COMMUNICATIONS

INVESTIGATION OF THE DYNAMIC PERFORMANCE OF HYSTERESIS MOTORS USING MATLAB/SIMULINK

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The hysteresis motor starts by virtue of hysteresis losses induced in its rotor. Hysteresis motors are widely used in small motor applications. In this paper the dynamic performance of hysteresis motors is investigated. A mathematical model is applied to analyze the transient and dynamic stability of this electrical machine type. The transient and dynamic stability studies examine the performance of the motor under transient condition and when the system is perturbed about some operating point, *ie* , small-signal analysis. In addition, in this paper, the mathematical based d, q axis model is incorporated in Matlab/Simulink tools for analysis and simulation purposes. This incorporation is considered a trend in applying new computerized tools in electrical machines simulation, design and analysis. The model offers a tool for studying the dynamic stability of the hysteresis motor for small-scale perturbations such as changes in motor's input supply voltage or frequency or in its output load torque as well as under other various conditions.

K e y w o r d s: hysteresis motor modelling, dynamic stability, Matlab/Simulink

1 INTRODUCTION

The hysteresis motor is a well-known type electrical machine. The construction of the hysteresis motor is of stator and rotor parts. The stator has conventional stator windings while the rotor comprises a solid rotor hysteresis ring of permanent-magnet material with no teeth or polar projection. The starting of the motor is due to the hysteresis losses induced in the rotor of the motor. The starting current is in the range of 180 to 200 % of its full load current, and it can pull into synchronism any load inertia coupled to its shaft. Due to its quiet operation, this motor gains a wide range of industrial applications.

The quasi-state analysis of the hysteresis motor had been investigated in [1–3]. The dynamic performance of hysteresis motors becomes important for the optimum design and stability performance. There has been little contribution in the literature on the dynamic characteristics of the hysteresis motors [4–7]. The application of new simulation and analysis techniques using different computer software such as Matlab/Simulink in simulation and analysis of conventional electrical machines has been reported in [8–10]. In this paper the application of Matlab/Simulink computer software is used in investigating the small signal stability performance and the non- linear simulation of the hysteresis motor. Such application can be considered as a trend for investigating the dynamic performance of this specific motor type.

This paper is organized as follows. In sections 2, a non-linear mathematical model representing the performance of the hysteresis motor under various conditions is outlined. This non-linear model is then linearized in section 3 to perform the necessary small signal stability analysis. The linearized model representation is derived from perturbing the non-linear model at some operating point, *ie* , at the steady-state point. In section 4, the non-linear simulation and eigenvalues computation of the tested hysteresis motor using Matlab/Simulink tools are reported and results are discussed. Finally, a conclusion and appendix are included. In the appendix, parameters of the motor under investigation [7, 11] are given.

2 MACHINE MATHEMATICAL MODEL

The non-linear approximated model for hysteresis type motor is outlined in this section. The nomenclature for different terms is illustrated at the end of the paper. Here, the mathematical model describing the hysteresis motor is referred to the stationary reference frame and is written as follows:

$$\mathbf{V} = \mathbf{RI} + \frac{1}{\omega_b}\mathbf{X}_1\dot{\mathbf{I}} + \frac{\omega_r}{\omega_b}\mathbf{X}_2\mathbf{I} \qquad (1)$$

$\mathbf{X}_1$, $\mathbf{X}_2$ are two matrices, their elements are detailed in equation (2).

This model gives the voltage equations for the motor. All bold symbols are n- or m-dimensional column vectors or $m \times n$-matrices of different electrical quantities. The overall detailed mathematical model of the hysteresis motor is given as in equation (2) and the electromagnetic developed torque equation is represented in equation (3).

$$
\begin{bmatrix} V_{Ds} \\ V_{Qs} \\ V_{dr} \\ V_{qr} \end{bmatrix} =
\begin{bmatrix} r_s & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 \\ 0 & 0 & r_r & 0 \\ 0 & 0 & 0 & r_r \end{bmatrix}
\begin{bmatrix} i_{Ds} \\ i_{Qs} \\ i_{dr} \\ i_{qr} \end{bmatrix} +
$$

$$
+ \frac{1}{\omega_b}
\begin{bmatrix} X_{ss} & 0 & X_{srm} & 0 \\ 0 & X_{ss} & 0 & X_{srm} \\ X_{ss} & 0 & X_{rr} & 0 \\ 0 & X_{srm} & 0 & X_{rr} \end{bmatrix}
\begin{bmatrix} \dot{i}_{Ds} \\ \dot{i}_{Qs} \\ \dot{i}_{dr} \\ \dot{i}_{qr} \end{bmatrix}
$$

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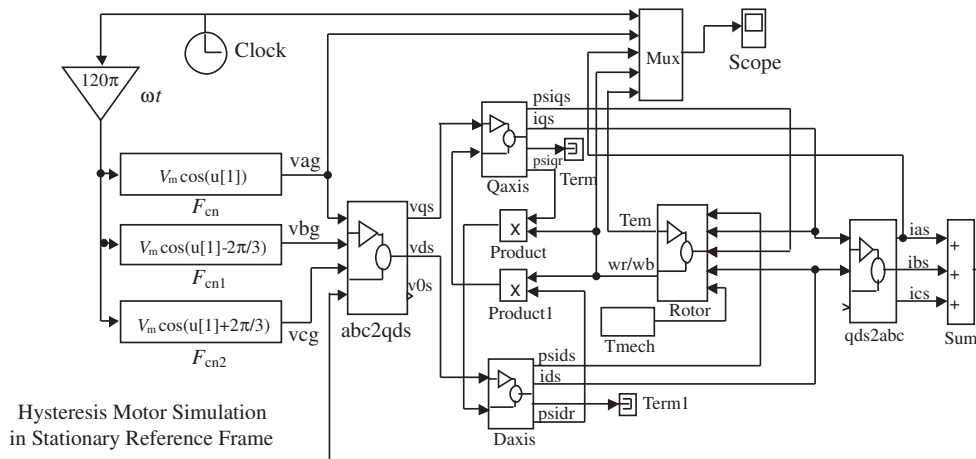
Fig. 1. Simulation of the hysteresis motor using Simulink

$$+ \frac{\omega_r}{\omega_b} \begin{bmatrix} 0 & -X_{ss} & 0 & -X_{srm} \\ X_{ss} & 0 & X_{srm} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{Ds} \\ i_{Qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}, \quad (2)$$

$$T_{em} = (3/2)(P/2)\big(i_Q \lambda_D - i_D \lambda_Q\big), \quad \text{where}$$

$$\lambda_D = X_{ss} i_D + X_{srm} i_d, \quad \lambda_Q = X_{ss} i_Q + X_{srm} i_q. \quad (3)$$

The equation of motion of the hysteresis motor can be finally written for a P-pole machine as in equation (4) as follows:

$$\frac{J}{P/2} \dot{\omega} = T_{em} - T_L. \quad (4)$$

In the model above the rotor is represented by its voltage and current d, q representation. The rotor voltage components are due to the stator magnetic flux which sweeps the rotor and established in it a distributed rotor flux density B_{rotor}. This flux density produces hysteresis losses in the permanent magnet rotor (disc) with a frequency proportional to the sweeping flux waves frequency. The rotor circuit now can be represented like as an equivalent field circuit similar to some extent to other machine types which are really have an actual field voltage ex synchronous motors. The axis of the rotor B_{rotor} lags the axis of the stator m.m.f. F_{stator} by angle δ because of the hysteresis effect in the rotor. As a result to this angle lag the torque is produced in the hysteresis motor. An assumption is made in this paper that the magnetic flux is radial in the air gap and tangential in the hysteresis rotor material, the effect of saliency may be neglected because of the approximately equal large air gap opposite to the direct axis and quadrature axis, respectively. This causes the stator self and mutual inductances to be almost independent of the rotor position. In addition, the rotor hysteresis ring is made of cobalt-steel material (as in permanent magnet machines) which has a reasonable electrical resistivity.

The rotor dimension used in this investigation is tabulated in reference (7). The material selected for the rotor is of type 36 % cobalt-steel.

The non-linear model of the hysteresis motor is simulated using the new trend in computer applications in simulation of different electrical machine types. In this paper, **SIMULINK**, an extension of Matlab program, is used as a simulation tool for determining the dynamics of the hysteresis motor. The simulation block diagram is shown in Fig. 1. The zero sequence loop can be added with no effect here as we assume that the stator has a sinusoidally distributed winding.

The results of the simulation are given under the section results and discussions. Unlike conventional synchronous motors, the hysteresis motor has no preferred synchronizing point. Also, in contrast to a reluctance motor, which must pull its load into synchronism from induction-motor torque-speed characteristics, the hysteresis motor can pull its load to synchronism, no matter what its load inertia.

3 LINEAR MODEL

Quite often, a linearized model of a non-linear system similar to equations (2–4) is needed in performing what is known as small-signal analysis of the electrical machine under investigation. The linearized model of the hysteresis motor can be obtained by performing a small perturbation on all motor variables of the full non-linear model in a synchronously rotating reference frame. The differential-algebraic equations of (2–4) are of the general form:

$$f\big(\dot{\mathbf{x}}, \mathbf{x}, \mathbf{u}, \mathbf{y}\big) = 0 \quad (5)$$

where $\mathbf{x}$ is an n-dimensional state variable vector, $\mathbf{u}$ is an m-dimensional input variable vector, $\mathbf{y}$ is the vector of desirable outputs. The dot accent on the $\mathbf{x}$ vector denotes the operator $\mathrm{d}/\mathrm{d}t$.

When a small displacement, denoted by Δ, is applied to each of the vectors above, and when higher order Δ terms are ignored, the system of equations (4) is written
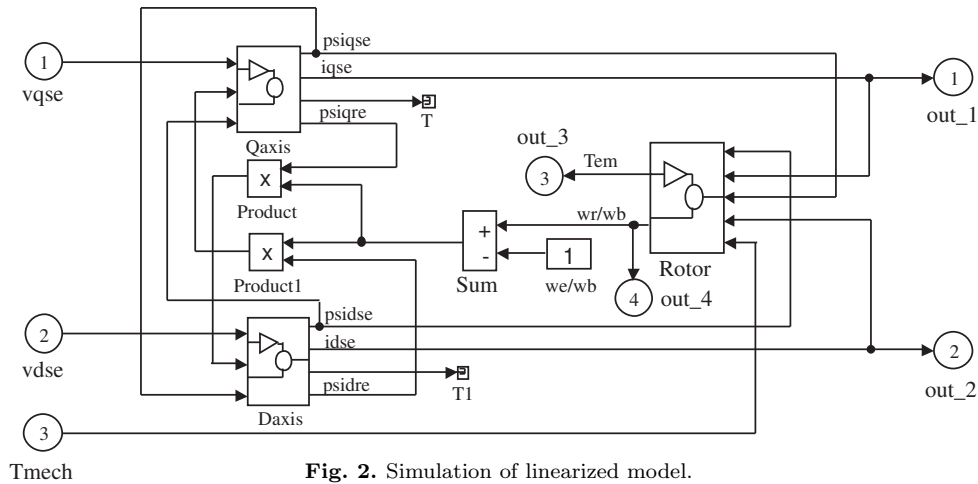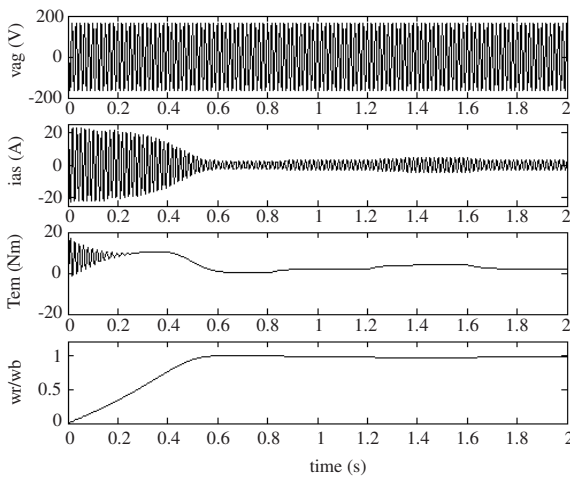
Fig. 2. Simulation of linearized model.

Fig. 3. Simulated phase voltage, phase current, developed torque, PU speed at rated voltage.

Table 1. Eigenvalues of the hysteresis motor at different loadings.

At $T_{mech} = 0.0$	
$-268.9 + j435.6$	$-268.9 - j435.6$
$-31.12 + j23.65$	$-31.12 - j23.65$
-50.40	
At $T_{mech} = -10\,\mathrm{N\,m}$ ($-$ ve sign for monitoring)	
$-287.6 + j441.1$	$-287.6 - j441.1$
$-42.11 + j25.65$	$-42.11 + j25.65$
-32.67	

in the following state-variable form:

$$\Delta\dot{x} = \mathbf{A}\Delta x + \mathbf{B}\Delta u\,,$$
$$\Delta\dot{y} = \mathbf{C}\Delta x + \mathbf{D}\Delta u\,, \tag{6}$$
$$\Delta\dot{x} = \left[\Delta i_{Ds}^{\cdot}\,,\ \Delta i_{Qs}^{\cdot}\,,\ \Delta i_{dr}^{\cdot}\,,\ \Delta i_{qr}^{\cdot}\,,\ \Delta\dot{\omega}\right]^{t},$$

where $\Delta u = [\Delta V_{Ds}\,,\ \Delta V_{Qs}\,,\ \Delta V\,,\ \Delta V\,,\ \Delta T_L]^t$, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ are constant matrices

To perform small-signal stability analysis of the hysteresis motor, the eigenvalues of the system matrix $\mathbf{A}$ is determined in addition to the values of constant matrices $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ if desired. **Matlab/Simulink** program

is used here to perform such an analysis. At the beginning, the steady state of the SIMULINK system of the hysteresis motor of Fig. 1 is determined at some desired operating point using the **trim** function. When the initial values are obtained, then **linmod** Matlab function is used to determine the $[\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}]$ matrices of the small-signal model of the non-linear system about the evaluated steady-state operating point. Other Matlab's useful functions, such as **eig** function or Matlab Control Toolbox, are used to determine eigenvalues, transfer functions and root locus plots. The linearized model is shown in Fig. 2.

4 RESULTS AND DISCUSSIONS

The hysteresis motor used in different theoretical tests has a standard three-phase, four-pole stator winding. The stator is fed with three phase-balanced voltage and at rated frequency. For non-linear simulation, the non-linear mathematical model has been solved numerically using the Runge-Kutta method for solving all differential equations after determining the steady state operating points. The torque angle, *ie* , the angle between the magnetic axes of stator and rotor of the motor and the one responsible for the developed torque has been considered in steady state calculations. The run-up responses for the hysteresis motor are shown in Fig. 3. The load torque has been varied between no load case and $\pm 50\,\%$ of base torque during simulation. The supply phase voltage, stator phase current, the electromagnetic developed torque and per unit motor speed are all recorded during the run-up period until synchronous speed is reached. Then, the supplied voltage has been decreased to half of its rated voltage, and the same variables of Fig. 3 have been recorded at this lesser voltage as in Fig. 4. Here, Fig. 4 shows how the motor can be synchronized at lower voltages (here voltages are taken as 0.5 pu). The results also indicate the high-induced starting torque. Other factors such as increasing resistances or reactance of the motor may affect the higher starting torque. In addition to the non-linear simulation, small-signal stability analysis is performed. Table 1 presents the eigenvalues computation under such dynamic stability analysis. The perturbation taken here
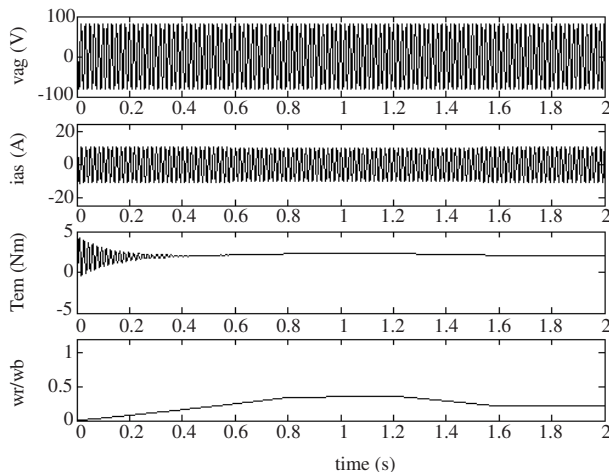
Fig. 4. Simulation results of phase voltage, phase current, developed torque and pu speed at 0.5 of rated voltage.

is by changing the mechanical load torque between no-load and 10 Nm load. Under heavy loading the motor departs from stability and looses synchronism. The motor remains stable as far as the loading is within the limits of the motor's developed torque.

5 CONCLUSION

This paper presents an investigation of the dynamic performance of a three-phase hysteresis motor fed from a three-phase balanced power supply. A mathematical model similar to those representing conventional machines has been adopted to reflect the hysteresis motor operation. Due to the new trend in electrical machine simulation, design and analysis, computer software Matlab/Simulink has been used as a tool to predict the hysteresis motor stability under various operating conditions. The transient and dynamic responses of the motor to different changes such as variation in load torque and reduction in supply voltages have been provided. The results indicate the smooth operation of the motor at a synchronous speed providing normal loading conditions. However, the output of the hysteresis motor is about one-quarter that of an induction motor of the same dimension making it suitable only for small rating applications.

Nomenclature

V_{Ds}, V_{Qs} d, q axis stator voltages (V),

V_{dr}, V_{qr} d, q axis rotor voltages (V),

r_s, r_r stator and rotor resistances (Ω),

X_{ss}, X_{rr} stator and rotor self reactances (Ω),

X_{srm} mutual reactance between the stator and rotor (Ω),

i_{Ds}, i_{Qs} d, q axis stator currents (A),

i_{dr}, i_{qr} d, q axis rotor currents (A),

ω_r rotor angular velocity (rad/s),

ω_b base angular frequency (rad/s),

T_{em} electromagnetic (developed) torque (N·m),

λ_D, λ_Q d, q stator flux linkages (Wb),

T_L load torque (N·m),

J rotor inertia (kg·m²).

Appendix

The tested motor is of 3-hp, 220 V, 3-phase motor. The stator is assumed to be supplied by balanced voltages and the torque angle is approximately equal to the hysteresis lag angle which depends on the B-H curve of the rotor material [7].

f	60 Hz	V_1-1	220 V
P	4 poles	r_r	5.34 Ω/phase
X_{srm}	20 Ω/phase	X_{1r}	3.3 Ω/phase
H	0.45 s	r_s	1.2 Ω/phase
X_{ss}	23.5 Ω/phase	X_{1s}	X_{1r}
ω_b	377	β	40–70 degrees, 0 at steady state.

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