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COMMUNICATIONS

TO DEFINITION OF PARAMETERS OF MEASURING DEVICES

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There are used a variety of often unfairly rigorous formulations when defining the precision of measuring converters and devices. Particularly significant is the case of a nonlinear converter, the conversion error of which depends on the type of curves used to approximate its transfer function. This contribution should be an impulse to those who are interested in standardization as well as to manufacturers of measuring devices.

K e y w o r d s: measuring device, measuring converter, additive error, multiplicative error, linearity error

1 INTRODUCTION

Let us analyze fundamental properties of a part of the measuring chain (MC), Fig. 1. In following text we use the term "converter" and the relation between output and input quantity $\mathbf{Y}$ and $\mathbf{X}$ will be called the "transfer function". The MC in this case can be a simple converter, measuring device or a part of it.

Fig 1 Part of s measuring chain - converter

The essential task of an MC is to provide a linear transfer-function given by the relation as

$$\mathbf{Y} = k \cdot \mathbf{X} \tag{1}$$

In practice it is usual to determinate such an relation by means of two variables with the input and output nominal value $\mathbf{X}_n$ and $\mathbf{Y}_n$. In this case we assume the characteristic to cross the initial point *ie* $\mathbf{X} = \mathbf{0}$, $Y = \mathbf{0}$. Such a relation is ideal, and deviations of it should be considered as the errors.

The manufacturers of MC usually declare: (i) some kind of complex maximal or limit error (class of the precision of electromechanical converters), (ii) additive and multiplicative error components, and (iii) nonlinearity in % of the reading and of the full scale of digital measuring instruments. The consequence is that the actual transfer characteristic of measuring device is unknown.

Real transfer characteristic we can obtain from the measured values of corresponding couples $\mathbf{X}$, $\mathbf{Y}$ and this is possible to approximate by a suitable function. It is obvious that the deviations from an ideal transfer function depend on the shape of approximating (continuous) function and on the method used to calculate appropriate coefficients. Namely the way used to determine the coefficients of the approximation polynomial can considerably affect the uncertainty definined as the "actual" error component.

2 NONLINEAR APPROXIMATION

Let us consider that in general all the three above mentioned error components are present *ie*the transfer characteristic is non-linear. One of suitable approximation is polynomial

$$\mathbf{Y}' = a_0 + a_1\mathbf{X} + a_2\mathbf{X}^2 + a_3\mathbf{X}^3 + \dots \tag{2}$$

The coefficients a_0, a_1, a_2, $a_3 \dots$ can be evaluated from the couples of measured values $(\mathbf{X}, \mathbf{Y})$ using any appropriate method (the most common being the least-mean-square method). In order to simplify the task and avoid a more detailed approach here we do not distinguish between the differential and integral nonlinearity. Let us consider function (2) as the actual characteristic of MC. It was obtained by more precise measurement, expecting deviations from an ideal transfer function.

Graphical interpretation of this characteristics is in Fig. 2. Let $\mathbf{X}$ be the input value of a converter. Since the conversion follows the actual characteristic, there is an output value of $\mathbf{Y}'$, whereas a correct value (according to an ideal characteristic) in this point should be $\mathbf{Y}$ (simple arrows in Fig. 2). The error of output value setting - the conversion error will be

$$\Delta\mathbf{Y} = \mathbf{Y}' - \mathbf{Y}\,. \tag{3}$$

Substituting (1) and (2) into (3) after simple calculation we obtain

$$\Delta\mathbf{Y} = a_0 + (a_1 - k) \cdot \mathbf{X} + a_2\mathbf{X}^2 + a_3\mathbf{X}_3 + \dots \tag{4}$$

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Fig 2 Transfer functions of converter

Fig 3 Linear approximation of actual transfer function

At first sight, in eqn. (4) there are three qualitatively different parts in. The first component (a_0) does not depend on the magnitude of input value, it can be considered as additive error. Component a_0 represents the magnitude of output value at zero input value condition, it is also called error of zero or the offset.

$$\Delta \mathbf{Y}_a = a_0 = \text{constant} \tag{5}$$

The second component, lineary depending on a measured value $\mathbf{X}$ is the multiplicative error or gain error or error of the transfer constant.

$$\Delta \mathbf{Y}_m = (a_1 - k)\mathbf{X} = \mathbf{KX} \tag{6}$$

denoting $\mathbf{K} = a_1 - k\mathbf{X}$.

The rest on the right side of formula (4) is the linearity error depending on the measured value too

$$\Delta \mathbf{Y}_L = a_2 \mathbf{X}^2 + a_3 \mathbf{X}^3 + \dots \tag{7}$$

Among several other possibilities of nonlinear modeling of the converter transfer function let us mention the interpolation covering all couples (X, Y) *eg* using the si-function or the abscissa interpolation. It is evident that for the above mentioned error components different values are obtained.

3 LINEAR APPROXIMATION

In practice it is often usual to use (for simplicity or because of a negligible nonlinearity) a linear approximation of the transfer function (Fig. 3).

Let us consider, at first, an approximation of the transfer function using the least-square method in form

$$\mathbf{Y}'' = b_0 + b_1 \mathbf{X}\,. \tag{8}$$

The additive error in this case can be recognized as

$$\Delta \mathbf{Y}_a = b_0\,. \tag{9}$$

Similarly to the previous case the multiplicative error is given as a difference between the slopes of linear approximation and the ideal transfer function.

$$\Delta \mathbf{Y}_m = (b_1 - k)\mathbf{X} \tag{10}$$

Because the coefficients in formulas (3) and (8) are different ($a_0 \neq b_0$, $a_1 \neq b_1$), in both kinds of approximation the values of error components will be different too. The result is shown in Fig. 3.

Using both analyzed approximations the values obtained by nonlinear and linear approximations should be consistently distinguished. As an error of zero (offset) we should consider the value a_0, because of $\mathbf{Y} = a_0$ occurs at output if $\mathbf{X} = \mathbf{0}$ at input, and as the additive error b_0, since it represent the shift of the linear approximation towards the ideal transfer function.

A similar problem occurs at definition and evaluation of the multiplicative error. Also this component should be evaluated as a difference of the slopes (10). There is a question if there is any difference between the slopes at the linear and nonlinear interpolations and what is a possible physical meaning of the coefficient a_1 in eqn. (2)

The linearity error we can evaluate as a difference of linear and nonlinear approximations

$$\Delta \mathbf{Y}_L = \mathbf{Y}' - \mathbf{Y}'' \tag{11}$$

giving in general another result as by eqn. (7). The situation can be more complex if one will distinguish the differential and integral nonlinearities.

There are no problems if we consider only the linear approximation, then formulas (9) and (10) are valid and the linearity error is zero.

4 LINEAR INTERPOLATION

The coefficients in eqn. (2) and (8) depend also on the method of the approximation. The most suitable is apparently the least-square method. Sometimes a linear interpolation through points $(0, a_0)$ and $(\mathbf{X}_n, \mathbf{Y}'_n)$, where $\mathbf{X}_n$ and $\mathbf{Y}'_n$ are nominal values of the input and corresponding (nominal) output quantities on the actual (precisely measured) transfer function (Fig. 3), is used. This, done perhaps to avoid the before mentioned discrepancy or from other reasons, may be not fully correct.

$$\mathbf{Y}''' = c_0 + c_1\mathbf{X}\,. \tag{12}$$

In this case is $a_0 = c_0$ and there no problems with the definition of the additive error are encountered. Definition of the multiplicative error is ambiguous as long as $a_1 \neq c_1$.

5 ERROR OF MEASUREMENT

If the converter fulfills its purpose of the use (measuring), from the value of output quantity $\mathbf{Y}$ the value of input quantity $\mathbf{X}$ can be determined. As labeled above, the real output value is $\mathbf{Y}'$ (Fig. 2). If the actual transfer function is not known and we determine the input quantity $\mathbf{X}'$ from an ideal transfer function, the measurement error of input quantity will be

$$\Delta\mathbf{X} = \mathbf{X}' - \mathbf{X}\,. \tag{13}$$

Let us multiply eqn. (13) by the slope of an ideal transfer function. At the right side we obtaine the corresponding difference of output quantity which according to (3) is the conversion error

$$k\Delta\mathbf{X} = k\mathbf{X}' - k\mathbf{X} = \mathbf{Y}' - \mathbf{Y} = \Delta\mathbf{Y} \tag{14}$$

and

$$\Delta\mathbf{Y} = k\Delta\mathbf{X} \qquad \Delta\mathbf{X} = \Delta\mathbf{Y}/k\,. \tag{15}$$

Because the total error is a sum of the additive multiplicative and the linearity errors (15) is applicable for theses errors too. It can be observed that it does no mater if we consider the input or output values, the results are easy to recalculate.

6 RELATIVE ERROR

Let us define the relative error of a measurement and the relative error of a conversion as the ratio of absolute error to corresponding quantity

$$\delta_X = \frac{\Delta X}{X} \tag{16}$$

$$\delta_Y = \frac{\Delta Y}{Y} \tag{17}$$

in view of the fact that $\Delta Y = k\Delta\mathbf{X}$ a $\mathbf{Y} = k\mathbf{X}$ follows

$$\delta_X = \delta_y \tag{18}$$

6 CONCLUSION

The aim of above mentioned contemplates was to point out that even such fundamental concepts as the additive and multiplicative errors or the nonlinearity may not be always simply and unambiguously defined. Products offered to us by the manufacturers of measuring devices in their users manuals *etc*should contain at least the information about used definitions. Even if using the marginal (maximum) errors only, their values will depend on their definition. It should be meaningful to precise the definitions of the error components and appropriate terminology or to provide some recommendations on this matter, as seems to be the simpler way.

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