained in a satisfactory way and there is no analytical relation known to us for computing the loss in this coil exactly or with sufficient preciseness. The source of increased eddy current losses is not clear and the radial part of the magnetic leakage field has been considered to be the source of the measured additional loss. In design practice, if the foil coil was used, only empirical constants and experience helped designers to foresee the test results. The biggest uncertainties were in those cases, when bigger transformer innovation was performed.

The market requirements force producers to use the foil more and more. This fact inspired us to start this work.

In this paper, all relations, expressions and formulas are expressed in the basic SI units only. All physical values used are so well known that we did not make any comment about what physical units belong to respective values.

2 TRANSFORMER MEASUREMENTS IN SHORT RUN CONNECTION

During a short run of the transformer or while measuring the short run values [1], the cable connectors of the secondary winding are bridged. The values of the secondary side are marked by lower index s and subscript p denotes the primary side. Wiring diagrams of single and three phase transformers are in Figs. 1 and 2.

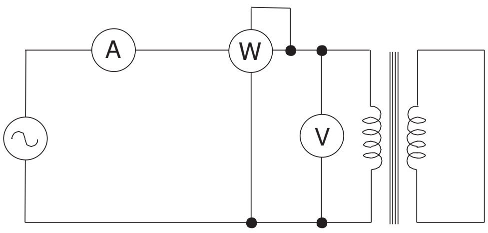

Fig. 1. Wiring diagram of a single phase transformer during short run measurement.

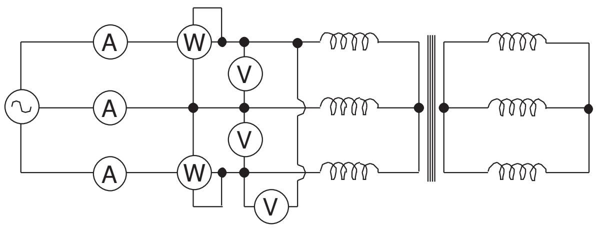

Fig. 2. Wiring diagram of a three phase transformer during short run measurement.

In the short run state, the dependences of $I_{pk} = f(U_k)$, $P_k = f(U_k)$ and $\cos\varphi_{pk} = f(U_k)$ are measured. Index k means the short run values. In this state, the magnetizing current is omitted. The following relation is valid:

$$U_0 = \frac{U_p}{n_p} = \frac{U_s}{n_s}\,. \tag{1}$$

U_0 is the voltage per turn, U_p and U_s are induced voltages in primary and secondary windings, and n_p and n_s are the numbers of turns in primary and secondary windings. For the currents on the primary and secondary sides, equation $I_p n_p = I_s n_s$ is valid. For the terminal potential measured on the input side, relation

$$U_{term} = U_p + Z_p I_p \tag{2}$$

can be written, where Z_p represents the input winding impedance and similarly on the output side Z_s represents the input winding impedance. The equation

$$Z_s I_s - U_s = 0 \tag{3}$$

shows that no other voltages are present in the secondary winding.

The loss generated in the real component of the short run impedance at rated currents at both sides is called the load loss, and its symbol is P_k. The load loss at rated currents means the full load loss [2]. This loss represents the basic load loss and can be expressed by the following relation:

$$P_k = R_k\, I_{pnom}^2 = R_k' I_{snom} I_{pnom} = \Big[\frac{n_p}{n_s} R_s + \frac{n_s}{n_p} R_p\Big] I_{snom} I_{pnom}\,. \tag{4}$$

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Suffix "*nom*" in the indices descends from nominal, which is usually used as a synonym for the rated values. Relation (4) can be modified to a formula that is utilized in transformer design:

$$P_k = \frac{n_p}{n_s} R_s I_{snom} I_{pnom} + \frac{n_s}{n_p} R_p I_{snom} I_{pnom} = R_s I_{snom}^2 + R_p I_{pnom}^2 . \quad (5)$$

3 EDDY CURRENT LOSS INFLUENCE ON MAGNETIC LEAKAGE FIELD

The additional loss has its origin in the basic physical nature of conductive materials. These, being put into a magnetic field, react to every change of the outer field by arising of eddy currents. These currents act against the change. Saying in another way, these currents create an additional magnetic field of opposite direction affecting the decrease of the basic field. Every change of the field affects the whole system, thus both sides of the transformer. Considering relations (4) and (5) from the point of view of the physical phenomena taking place in the transformer, relation (4) is more authentic. Eddy currents in the coil act against the current component I_s so that the inequality $I_s < I_{snom}$ is valid. This depression according to relation (4) affects both sides, which can be seen if index substitution is made and I_s instead of I_{snom} is put into relation (4). The basic loss is decreasing on both sides but current I_{pnom} is unchanged.

If any secondary winding parameter causes an increase of R_s by ΔR_s, the eddy current loss grows up and the overall load loss can also grow at the same time. Nevertheless, the basic loss falls down by the effect of the eddy current loss and the value of it is lower than its value at the rated current. This principle does not allow growing the eddy current loss to infinity because its source is reduced simultaneously.

Let us substitute the eddy currents by the dummy longitudinal current I_{loss} having the same effect upon the magnetic field and I_s current. In resistor R_s, current I_{loss} generates in the same loss as eddy currents and there is a 90° electrical angular shift between current I_s and current I_{loss}. The rated current I_{snom} is a vector sum of I_s and I_{loss} so that relation of $I_{snom}^2 = I_s^2 + I_{loss}^2$ is valid. It gives the following result:

$$P_s = P_s^k + P_{loss}^k = R_s I_s^2 + R_s I_{loss}^2 = R_s I_{snom}^2 . \quad (6)$$

According to relation (4), the basic loss in the winding of the secondary side is not $P_s^k = R_s I_s^2$ but it assumes a value in accordance with relation (7), where the variable is I_s.

$$P_s^k = R_s I_s \frac{n_p}{n_s} I_{pnom} = R_s I_s I_{snom} . \quad (7)$$

Let us define a dimensionless factor of the loss increment by the following expression:

$$\chi = \frac{R_s I_{snom}^2 - R_s I_s^2}{R_s I_{snom}^2} = \frac{I_{snom}^2 - I_s^2}{I_{snom}^2} . \quad (8)$$

This directly yields

$$I_s = I_{snom} \sqrt{1-\chi} . \quad (9)$$

After substituting relation (9) into relation (7), the dependence of the basic loss in the secondary winding on the loss increment factor can be expressed:

$$P_s^k = R_s I_{snom}^2 \sqrt{1-\chi} = P_{snom}^k \sqrt{1-\chi} . \quad (10)$$

Loss P_{snom}^k is the load loss in the secondary winding at zero point of eddy current losses. Analyses and deriving relations for eddy current losses [1, 3, 4, 5, 6] come out from the P_{snom}^k state and lead to the factor of added losses χ_0. But there are no regards to the changes of the initial conditions later on. As a result, the value χ_0 can exceed the value of 1 even couple of times. The validity is, therefore, limited to $\chi_0 \le 0.2$. In the range of the limited validity of the added loss factor χ_0, expression (11) is valid:

$$\chi_0 = \frac{\Delta R_s I_{snom}^2}{R_s I_{snom}^2} = \frac{\Delta R_s}{R_s} = \frac{\Delta \rho_s}{\rho_s} . \quad (11)$$

Value ΔR_s ($\Delta \rho_s$) is an increase of the winding resistance for alternating current. It is analytically derived from the magnetic leakage field shape, wire sizes and their layout in the leakage field. The loss increment factor is lower because the winding current is $I_s < I_{snom}$.

$$\chi = \frac{\Delta R_s I_{snom} I_s}{R_s I_{snom}^2} = \frac{\Delta R_s I_{snom}^2 \sqrt{1-\chi}}{R_s I_{snom}^2} = \chi_0 \sqrt{1-\chi} . \quad (12)$$

Quadratic equation (13) is derived from relation (12) and in this way the loss increment factor χ is obtained in terms of the added loss factor χ_0:

$$\chi^2 + \chi_0^2 \chi - \chi_0^2 = 0 . \quad (13)$$

One of the roots of the quadratic equation is the solution allowing to compute the eddy current loss:

$$\chi = \frac{\chi_0^2}{2} \left[\sqrt{1 + \frac{4}{\chi_0^2}} - 1 \right] . \quad (14)$$

Factor χ is a better approximation of the real increase of the loss in comparison with factor ?0. It involves the change of initial conditions automatically. The value χ for $\chi_0 \to \infty$ is converging towards 1. For low values of χ_0, formula (14) can be modified to $\chi = \chi_0$ identity. Factor χ_0 used in formula (14) has a wider range of validity than interval $0 \le \chi_0 \le 0.2$. The overall loss in the secondary winding is computed from relation (15).

$$P_s = (1+\chi) R_s I_{snom}^2 = (1+\chi) P_{snom}^k . \quad (15)$$

Factor χ is not additive. During computing, the first step is to make the sum of all partial added loss factors $\chi_0 = \sum_i \chi_{0i}$ and then to compute χ factor.

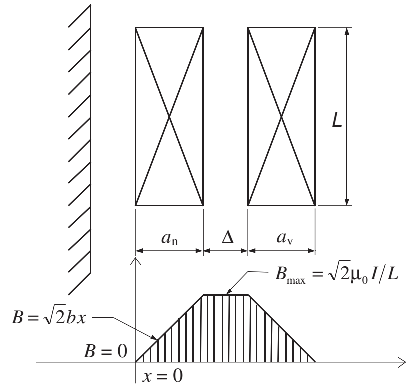

Fig. 3. Axial magnetic leakage field in transformer window.

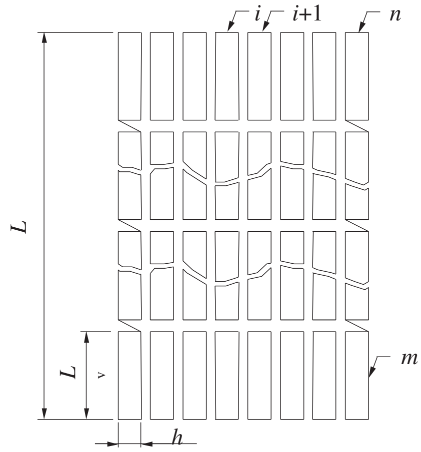

Fig. 4. The axial cross-sectional foil coil with m parallel windings.

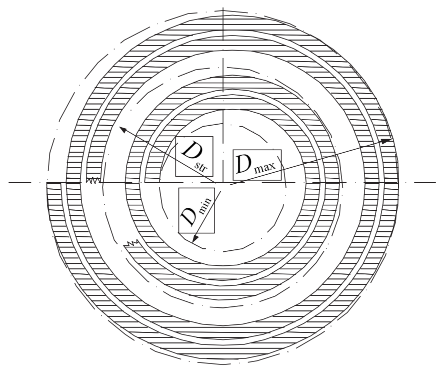

Fig. 5. The axial cross-sectional foil coil with m parallel windings.

4 LOAD LOSS TEMPERATURE DEPENDENCE

Since the real part of the short run impedance depends on temperature, the values measured in the transformer at the short run state are temperature dependent. The value depending on temperature in the strongest way is the load loss value. According to standard [7], the values of load loss must be converted to those for temperature 75 °C in the case of oil transformers and for a temperature according to general requirements for tests in IEC 726 in the case of dry transformers. For dry transformers, the loss is converted to that for temperature 75 °C every time in order to get a compatibility of all types of transformers. The procedure for load loss conversion to the reference temperature is normative and is described in detail in appendix E [7].

The standard presupposes implicitly the correctness of the load loss conversion to a temperature different from the temperature at the measurement. According to the discussed theory, this is not true. The result of computation or measurement represents the loss correctly only at the temperature at which appropriate works are done. To make the result of computing comparable with transformer measurement protocols, some procedure must be utilized as with the measured values. Computations must be executed at the temperature of measurement and the result shall be converted according to the standard procedure. In transformer design, computations should be made for the expected measured temperature.

5 VERIFICATION EXAMPLES

Examples of utilizing the loss increment factor χ in transformer design are in Tab. 1 and they are presented to verify the discussed theory.

All examples are real machines selected to cover a certain range of power. In all cases the material of both windings was aluminium. In the procedure presented in Tab. 1, the value of $P_{pnom+p}^{kt=75\,°C}$ was increased by a factor of 1.05 in all cases to respect the added loss in the primary winding. The overall load loss was computed by the formula $P_k^{t=75\,°C} = P_{pnom+p}^{t=75\,°C} + P_{snom}^{t=75\,°C} + P_p^{t=75\,°C}$. In the last column, $P_{km}^{t=75\,°C}$ values represent the measured values.

6 EDDY CURRENT LOSS IN TRANSFORMER WITH FOIL COIL

As it can be seen above, solving the problem of eddy current losses in foil coils and at the same time the problem of the added losses factor χ_0 means finding the main source of these losses correctly. The cross-sectional eddy currents that arise because of the radial gradient of the magnetic leakage field do not seem to be a powerful source. It is also supposed that the influence of the radial part of the magnetic leakage field, especially in the case of high and slim coils, and of the skin effect at frequencies 50−60 Hz in thin foils is negligible.

The foil coil has one special feature that makes it unlike others. The turns look from the top view look like a spiral and they grow up constantly along with the outer magnetic field. Direction of this growth is not changed within the whole length of the foil. It leads us to the model, where the magnetic field grows along with the foil length dimension. The winding can be represented by a thin straight line of certain thickness. This length and thickness determine the area whereto the magnetic field is perpendicular. This model, assuming the magnetic field within the thickness to be constant, simplifies the following calculations below.

Figure 3 shows a typical trapezoid of the axial magnetic leakage field in the transformer window. Only the increasing part of the magnetic field is transformed to our model. Figure 4 describes the most important parameters of the foil coil and Fig. 5 shows the spiral shape of the foil coil. The legend of symbols is in Tab. 2.

Table 1.

P (kVA)	$P_{vnom}^{kt=20\,°C}$ (W)	$P_{nnom}^{kt=20\,°C}$ (W)	$P_{vnom+p}^{kt=75\,°C}$ (W)	$P_{nnom}^{kt=75\,°C}$ (W)	$\chi_0^{t=20\,°C}$	$\chi^{t=20\,°C}$	$P_p^{kt=20\,°C}$ (W)	$P_p^{kt=75\,°C}$ (W)	$P_k^{t=75\,°C}$ (W)	$P_{kmer}^{t=75\,°C}$ (W)
630	3218	2243	4139	2747	0.296	0.255	573	468	7354	7231
1000	3175	2527	4084	3095	1.498	0.75	1895	1547	8726	8778
1600	4539	3268	5838	4003	2.776	0.896	2928	2390	12231	12502

Table 2.

Symbol (Symbol=Expression)	**Symbol meaning**
$D_{ave} = (D_{\max} + D_{\min})/2$	average (mean) coil diameter
$a = n\pi D_{ave}$	foil length
σ	basic current density
n	number of current turns (foil turns in radial direction)
m	number of parallel coils in axial direction
L_c	foil highness
$L \cong mL_c$	coil highness
h	foil thickness
$L_{mf} \cong L$	mean length of induction force line
$B_{\max} = \dfrac{\sqrt{2}\mu_0 mnhL_c\sigma}{L_{mf}}$	induction maximal value in leakage canal
$b = B_{\max}/a$	magnetic field foil length density

Now it is possible to express the magnetic field as a function of variable x.

$$B(x) = \frac{B_{\max}}{a}x = bx\,. \tag{16}$$

The magnetic flux that is bound to little area $\mathrm{d}S = h\mathrm{d}x$ of foil and in interval $\langle x_0, x\rangle$ can be written as follows:

$$\Phi(x) = \int_{x_0}^{x} B(x)h\mathrm{d}x = bh\Big(\frac{x^2}{2} - \frac{x_0^2}{2}\Big). \tag{17}$$

Similarly, an induced voltage arises around the flux area.

$$U(x) = -\frac{2\pi f}{\sqrt{2}}\Phi(x) = -\frac{2\pi f}{\sqrt{2}}bh\Big(\frac{x^2}{2} - \frac{x_0^2}{2}\Big). \tag{18}$$

As the terminal voltage of $U(x)$ must be zero and no other induced voltage except of U_s can be between the coil terminals, the mean value of $U(x)$ along the whole foil must be zero. For $U(x)$ satisfying the condition above, the following expression is obtained:

$$U(x) = -\frac{2\pi fbh}{2\sqrt{2}}\Big(x^2 - \frac{a^2}{3}\Big) = -k\Big(x^2 - \frac{a^2}{3}\Big). \tag{20}$$

A simple substitution is used to simplify this expression:

$$k = \frac{2\pi fbh}{2\sqrt{2}}\,. \tag{21}$$

The electric resistivity is marked by symbol ρ in the next relations. The eddy current density in element $\mathrm{d}x$ can be expressed as

$$\sigma_{curl}\mathrm{d}x = -\frac{1}{\rho}\frac{\partial U(x)}{\partial x}\mathrm{d}x = \frac{k}{\rho}2x\mathrm{d}x\,. \tag{22}$$

To make a difference between the current density σ_{curl} arising from the local gradient and the current density σ_{circ} due to the gradient between two different points, the second one is called the circulating current density. It has its origin in different voltage levels between position x and position with zero voltage level $x_0 = a/\sqrt{3}$, which for current density in $\mathrm{d}x$ element gives

$$\sigma_{circ}\mathrm{d}x = -\frac{1}{\rho}\frac{\Delta U(x)}{\Delta x}\mathrm{d}x = \frac{k}{\rho}\Big(x + \frac{a}{\sqrt{3}}\Big)\mathrm{d}x\,. \tag{23}$$

The sum of these two parts of current density causes the added load loss in the foil winding. The overall current density can be expressed as follows:

$$\sigma(x)\mathrm{d}x = \frac{k}{\rho}\Big(3x + \frac{a}{\sqrt{3}}\Big)\mathrm{d}x\,. \tag{24}$$

The loss in a unit volume is

$$\begin{aligned}\rho\sigma(x)^2\mathrm{d}x &= \frac{1}{\rho}\Big[\frac{k}{3}\big(9x + a\sqrt{3}\big)\Big]^2\mathrm{d}x \\ &= \frac{1}{\rho}\Big[\frac{k^2}{3}\big(27x^2 + 6a\sqrt{3}x + a^2\big)\Big]\mathrm{d}x \end{aligned} \tag{25}$$

and the mean value of eddy current loss in a unit volume is

$$\begin{aligned} p_p^k &= \frac{1}{a}\int_0^a \rho\sigma(x)^2\mathrm{d}x = \frac{1}{\rho}\frac{k^2a^2(9 + 3\sqrt{3} + 1)}{3} \\ &= 1.266\Big(\frac{2\pi\mu_0 fmL_cnh^2\sigma}{L_{mf}}\Big)^2\frac{1}{\rho}\,. \end{aligned} \tag{26}$$

Taking into account that $L_{mf} \cong mL_c$ and $p_{snom}^k = \rho\sigma^2$, the added loss factor can be expressed as follows:

$$\begin{aligned}\chi_0 &= \frac{p_p^k}{p_{snom}^k} = 1.266\Big(\frac{2\pi\mu_0 fh^2mL_cn}{\rho L_{mf}}\Big)^2 \\ &= 1.266\Big(\frac{7.9fh^2n}{10^6\rho}\Big)^2. \end{aligned} \tag{28}$$

Now we take into account perpendicular, cross-sectional eddy currents and because they are normal to the length

Table 3.

Power P (kVA)	630	1000	1000	1500	1600	2000	2500	2500	1000	1600
Foil material	cu	cu	cu	cu	cu	cu	cu	cu	al	al
No. of turns n	24	16	16	15	27	15	22	22	18	14
Foil thick. h (m)	0.0006	0.0012	0.0012	0.0022	0.0007	0.0014	0.0009	0.0009	0.0009	0.0018
P_p^{20} (W)	2581	2536	2513	3730	7875	11780	13343	10106	3825	4615
P_s^{20} (W)	2865	2561	2561	2736	5747	6621	8719	8796	3416	3673
χ_0^{20}	0.048	0.338	0.338	3.355	0.111	0.550	0.202	0.202	0.050	0.483
χ^{20}	0.046	0.286	0.286	0.924	0.105	0.419	0.183	0.183	0.049	0.380
$P_{k\text{-computed}}^{75}$ (W)	6699	6772	6745	9914	16983	24560	28012	24200	9006	11294
$P_{k\text{-measured}}^{75}$ (W)	6830	6875	6839	10157	16500	24279	27809	23757	9016	12052

of the foil, the factors can be derived separately and the results are additive. The next relation is published in [1, 3, 4, 5, 6].

$$\chi_0 = \frac{n^2-0.2}{9}\Big(2\pi h\sqrt{\frac{f}{10^7\rho}}\Big)^4 = 0.0278\Big(\frac{7.9fh^2n}{10^6\rho}\Big)^2. \tag{29}$$

Hence, the final added factor is

$$\chi_0 = 1.3\cdot\Big(\frac{7.9fh^2n}{10^6\rho}\Big)^2. \tag{30}$$

As it can be seen according to our presupposition, the effect of cross-sectional eddy current makes a negligible error.

7 COMPUTED VERSUS MEASURED VALUES

In Tab. 3, the values from an archive of transformer test protocols are used to compare them with computed values.

The computed values given in Tab. 3 match with the measured values satisfactorily. There is lot of tolerance and production factors that affect the resulting resistance. For example, in a 630 kVA transformer a foil of $h = 0.6$ mm was used. The computed overall load loss is 6699 W. If the thickness is changed to $h = 0.55$ mm, then the value of 6990 W is achieved.

8 CONCLUSION

The last table above shows that the goal to find a theory and explanation of eddy current loss in foil coils was achieved. The computed results correspond with measured ones well enough. On this basis we can state that the source of the eddy current loss in the foil coil is the axial magnetic leakage field and the radial part of the magnetic leakage field plays a negligible role in the shape of coil. The presented theory brings a new concept in the form of a loss increment factor. It is added to the loss factor. Although the new factor was derived for the secondary winding, it is valid for the primary winding, too. Of course, if the eddy current loss is small enough, then there is no need to use any correction.

As we hope, the presented theory is a contribution to the transformer theory and it gives the missing tool for transformer designers.

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