Journal of ELECTRICAL ENGINEERING, VOL. 52, NO. 5-6, 2001, 134–138

A NEW METHODOLOGY OF SIMPLE CONTROLLER DESIGN FOR TIME DELAY SYSTEMS PART ONE — STABLE SYSTEM

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The paper presents a new methodology of the controller design for time delay systems. The proposed method based on a polynomial approach and LQ control theory yields a class of CT controllers ensuring setpoint tracking as well as load disturbance attenuation. The 2DOF control configuration was considered. The resulting controller obtained via polynomial Diophantine equations and spectral factorization technique are stable. Time delay terms are approximated by two methods and they are incorporated into the control design as a perturbation. The obtained control algorithms were compared and analysed for a stable first order time delay system.

K e y w o r d s: Time delay system, time delay approximation, polynomial method, LQ control.

1 INTRODUCTION

The existence of a time delay in input-output relations is a common property of many technological processes. Plants with a time delay often can not be controlled using usual controllers designed without a consideration for the presence of a dead-time. The control responses using such controllers then tends to destabilize the closed-loop system.

It is well known that as an effective time-delay compensator especially for stable systems with long time-delays the Smith predictor can be used. The control system configurations based on the Smith predictor inclusive of various methods for tuning the controller parameters have been developed by many authors [1], [3], [6] and [8] for stable and also unstable systems. Another possible approach to the time-delay compensation in stable systems based on the inversion of dynamics can be found *eg*in the work [10]. The procedure based on operations in the ring of rational functions and on the norm minimization in the frequency domain has been described in [9].

Here presented procedure is proposed for a stable first order time delay system and follows from two different approximations of the time delay. Then, the polynomial approach is used to obtain the controllers in the 2DOF control configuration, see, *eg*[5], [7]. For a tuning of the controller parameters the LQ control technique described in detail in [4] is employed. The simulation results show a satisfactory control quality also for a higher ratio between the time delay and the controlled system time constant.

2 THEORETICAL PART

2.1 Approximate transfer functions

Consider the transfer function of a stable first order time delay controlled system having the form

$$G(s) = \frac{y(s)}{u(s)} = \frac{K}{\tau s + 1} e^{-\tau_d s} \tag{1}$$

where $K > 0$ is the gain, $\tau > 0$ is the time constant and $\tau_d > 0$ is the time delay. Time delay transforms are approximated by

- the Taylor numerator expansion

$$e^{-\tau_d s} \approx 1 - \tau_d s \tag{2}$$

- the Padé approximation

$$e^{-\tau_d s} \approx \frac{1 - \frac{\tau_d}{2} s}{1 + \frac{\tau_d}{2} s}. \tag{3}$$

Using approximation (2), the approximate transfer function takes the form

$$G_{A1}(s) = \frac{K(1 - \tau_d s)}{\tau s + 1} = \frac{b_0 - b_1 s}{s + a_0} \tag{4}$$

where $b_0 = \frac{K}{\tau}$, $b_1 = \frac{K\tau_d}{\tau}$ and $a_0 = \frac{1}{\tau}$. Using approximation (3) the approximate transfer function is expressed as

$$G_{A2}(s) = \frac{K(1 - \frac{\tau_d}{2} s)}{(\tau s + 1)(1 + \frac{\tau_d}{2} s)} = \frac{b_0 - b_1 s}{s^2 + a_1 s + a_0} \tag{5}$$

where $b_0 = \frac{2K}{\tau \tau_d}$, $b_1 = \frac{K}{\tau}$, $a_0 = \frac{2}{\tau \tau_d}$ and $a_1 = \frac{2\tau + \tau_d}{\tau \tau_d}$.

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ISSN 1335-3632

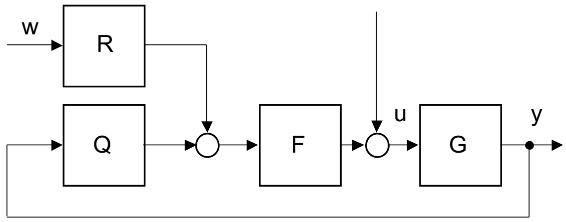

Fig. 1. The 2DOF control system configuration.

2.2 Control system description

The 2DOF control system configuration is depicted in Fig. 1. There the controller contains both feedback and feedforward parts. In Fig. 1, w is the reference signal, v is the load disturbance and u is the control variable. Both w and v are considered to be step functions. The transfer function of the controlled system is assumed as

$$G(s) = \frac{b(s)}{a(s)} \tag{6}$$

where b and a are coprime polynomials in s that fulfill the inequality $\deg b \leq \deg a$. In the next sections, G will represent approximate transfer functions (4) and (5). Transfer functions Q and R of the controller feedback and feedforward parts without the compensator F have the form

$$Q(s) = \frac{q(s)}{p(s)}\,, \tag{7}$$

$$R(s) = \frac{r(s)}{p(s)} \tag{8}$$

where q, r, p are polynomials in s. For the stepwise reference and the step load disturbance, F is an integrator with the transfer function

$$F(s) = \frac{1}{s} \tag{9}$$

and in Fig. 1, the equality $\tilde{u} = \dot{u}$ holds. Considering (7), (8) and (9), the complete transfer functions of the controller take the forms

$$\tilde{Q}(s) = Q(s)F(s) = \frac{q(s)}{s\,p(s)} \tag{10}$$

$$\tilde{R}(s) = R(s)F(s) = \frac{r(s)}{sp(s)}\,. \tag{11}$$

A stable controller having no positive exponential modes at its output under the step input is required. This condition is fulfilled when polynomial p in the denominators of (10) and (11) is a stable polynomial.

2.3 Polynomial method

The controller design described in this section follows from the polynomial approach. The general conditions required to govern the control system properties are formulated as internal properness and stability of the control system, asymptotic tracking of a reference and a load disturbance attenuation. The procedure to derive admissible controllers can be carried out as follows:
The feedback part of the controller given by a solution of the polynomial equation

$$a(s)sp(s) + b(s)q(s) = d(s) \tag{12}$$

with a stable polynomial d on the right side ensures the stability and the load disturbance attenuation in the control system. Asymptotic tracking provides the controller feedforward part given by a solution of the polynomial equation

$$z(s)s + b(s)r(s) = d(s)\,. \tag{13}$$

The control system satisfies the condition of internal properness when the transfer functions of all components are proper. The degrees of the controller polynomials (inclusive of the integrator) then fulfill inequalities

$$\deg q \leq \deg p + 1 \tag{14}$$

$$\deg r \leq \deg p + 1\,. \tag{15}$$

Taking into account (14) and (15), the condition $\deg b \leq \deg a$ and the solvability of (12) and (13), the degrees of polynomials q and r can be derived to be

$$\deg q = \deg a\,, \quad \deg r = 0\,. \tag{16}$$

Moreover, the equality $r_0 = q_0$ can be easily obtained.

The controller parameters then follow from solutions of polynomial equations (12) and (13) and depend upon coefficients of polynomial d. The next problem here means to find a stable polynomial d that allows to obtain the acceptable stabilizing and stable controller.

2.4 Application of the LQ control theory

The idea of the procedure proposed here issues from the *optimal* control theory. The goal of optimal deterministic LQ tracking and load disturbance attenuation is to design controllers that enable the control system to satisfy the above basic requirements and, in addition, the control law minimizes the cost function

$$J = \int_0^\infty \left\{\mu e^2(t) + \varphi \dot{u}^2(t)\right\} \mathrm{d}t \tag{17}$$

where $\mu \geq 0$ and $\varphi \geq 0$ are weighting coefficients. Note that other quadratic performance criterions to obtain the controller parameters are used *eg* in the work [2]. However, these only penalize the tracking error. The design procedure can be now realized as follows:

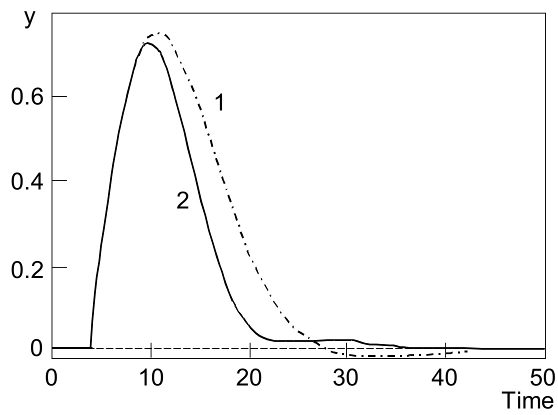

Fig. 2. Load disturbance responses: $\tau_d = 4$, $\varphi = 1$. (1)-the numerator, (2)-the Padé approximation.

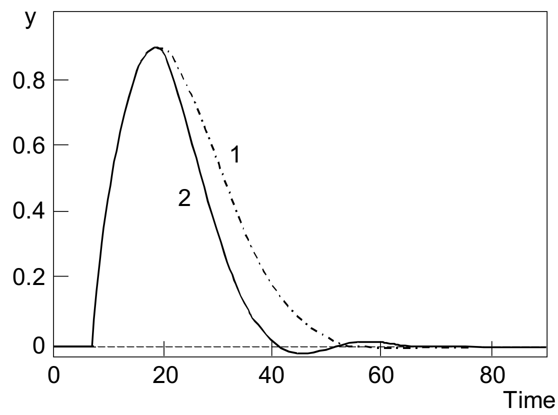

Fig. 3. Load disturbance responses: $\tau_d = 8$, $\varphi = 1$. (1)-the numerator, (2)-the Padé approximation.

Calculate stable polynomials g and n as the results of the spectral factorizations

$$(sa(s))^* \varphi (sa(s)) + b^*(s)\mu b(s) = g^*(s)g(s) \tag{18}$$

$$n^*(s)n(s) = a^*(s)a(s) \tag{19}$$

where the asterisk denotes a conjugate polynomial. The polynomials p, q and r then are given by solutions of polynomial equations (12) and (13) with $d(s) = g(s)n(s)$. Note that the resulting control is optimal only for zero initial conditions of the signals in the control system. However, this procedure makes possible to obtain always stable polynomials on the right sides of polynomial equations (12), (13) and, consequently, the resulting feedback controller stabilizes the control system. Some details of the procedure are described in the work [4].

The transfer functions of both controller parts (inclusive the integrator) are strictly proper. The degree of the right side of (12) and (13) is given by $\deg[g(s)n(s)] = 2\deg a(s) + 1$. Taking into account (16) and the relation $\deg[sp(s)] = \deg[g(s)n(s)] - \deg a(s) = \deg a(s) + 1$, the strict properness of both (10) and (11) is evident.

2.5 Controller design

After some manipulations, the structures of controllers for the time delay approximations together with formulas for their parameter calculation can be obtained. All formulas contain parameters K, τ and τ_d of the transfer function (1). Moreover, the polynomial $a(s)$ in (4), (5) is always stable and, consequently, $n(s) = a(s)$ and the spectral factorization (19) need not be solved.

For the numerator approximation with approximate transfer function (4) and $\deg a = 1$, the spectral factor takes the form

$$g(s) = s^2 + g_1 s + g_0 \tag{20}$$

where

$$g_0 = \frac{K}{\tau}\sqrt{\frac{\mu}{\varphi}}, \; g_1 = \frac{1}{\tau}\sqrt{K^2\tau_d^2\frac{\mu}{\varphi} + 2K\tau\sqrt{\frac{\mu}{\varphi}} + 1}\,. \tag{21}$$

The transfer functions of both parts of the controller are

$$\tilde{Q}(s) = \frac{q_1 s + q_0}{s(s+p_0)}, \quad \tilde{R}(s) = \frac{r_0}{s(s+p_0)} \tag{22}$$

with parameters

$$p_0 = g_1 + \tau_d g_0, \quad q_0 = \frac{1}{K}g_0, \quad q_1 = \frac{\tau}{K}g_0\,. \tag{23}$$

Evidently, $p_0 > 0$ and the resulting controller is stable.

For the Padé approximation with the approximate transfer function (5) and $\deg a = 2$, the spectral factor normed on the highest power of s results

$$g(s) = s^3 + g_2 s^2 + g_1 s + g_0 \tag{24}$$

where

$$g_0 = \frac{2K}{\tau\tau_d}\sqrt{\frac{\mu}{\varphi}},$$
$$g_1 = \frac{1}{\tau\tau_d}\sqrt{4\Big(K\tau\tau_d\sqrt{\frac{\mu}{\varphi}}\,g_2 + 1\Big) + K^2\tau_d^2\frac{\mu}{\varphi}}, \tag{25}$$
$$g_2 = \frac{1}{\tau\tau_d}\sqrt{2\tau^2\tau_d^2\sqrt{\frac{\mu}{\varphi}}\,g_1 + 4\tau^2 + \tau_d^2}\,.$$

The transfer functions of the controller are

$$\tilde{Q}(s) = \frac{q_2 s^2 + q_1 s + q_0}{s(s^2 + p_1 s + p_0)}, \quad \tilde{R}(s) = \frac{r_0}{s(s^2 + p_1 s + p_0)} \tag{26}$$

with parameters

$$p_0 = g_1 + \frac{\tau_d}{2}g_0, \quad p_1 = g_2,$$
$$q_0 = \frac{1}{K}g_0, q_1 = \frac{2\tau + \tau_d}{2K}g_0, q_2 = \frac{\tau\tau_d}{2K}g_0\,. \tag{27}$$

Evidently, $p_0 > 0$, $p_1 > 0$ and the resulting controller is stable.

It can be seen that for any controlled system with given parameters, and by choosing $\mu = 1$, all parameters depend upon the only one selectable parameter φ. The control responses then can be tuned just through its choice.

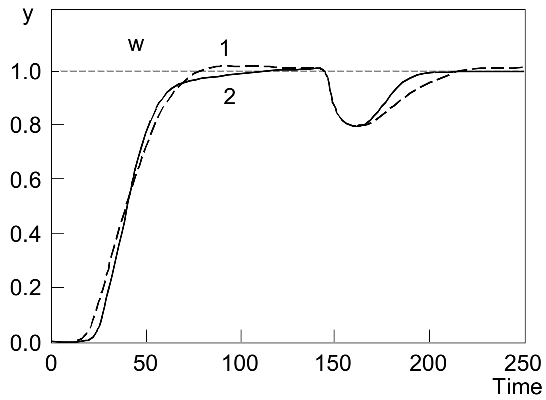

Fig. 4. Step setpoint and load disturbance responses: $\tau_d = 16$, $\varphi = 100$. (1)-the numerator, (2)-the Padé approximation.

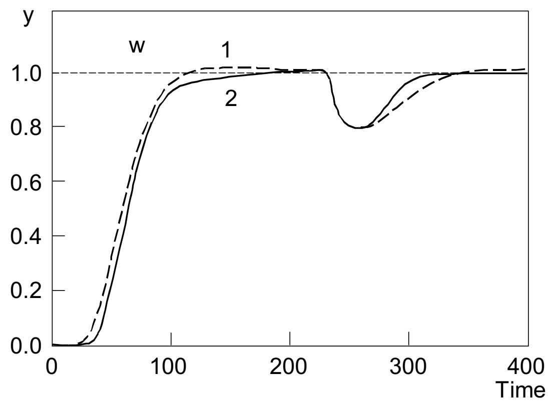

Fig. 5. Step setpoint and load disturbance responses: $\tau_d = 24$, $\varphi = 400$. (1)-the numerator, (2)-the Padé approximation.

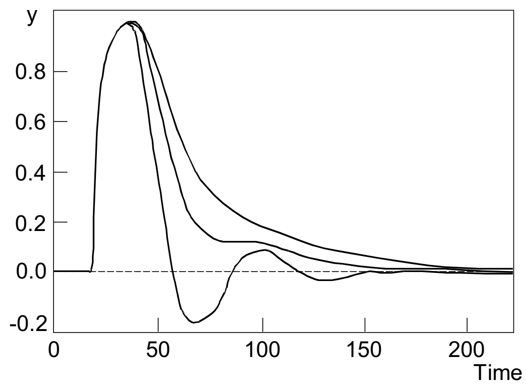

Fig. 6. Load disturbance responses using the Padé approximation: $\tau_d = 20$.

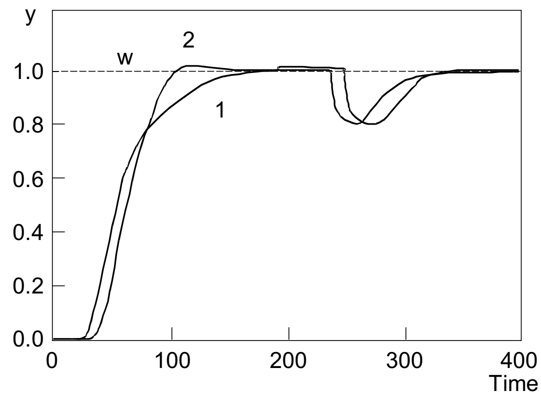

Fig. 7. Step setpoint and load disturbance responses using the Padé approximation: $\varphi = 400$. (1) – $\tau_d = 19.2$, (2) – $\tau_d = 28.8$.

3 EXAMPLE

Consider the first order time delay system with transfer function (1) where $K = 1$ and $\tau = 4$. All simulations were performed by MATLAB-Simulink tools.

The unit load disturbance responses for $\tau_d = \tau$ and $\tau_d = 2\tau$ are shown in Figs. 2, 3. The responses clearly show the usability of both approximations. There are no strong differences between responses. A slightly higher overshoot for the numerator approximation can be judged to be insignificant. The coresponding results provide the unit setpoint and load disturbance responses for $\tau_d = 4\tau$ and $\tau_d = 6\tau$ shown in Figs. 4, 5 where the disturbance $v(t) = -0.2$ was incorporated into the controlled system at times $t = 130$ and $t = 220$.

The effect of parameter φ upon the control responses is shown in Fig. 6. An increasing φ improves the control stability and by choosing its higher value aperiodic responses can be obtained.

The unit setpoint and load disturbance responses in Fig. 7 demonstrate the robustness of the proposed method against changes of τ_d. The controller parameters were computed for a nominal model with $\tau_d = 24$ and subsequently used for perturbed models with a $\pm 20\,\%$ error in estimating τ_d ($\tau_d = 28.8$ and $\tau_d = 19.2$).

4 CONCLUSION

The problem of controlling the time delay systems has been solved and analysed. The proposed method is based on time delay approximations. The controller design makes use of the polynomial synthesis and the controller setting employs the results of the LQ control theory. The presented results demonstrate the usability of the method and the control of good quality also for a relatively high ratio of the time delay to the time constant ($\tau_d/\tau \approx 6$). Moreover, the method provides robust controllers for an estimation error in τ_d. The methodology can be easily extended to higher order controlled systems, higher degree of approximation and for unstable time delay systems.

Acknowledgements

This work was supported in part by the Grant Agency of the Czech Republic under grants No. 102/00/0526 and

No. 102/99/1292 and by the Ministry of Education of the Czech Republic under grant MSM 2811 00001.

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Received 7 November 2000

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