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REFLECTION OF PLANE WAVE FROM AN INFINITE MICROSTRIP ARRAY

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A solution is presented for the problem of the incidence and reflection of a plane wave from an infinite microstrip array for various directions of incidence. A full-wave treatment of the electric current surface density excited on the array patches is accomplished using the moment method in the spectral domain. As a result, the phase and absolute value of reflection coefficients for various polarizations are calculated.

K e y w o r d s: microstrip array, reflectarray, moment method, spectral domain.

1 INTRODUCTION

The concept of the probe-fed phased microstrip array [1], [2] has been studied widely as a possible candidate for the integrated phased array at millimeter wave band. Those arrays combine the best features of microstrip antennas and far-field pattern synthesis of phased arrays. One of the recent advances in this field is a microstrip reflectarray [3], which seems to be a good alternative to the parabolic antenna thanks to its flat conform shape and low cost. This approach is especially convenient for high-gain millimeter wave antennas because losses in a microstrip feed network at high frequencies are often unacceptable.

One of the key steps when analysing a microstrip reflectarray is a full-wave moment-method solution for the amplitude and phase of the field reflected from an infinite uniform array of microstrip patches versus patch length for various incidence angles [3]. The required design data for the microstrip reflectarray can be then determined from the phase-delay condition (which is essentially the same as that for a parabolic reflector) with interpolating the above mentioned reflection coefficients.

The purpose of this paper is to give a comprehensive insight into the moment-method solution of the electric current surface density excited on the array patches and consequently to the computation of the reflected and scattered electric field from an infinite microstrip array.

An alternative approach to the plane wave reflection has been given in [4], where the electric field is expanded in terms of a complete set of TM and TE orthogonal Floquet modes instead of using the rigorous spectral domain Green's function of the grounded dielectric slab, as considered in our approach.

2 THEORY

2.1 Some definitions

Let us assume an infinite uniform microstrip array with element size L, W and element spacing a, b in x and y directions respectively (Fig. 1). The origin is located at the bottom of the grounded substrate, in the centre of an arbitrary patch, z-axis is perpendicular to the substrate. The patches are assumed to be made of a perfect conductor and to resonate along the x direction. The substrate is of thickness d, relative permittivity ε, and loss-tangent $\tan\delta$. The dielectric losses can be readily accounted for by introducing complex permittivity $\hat{\varepsilon}_r = \varepsilon_r(1-j\tan\delta)$. The incident plane wave shall be

$$\vec{E}^i = \vec{E}_0 e^{jk_0(xu_i+yv_i+z\cos\theta_i)} \tag{1}$$

where $\vec{E}_0$ defines the amplitude and polarization of the incident plane wave and

$$u_i = \sin\theta_i\cos\phi_i, \qquad v_i = \sin\theta_i\sin\phi_i \tag{2}$$

are the direction cosines of the wave (θ_i, ϕ_i are the angles of incidence). The phase reference for all of the fields is at the origin (bottom of substrate). Then the total field reflected from the array in the specular direction $(\theta_i, \phi_i+\pi)$ can be written as [3]

$$\begin{bmatrix} E^T_\theta \\ E^T_\phi \end{bmatrix} = \begin{bmatrix} S_{\theta\theta}+R_{\theta\theta} & S_{\theta\phi} \\ S_{\phi\theta} & S_{\phi\phi}+R_{\phi\phi} \end{bmatrix} \cdot \begin{bmatrix} E_{0\theta} \\ E_{0\phi} \end{bmatrix} e^{jk_0(xu_i+yv_i-z\cos\theta_i)} \tag{3}$$

which can be rewritten in vector form

$$\vec{E}^T = \left[\overline{\overline{S}}(\theta_i,\phi_i) + \overline{\overline{R}}(\theta_i,\phi_i)\right] \cdot \vec{E}_0 e^{jk_0(xu_i+yv_i-z\cos\theta_i)} \tag{4}$$

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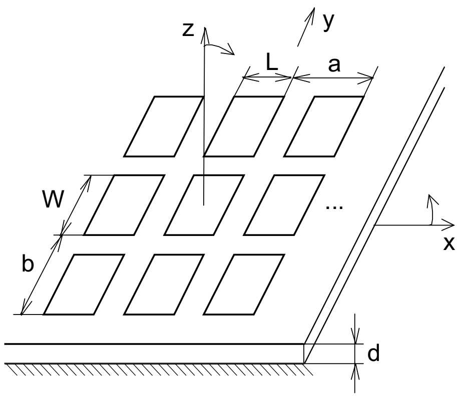

Fig. 1. Geometry of an infinite array of microstrip patches. The ground plane is in the $z = 0$ plane and the top of the substrate is in the $z = d$ plane.

The diagonal matrix $\overline{\overline{R}}(\theta_i, \phi_i)$ represents the reflection of the plane wave in absence of microstrip patches and is given in [3]. The elements of the scattering matrix $\overline{\overline{S}}(\theta_i, \phi_i)$ represent the scattering of the plane wave by microstrip patches and will be determined in the following sections.

2.2 The moment method formulation

In this section we are going to compute the electric current surface density excited on the array patches by using the moment method in the spectral domain. The surface current is now expanded in a set of basis functions as [5]

$$\vec{J}(x,y) = \sum_j I_j \vec{J}_j(x,y) \tag{5}$$

where $\vec{J}_j$ is an expansion mode representing current flow in either x or y direction, and I_j is an unknown complex coefficient. In interest of clarity we define x or y directed expansion mode as

$$\vec{J}^x_m(x,y) = \begin{bmatrix} J^x_m(x,y) \\ 0 \\ 0 \end{bmatrix}, \quad \vec{J}^y_n(x,y) = \begin{bmatrix} 0 \\ J^y_n(x,y) \\ 0 \end{bmatrix}. \tag{6}$$

In the following we will use piecewise sinusoidal (PWS) expansion modes defined as [5]

$$\begin{aligned} J^x_m(x,y) &= \frac{\sin k_e(h_x/2 - |x - x_m|)}{h_y \sin(k_e h_x/2)} \\ J^y_n(x,y) &= \frac{\sin k_e(h_y/2 - |y - y_n|)}{h_x \sin(k_e h_y/2)} \end{aligned} \tag{7}$$

for $|x - x_m| < h_x/2$, $|y - y_n| < h_y/2$, where h_x, h_y are the lengths of the expansion modes in x and y directions respectively, x_j, y_j are the x, y co-ordinates of the center of mode j (respectively to the center of the patch), and $k_e = k_0\sqrt{(\varepsilon_r + 1)/2}$ is the effective wavenumber in the substrate. Fourier transforms of the PWS expansion modes are given [2]

$$\tilde{J}^x_m(k_x,k_y) = \tilde{F}_1(k_x,h_x)\tilde{F}_2(k_y,h_y)e^{jk_x x_m}e^{jk_y y_m} \tag{8a}$$

$$\tilde{J}^y_n(k_x,k_y) = \tilde{F}_1(k_y,h_y)\tilde{F}_2(k_x,h_x)e^{jk_x x_n}e^{jk_y y_n} \tag{8b}$$

where k_x, k_y are spectral domain variables and

$$\tilde{F}_1(k_q,h_q) = \frac{2k_e\left(\cos\frac{k_q h_q}{2} - \cos\frac{k_e h_q}{2}\right)}{(k_e^2 - k_q^2)\sin\frac{k_e h_q}{2}} \qquad q = x, y \tag{9a}$$

$$\tilde{F}_2(k_q,h_q) = \frac{\sin\frac{k_q h_q}{2}}{\frac{k_q h_q}{2}} \qquad q = x, y \tag{9b}$$

It is understood that

$$\tilde{\vec{J}}^x_m(k_x,k_y) = \begin{bmatrix} \tilde{J}^x_m(k_x,k_y) & 0 & 0 \end{bmatrix}^\top, \tag{10a}$$

$$\tilde{\vec{J}}^y_n(k_x,k_y) = \begin{bmatrix} 0 & \tilde{J}^y_n(k_x,k_y) & 0 \end{bmatrix}^\top. \tag{10b}$$

We now follow the moment method procedure by calculating the impedance matrix elements [3]

$$Z_{ij} = \frac{jZ_0}{abk_0}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\left[\tilde{\vec{J}}_i(k_{xm},k_{yn})\right]^{\boldsymbol{H}} \cdot \overline{\overline{G}}(k_{xm},k_{yn}) \cdot \tilde{\vec{J}}_j(k_{xm},k_{yn}) \tag{11}$$

where $\boldsymbol{H}$ means Hermit's adjoint matrix (transposed and conjugated), Z_0, k_0 are the wave impedance and wavenumber of the air and the Floquet wavenumbers are defined as

$$k_{xm} = \frac{2\pi m}{a} + k_0 u_i, \qquad k_{yn} = \frac{2\pi n}{b} + k_0 v_i. \tag{12}$$

$\overline{\overline{G}}(k_{xm},k_{yn})$ is the dyadic Green's function for the grounded dielectric slab

$$\overline{\overline{G}}(k_{xm},k_{yn}) = \begin{bmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{yx} & G_{yy} & G_{yz} \\ G_{zx} & G_{zy} & G_{zz} \end{bmatrix} \tag{13}$$

where its components are defined as [7]

$$G_{qz} = G_{zq} = \frac{k_1 k_q \sin k_1 d}{T_m} \qquad \text{for } q = x, y \tag{14}$$

where T_m represents transverse electric surface wave poles [2] (distinction must be made between T_m and reflection coefficients $\overline{\overline{T}}(\theta_i, \phi_i)$ defined in (20)). The other components $G_{xx}, G_{yy}, G_{xy} = G_{yx}, k_1$ can be found in [2] and G_{zz} will not be used in our computations. The voltage vector elements can be expressed using the reciprocity theorem as [3]

$$V_i = \frac{-j}{k_0}\left[\tilde{\vec{J}}_{s0}(\theta_i,\phi_i)\right]^\top \cdot \overline{\overline{G}}(-k_0u_i, -k_0v_i) \cdot \tilde{\vec{J}}_i(-k_0u_i, -k_0v_i)e^{jk_0 d\cos\theta_i} \tag{15}$$

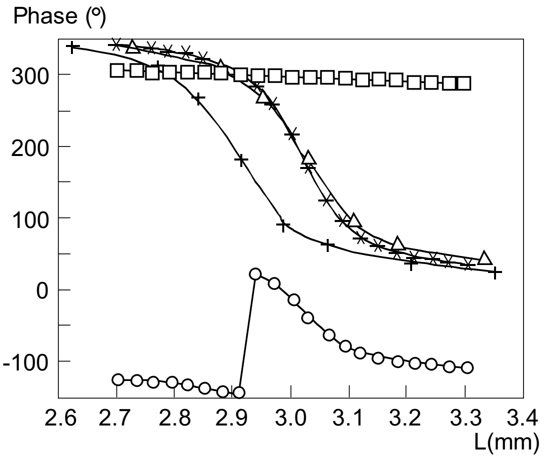

Fig. 2. Phase of the total reflected field from an infinite array of microstrip patches versus patch length. $\theta_i = \phi_i = 0$ (normal incidence), $f = 28$ GHz, $\varepsilon_r = 2.95$, $d = 0.254$ mm, $\tan\delta = 0$, $a = b = 0.536$ cm, $W = 0.2915$ cm. Legend: $*$ Phase$(T_{\theta\theta})$, □ Phase$(T_{\phi\phi})$, ○ Phase$(T_{\theta\phi})$ =Phase$(T_{\phi\theta})$, + Phase$(T_{\theta\theta})$ [3] for $L_0 = W = 0.2915$cm, △Phase$(T_{\theta\theta})$ [3] for $L_0 = 0.303$cm.

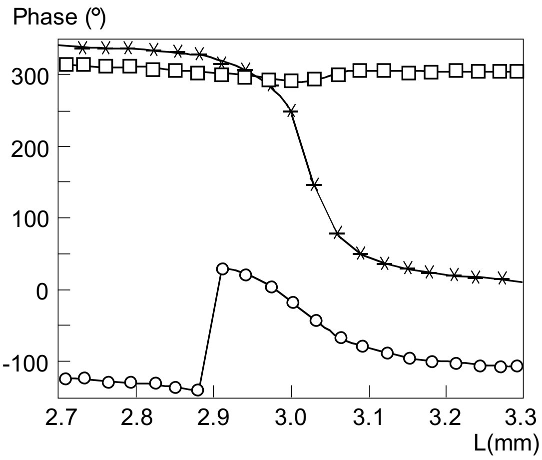

Fig. 3. Phase of the total reflected field from an infinite array of microstrip patches versus patch length. $\theta_i = \phi_i = \pi/8$, $f = 28$ GHz, $\varepsilon_r = 2.95$, $d = 0.254$ mm, $\tan\delta = 0$, $a = b = 0.536$ cm, $W = 0.2915$ cm. Legend: $*$ Phase$(T_{\theta\theta})$, □ Phase$(T_{\phi\phi})$, ○ Phase$(T_{\theta\phi})$ =Phase$(T_{\phi\theta})$.

where

$$\widetilde{\vec{J}}_{s0}(\theta_i, \phi_i) = -2 \begin{bmatrix} E_{0\theta}\cos\phi_i - E_{0\phi}\cos\theta_i\sin\phi_i \\ E_{0\theta}\sin\phi_i + E_{0\phi}\cos\theta_i\cos\phi_i \\ 0 \end{bmatrix}. \quad (16)$$

The vector of unknown expansion coefficients $[I]$ can be found as [5]

$$[I] = [Z]^{-1}[V] \quad (17)$$

We have found that good results can be obtained using just one PWS expansion mode for each current polarisation per patch, which gives $x_j = y_j = 0$ and $h_x = L$, $h_y = W$. In this case it is sufficient to use about 100 terms $(-50 < m, n < 50)$ in each series in expression (11).

2.3 Reflected and scattered field

Once we have obtained expansion coefficients I_j, the current distribution on the array is determined. Note that for incident plane wave arriving from direction (θ_i, ϕ_i), the specular direction of the reflected plane wave is $(\theta_r = \theta_i, \phi_r = \phi_i + \pi)$. That implies the phasing of the currents induced on m, n-th microstrip patch as $e^{jk_0(mau_r+nbv_r)}$, where u_r, v_r are the direction cosines for the reflection angle (θ_r, ϕ_r) similar to those in equation (2). Now we are able to compute the field scattered by the microstrip patches

$$\vec{E}(x,y,z) = \frac{-jZ_0}{abk_0}\sum_m\sum_n \overline{\overline{G}}(k_{xm}, k_{yn}) \cdot \sum_{j=1}^{N} I_j \widetilde{\vec{J}}_j(k_{xm}, k_{yn}) e^{-jk_{xm}x} e^{-jk_{yn}y} e^{-jk_2(z-d)} \quad (18)$$

where N is the total number of expansion modes per patch,

$$k_{xm} = \frac{2\pi m}{a} + k_0 u_r, \quad k_{yn} = \frac{2\pi n}{b} + k_0 v_r, \quad (19a)$$

$$k_2 = \sqrt{k_0^2 - \beta^2}, \quad \beta = \sqrt{k_{xm}^2 + k_{yn}^2}, \quad (\operatorname{Im} k_2 < 0) \quad (19b)$$

Equation (18) is generalisation of [6, Eq.3] for observation point (x, y, z) located above the substrate. We have arrived to this expression by considering the exact electric field at (x, y, z) due to a unit x or y directed current source located on the surface of the substrate as given in [7, Eq.18].

The computation of elements of scattering matrix $\overline{\overline{S}}(\theta_i, \phi_i)$ is straightforward (for given polarisation vector $\vec{E}_0$ we have to determine the scattered field $\vec{E}(n\lambda_0, \theta_r, \phi_r)$ in spherical co-ordinates where $n = 1, 2, \ldots$ and λ_0 is the free space wavelength). We have found that 20 terms in each series in (18) are sufficient $(-10 < m, n < 10)$.

3 SOME RESULTS

We have numerically computed the phase and magnitude of the total reflection coefficients for two incidence angles versus various patch lengths. These coefficients are defined as

$$\overline{\overline{T}}(\theta_i, \phi_i) = [\overline{\overline{S}}(\theta_i, \phi_i) + \overline{\overline{R}}(\theta_i, \phi_i)] = \begin{bmatrix} T_{\theta\theta} & T_{\theta\phi} \\ T_{\phi\theta} & T_{\phi\phi} \end{bmatrix}. \quad (20)$$

The data obtained for normal incidence reveal that the magnitudes of $T_{\theta\theta}$ and $T_{\phi\phi}$ are constantly equal to 1, while the phase of $T_{\theta\theta}$ (Fig. 2) shows a behaviour similar to those of [3, Fig. 5]. According to our calculation, the resonant length of the patch L_0 for a given frequency is equal to 0.303 cm, instead of 0.2915 cm as stated in [3]. Both values of L_0 are considered in Fig. 2, the former value gives the reflection coefficients almost equal to our results.

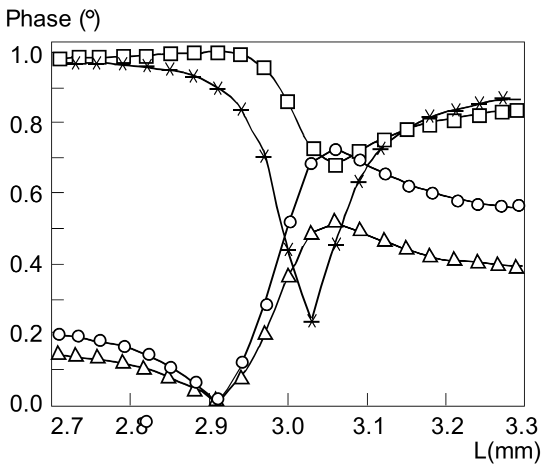

Fig. 4. Magnitude of the total reflected field from an infinite array of microstrip patches versus patch length. $\theta_i = \phi_i = \pi/8$, $f =$ 28 GHz, $\varepsilon_r = 2.95$, $d = 0.254$ mm, $\tan\delta = 0$, $a = b = 0.536$ cm, $W = 0.2915$ cm. Legend: $*$ $|(T_{\theta\theta}|$, □ $|T_{\phi\phi}|$, ◯ $|T_{\phi\theta}|$, △ $|T_{\theta\phi}|$.

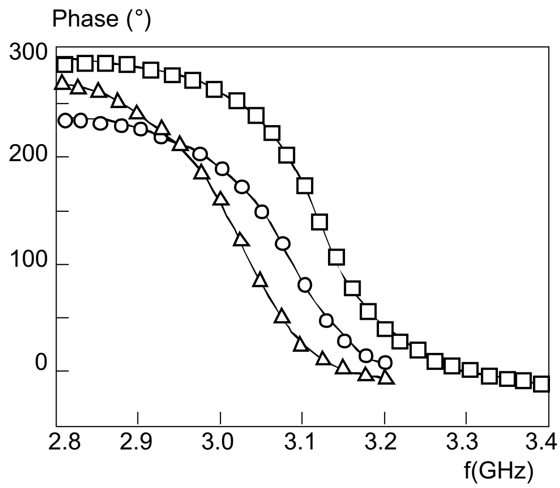

Fig. 5. Phase of the total reflected field from an infinite array of microstrip patches versus frequency compared with the experiment in a waveguide simulator [4]. $\sin\theta_i = \lambda_0/4b$, $\phi_i = \pi/2$, $\varepsilon_r = 2.2$, $d = 3.175$ mm, $\tan\delta = 0$, $a = 7.2898$ cm, $b = 4.572$ cm, $L = W =$ 3.048 cm. Legend: □ Phase$(T_{\phi\phi})$ theory, △ Phase$(T_{\phi\phi})$-theory [4], ◯ Phase$(T_{\phi\phi})$ -experiment [4].

More interesting are the results for incidence angle $\theta_i = \phi_i = \pi/8$ (Fig. 3, 4) where the magnitudes of reflection coefficients show an abrupt change when the length of patches is near the resonant length L_0.

Comparison with experimental results is given in Fig. 5. The experiment had been conducted in waveguide simulator [4] with dimensions 3.6 in×2.87 in. From the simulator geometry it was found that the plane of incidence is H-cardinal plane ($\phi_i = \pi/2$) and θ_i is frequency dependent as $\sin\theta_i = \lambda_0/4b$ (the incidence angle θ_i at 3.3 GHz is approximately 29°). Note that $T_{\phi\phi}$ in [4] is defined as $-T_{\phi\phi}$ in our approach (which leads to 180° phase shift in all of the phase data).

We have found that our method of computation of the scattered field gives good results for angles of incidence $\theta_i < \pi/4$, which is not surprising because for higher values of θ_i the observation point $(n\lambda_0, \theta_r, \phi_r)$ is located in the near-field zone. From our results it can be seen that the reflection phase data are critically sensitive to precise patch fabrication (in our first example the 0.05 mm (1.7 %) error in the patch length could lead to 180° phase error).

4 CONCLUSION

The well-known moment method in the spectral domain was introduced in order to compute the current distribution on the uniform infinite microstrip array induced by plane wave incidence. The method presented in this paper offers accurate computation of electric field scattered by a microstrip array $\vec{E}(x,y,z)$, in terms of the rigorous spectral domain dyadic Green's function of the grounded dielectric slab, which is a generalisation of the known field solutions for an observation point located on the surface of the substrate $\vec{E}(x,y,d)$.

The analysis of the reflection and scattering by uniform infinite microstrip array is of prime importance in a successful microstrip reflectarray design. Attention must be paid to precise patch fabrication in order to avoid performance deterioration caused by phase errors.

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