

# ASSIGNMENT MODIFICATION IN THE DESIGN OF MULTIPOINT COMMUNICATION NETWORKS

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A typical way to design a hierarchical communication network is first to solve the concentrator quantity problem, the concentrator location problem and the assignment problem. Afterwards the concentrator layout problem is solved and the designed star topology is converted into a tree topology. This problem is often mentioned in literature but without considering the advantage of modifying the existent assignment. In this paper a heuristic algorithm that solves this problem and finds a low cost multipoint solution using few CPU time is proposed.

**Key words:** multipoint, tree topology, heuristic, algorithm, assignment problem, terminal layout problem, hierarchical network planning

## 1 INTRODUCTION

The cost optimal conversion of a tree to a star network topology has been considered in many communication and operation research papers. The problem is known as the Capacitated Minimal Spanning Tree Problem (CMST, [9]). For simplicity many authors consider the simplified situation, when all nodes generate equal traffic [4], [5], [11], [8], [3], [1], [7]. The problem is rarely solved in case one can select several link types and in case the network nodes may generate any traffic [6]. The authors of [6] used the name CTP (Capacitated Tree Problem) for this type of problem. The algorithm denoted in [6] finds the optimal solution and needs approximately 1 minute to solve the problem with 10 Nodes and an hour to solve the problem with 20 Nodes (HP9000/715 workstation). If the concentrator connection capacity is lower than 20, the calculation time seems to be acceptable especially for smaller networks.

Our goal was to propose a very quick algorithm that finds a nearly optimal solution with a small gap to the optimal one for CMST. Using our assignment modification the final solution may be cheaper than the optimal solution for the CMST problem. The assignment modification is the re-assigning of a terminal to another concentrator after the conversion of the star to the tree topology has been done if it leads to economical profit.

Most CMST algorithms do not regard the possibility of changing given link capacity. The algorithm described here can do this in case it leads to economical profit. If this is not the case, new nodes will be connected to a branch of a tree as long as the maximal traffic flow capacity of the branch is not exceeded. The economy of the network topology is achieved due to proper usage of installed lines.

The traffic and economical loss due to link failure could be considered in the design problem too. The introducing of redundant links may solve the reliability problems [2]. Reliability aspects are not considered in this paper.

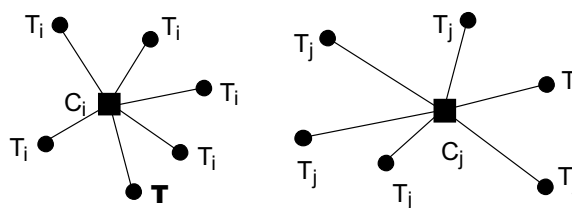


Fig. 1. Local situation for the node T

The CMST is a well known NP-hard problem [10]. The order of computational complexity of the algorithm proposed in this paper is better than  $O(N^2)$ , where  $N$  is the number of terminals.

## 2 DESCRIPTION OF THE ALGORITHM

The proposed algorithm consists of the following steps (Fig. 1):

1. For each terminal  $T$  the next (max. 10) other terminals  $T_i$  are found that are assigned to the same concentrator  $C_i$  and for which is

$$\text{dist}(T, T_i) < \text{dist}(T, C_i) \quad (1)$$

where  $\text{dist}(\text{node1}, \text{node2})$  is the distance between nodes  $\text{node1}, \text{node2}$ . Only in case a station  $T_i$  exists that fulfils

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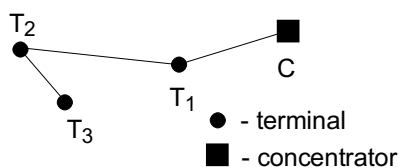


Fig. 2. Problem of the Essau-Williams algorithm

(1), it is reasonable not to directly connect  $T$  to  $C_i$  (as in star topology), but to use another terminal  $T_i$ .

2. For each terminal  $T$  the next (max. 10) other terminals  $T_j$  are found that are assigned to another concentrator  $C_j$  and fulfill

$$\text{dist}(T, T_j) < \text{dist}(T, C_i). \quad (2)$$

3. The base idea comes from the well known Essau-Williams algorithm [3]. For each terminal station  $T$  the following procedure is done:

First the costs  $\text{costs}(T \rightarrow C_i)$  of directly connecting  $T$  to  $C_i$  are calculated. Then  $T$  is temporarily connected to  $C_i$  via a  $T_i$  found in step 1. In contrast with the Essau-Williams algorithm the algorithm is able to change the capacity of the link(s) between  $T_i$  and  $C_i$ . By repeating this process for each  $T_i$ , the  $T_m$  is found for which the costs of connecting  $T$  to  $C_i$  via  $T_m$  are minimal. The value of the parameter *save* is found:

$$\text{save}_T = \text{costs}(T \rightarrow C_i) - \text{costs}(T \rightarrow T_m \rightarrow C_i) \quad (3)$$

where  $\text{save}_T$  are the saved costs after connecting  $T$  to  $C_i$  via another station  $T_m$ . Now the original solution is recovered — the station  $T$  is to be connected directly to concentrator  $C_i$  like in star topology. The procedure in step 3 was done to sort all terminal stations according to the  $\text{save}_T$  parameter. If the parameter  $\text{save}_T$  is negative, the station  $T$  will be definitely connected directly to concentrator  $C_i$ .

4. The technique used in step 3 is applied again but at the end  $T$  stays connected to  $C_i$  via  $T_m$ . First the station with the biggest  $\text{save}_T$  is connected. The basic problem of this method is the occurrence of non-optimal solutions like in Fig. 2. The station  $T_2$  has the highest value of the  $\text{save}_{T_2}$  parameter. Therefore it is connected to concentrator  $C$  via  $T_1$  first. The station  $T_3$  is connected via  $T_2$  to  $C$  afterwards. If the traffics of nodes  $T_2$  and  $T_3$  are equal this solution will be obviously not optimal. Therefore step 5 is proposed.

5. Terminals  $T_k$  are found to which only one link is connected. In Fig. 2  $T_3$  is such a node. Thereafter the path between  $T_k$  and  $C_k$  is found. Let the path be an ordered list

$$\{N_0, N_1, \dots, N_N\}. \quad (4)$$

In Fig. 2,  $N_0 = T_3, N_1 = T_2, \dots, N_N = C$ . The costs of the present tree topology are calculated and saved as

*mincosts*. Afterwards the following steps can be identified:

- a) Set counter  $i = 0$
- b) The link between  $N_{i+1}$  and  $N_{i+2}$  is removed. A link between  $N_i$  and  $N_{i+2}$  is added.
- c) The links are dimensioned and the new costs *newcosts* are calculated. If  $\text{newcosts} < \text{mincosts}$ , *mincosts* will be updated and the new solution will be accepted. Otherwise the original path is recovered.
- d) If  $i = N - 2$  terminate and continue with step 6. Otherwise  $i := i + 1$  and continue with b).

6. The assignment modification follows. The exchange procedure is done in the following way: For each terminal  $T_k$  and terminals  $T_i$  found in step 2, the following steps are executed:

- a) Set counter  $i = 0$ .
- b) Let  $T_k$  be assigned to  $C_A$  and  $T_i$  be assigned to another concentrator  $C_B$ . The sum total of the link costs in both trees of  $C_A$  and  $C_B$  is calculated:

$$\text{costs}_{\min} = \text{costs}_A + \text{costs}_B. \quad (5)$$

If  $C_B$  has enough spare capacities<sup>1</sup>,  $T_k$  will be connected to  $C_B$  via  $T_i$ . Otherwise the algorithm tries to connect a terminal assigned to  $C_B$  to another neighboring concentrator or finds a terminal  $T_j$  that may be exchanged with  $T_k$ <sup>2</sup>. All links in both trees are properly dimensioned and the costs of the new solution  $\text{costs}_{\text{new}}$  are calculated. If  $\text{costs}_{\text{new}} < \text{costs}_{\min}$ ,  $\text{costs}_{\min}$  will be updated and  $\text{candidate}_C = C_A$  and  $\text{candidate}_T = T_i$ .

- c)  $i := i + 1$ . If  $T_i$  does not exist, continue with d). Else continue with b)
- d) If  $\text{candidate}_C$  exists,  $T_k$  will be connected to  $\text{candidate}_C$  via  $\text{candidate}_T$ . If a  $T_j$  assigned to  $\text{candidate}_C$  must be disconnected, it is connected to a neighboring concentrator using the cost-optimal way according to the step 4.

### 3 RESULTS

The network in Fig. 3a. has been created for a given set of terminal positions using another algorithm [12]. Let the concentrator connection capacity be equal to 10. A concentrator is placed together with a terminal. There are 30 terminals present in the network. The monthly link costs are 175.200 DM. Running steps 1–5 leads to the network in Fig. 3b. The monthly link costs are 151.480 DM. Due to the cost functions of the link types and the node traffics for this network it is profitable to connect at the utmost 3 terminals to a branch<sup>3</sup>.

<sup>1</sup>the connection and traffic flow capacities are not exceeded after connecting  $T_k$  to  $C_B$

<sup>2</sup> $T_k$  is connected to  $C_B$  and  $T_j$  is connected to  $C_A$

<sup>3</sup>links  $n \cdot 2$  Mbit/s, traffic 10 Erlang for each node is set, no traffic loss formula, PCM used

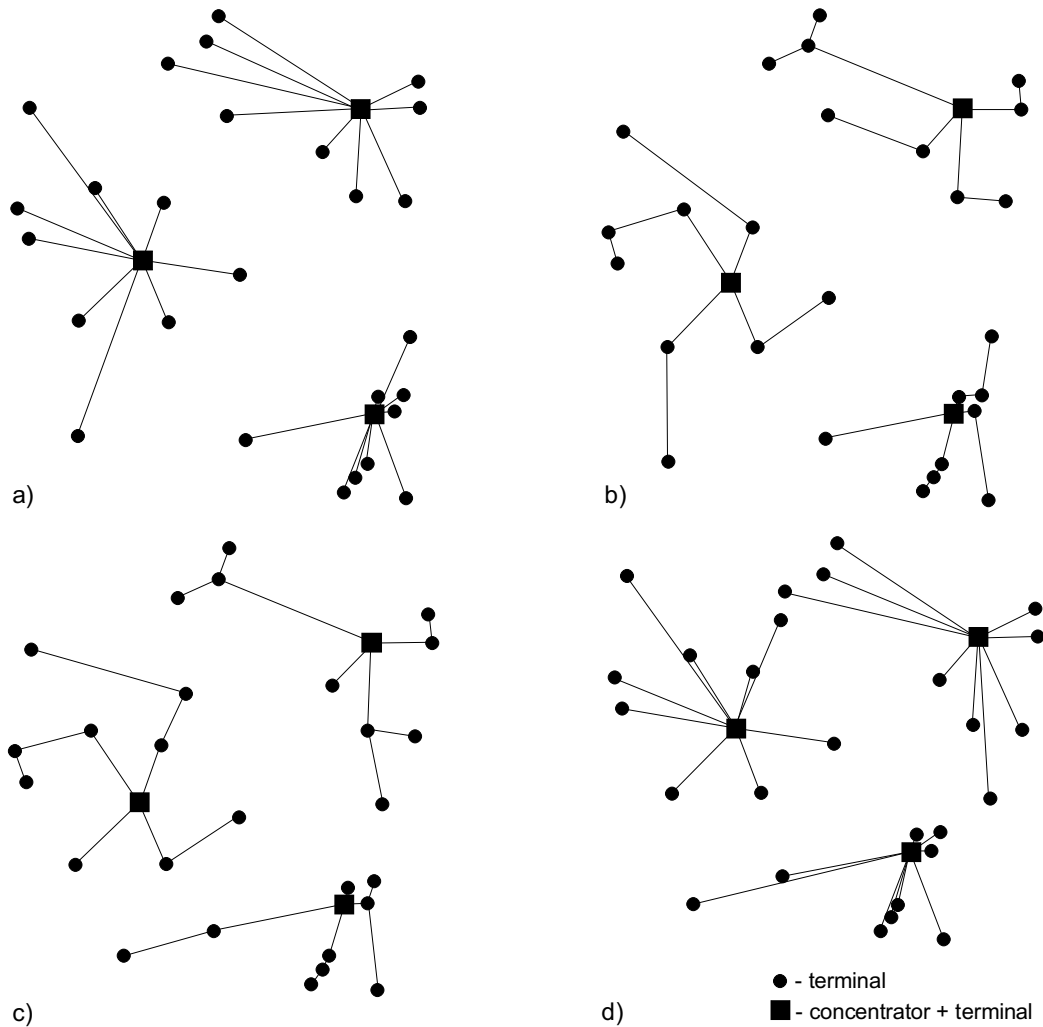


Fig. 3. Demonstration of assignment modification profit

The final solution after the assignment modification in step 6 is depicted in Fig. 3c. The monthly costs are 149.666 DM. That is approximately a 1.2% gain compared to the network in Fig. 3b. In Fig. 3d is the network of Fig. 3c converted to star topology. The monthly link costs would be 176.800 DM. This amount is higher than the monthly link costs of the network in Fig. 3a.

#### 4 CONCLUSIONS

In this paper an algorithm for solving the CTP problem has been proposed. The algorithm solves the classical CMST problem very well. Due to the assignment modification ability the algorithm finds solutions with lower link costs (gain about 0.5%-1%) than the best CMST algorithms proposed to convert tree to star topology for only one concentrator. The ability to change link dimensioning could result in further profit. The algorithm has been integrated into "COMPOSIS" - the communication

network planning tool developed at Comnets, University of Aachen. The runtime is less than 1 hour (SUN Ultra1) for a network with 4000 terminals and 209 concentrators.

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