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ROBUST OUTPUT FEEDBACK CONTROLLER DESIGN FOR LINEAR PARAMETRIC UNCERTAIN SYSTEMS

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This paper proposes the guaranteed cost design of a robust output feedback controller for continuous linear parametric uncertain systems. New necessary and sufficient conditions for static output feedback stabilizability of linear continuous time systems underlay the design procedure. The proposed algorithms are computationally simple and tightly connected with the Lyapunov stability theory and the LQR optimal state feedback design. The proposed approach allows for prescribing the structure of the output feedback gain matrix (including the decentralized one) by the designer. Two design methods have been proposed in this paper, leading to two classical iterative algorithms; the *V-K LMI* based iterative algorithm with guaranteed convergence and the *non iterative* LMI based algorithm. Numerical examples are given to illustrate the performance of the proposed robust controllers.

K e y w o r d s: linear continuous system, robust control, guaranteed cost

1 INTRODUCTION

In the excellent survey of static output feedback controller design in [19] it is stated that the static output feedback problem is one of the most important open questions in control engineering. Simply stated, the problem is as follows: given a dynamic system, find a static output feedback so that the closed loop system has some desirable characteristics. During the last two decades numerous papers dealing with the design of robust output feedback control schemes have been published [2], [4], [7], [9], [10], [11], [12], [13], [17], [16], [21], [22], [23]. Various approaches have been applied to study two aspects of this stabilization problem, namely the conditions under which a linear system described in the state space can be stabilized via output feedback and the respective procedure for obtaining a stabilizing control law [11], [14], [19]. In the above papers, the authors basically conclude that despite the availability of many approaches and numerical algorithms the static output feedback problem is still open. This is justified by the fact that up to now there are no testable necessary and sufficient conditions available to test stability of a static output feedback system.

Recently, it has been shown that an extremely wide array of robust controller design problems can be reduced to the problem of finding a feasible point under a Biaffine Matrix Inequality (BMI) constraint. The BMI has been introduced in [18] and [8] as a geometric reformulation of many robust control problems. However it is known that BMI problems are *NP*-hard [20]. The main result of [20] shows that it is rather unlikely to find an algorithm for solving general BMI problems and it has also been shown that simultaneous stabilization of N plants with static output feedback is an *NP*-hard problem. The BMI feasibility problem is discussed and a branch and bound global optimization algorithm to find an $\varepsilon-$ global minimum in a finite number of iterations is presented in [8]. The BMI for the μ/K_m robust controller synthesis formulation are adapted in [7]. In the above paper, the authors explain why the BMI feasible problem inevitably holds such a central place in the robust control synthesis problem.

In this paper, the BMI problem of robust controller design with output feedback is reduced to a LMI problem. The theory of linear matrix inequalities (LMIs) [3] has been used to design robust output feedback controllers in [2], [23], [11], [16]. Most of the above works present iterative algorithms in which a set of equations, or set of LMI problems, are repeated until certain convergence criteria are met. In [17] the authors study the problem of designing robust state controllers using the Riccati function approach. In [23] a necessary and sufficient condition for simultaneous stabilizability via static output feedback is obtained and an iterative LMI algorithm is proposed to obtain the output feedback gain. In [11] necessary and sufficient conditions for the existence of an H_∞ controller of any order are given in terms of three LMIs. The authors in [12] study conditions under which the designed output feedback controllers can be divided into two stages and the dynamic output feedback can be obtained. In [2] a LMI based algorithm has been proposed which does not require iteration of LMI solution. The goal is to eliminate the need for iteration by an appropriate choice of an initializing state feedback matrix. The proposed algorithm can be used to robustly stabilize a polytopic system via static output feedback. The V-K iteration algorithm proposed in [4] is based on an alternative solution of two convex LMI optimization problems obtained by fixing the Lyapunov matrix or the gain controller matrix. This algorithm is guaranteed to converge, but not necessarily,

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to the global optimum of the problem depending on the starting conditions.

In this paper two approaches have been developed. New necessary and sufficient conditions to stabilize continuous time systems via static output feedback have been used to design a robust controller for different types of uncertainty. In the first proposed method, we pursue the idea of the static structurally constrained state feedback, proposed in [6], to design a robust output feedback controller. The structure of such a controller including the decentralized one can be prescribed by the designer. The second proposed method is based on a necessary and sufficient condition for output feedback stabilizability of linear continuous time systems; for guaranteed cost it leads to a V-K iterative or non iterative LMI based algorithm for polytope systems.

The paper is organized as follows. In Section 2 the problem formulation and some preliminary results are brought. The main results are given in Section 3. In Section 4 the obtained theoretical results are applied to some examples. We have used the standard notation. A real symmetric positive (negative) definite matrix P is denoted by $P > 0$ ($P < 0$). Much of the notation and terminology follows references [2], [3], [14], [23].

2 PRELIMINARIES AND PROBLEM FORMULATION

In the context of robustness analysis and robust controller synthesis for linear time invariant systems the following uncertain model is commonly used

$$\begin{aligned}\dot{x} &= (A+\delta A)x + (B+\delta B)u \\ y &= Cx\,, \quad x(0)=x_0\end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^m$ and $y \in R^l$ are the state, control and output vectors, respectively; A, B and C are known matrices of appropriate dimensions; $\delta A = \{\delta a_{ij}\}$, $\delta B = \{\delta b_{lk}\}$ are unknown but norm bounded uncertainties. The following types of uncertainty descriptions are often used in the robustness investigations.

- Norm bounded uncertainties, unstructured model
$$\|\delta A\| \le q_a\,, \quad \|\delta B\| \le q_b \tag{2}$$
where $\|\cdot\|$ represents any matrix norm, and q_a and q_b are nonnegative constants.
- Element bounded uncertainties, structured model
$$|\delta A|_m \le A_m\,, \quad |\delta B|_m \le B_m \tag{3}$$
where $|\cdot|$ represents the modules of corresponding matrix, $A_m = \{a^m_{ij}\}$, $B_m = \{b^m_{lk}\}$ are matrices with nonnegative entries and corresponding dimensions, respectively, and
$$a^m_{ij} \ge |\delta a_{ij}|\,, \quad b^m_{lk} \ge |\delta b_{lk}|$$
- Matrix affine type uncertain structure
$$\begin{gathered}\delta A = \sum_{i=1}^{p} \epsilon_i A_i\,, \quad \delta B = \sum_{j=1}^{s} \gamma_j B_j \\ \underline{\epsilon_i} \le \epsilon_i \le \overline{\epsilon_i}\,, \quad \underline{\gamma_j} \le \gamma_j \le \overline{\gamma_j}\end{gathered} \tag{4}$$
where A_i, B_j are known matrices, ϵ_i, γ_j are unknown parameters. The ϵ_i, γ_j can vary in time arbitrarily fast [5], provided that each element is within given bounds. In general, a polytope description of uncertainties results in less conservative controller designs than other characterizations of uncertainty [3].
- Matrix bounded uncertain structure
$$\delta A^\top \delta A \le \gamma_a Q_a\,, \quad \delta B^\top \delta B \le \gamma_b Q_b \tag{5}$$
where γ_a, γ_b are nonnegative constants, Q_a, Q_b are nonnegative definite matrices.
- The uncertainties satisfying "matching conditions"
$$\delta A = UWA_1\,, \quad \delta B = UWA_2 \tag{6}$$
where U, A_1 and A_2 are known matrices and W is an unknown matrix satisfying $W^\top W \le I$, I is an identity matrix of corresponding dimension.

The problem studied in this paper can be formulated as follows. For a continuous linear time invariant system described by (1) a robust static output feedback controller is to be designed with the control algorithm in the form
$$u = FCx \tag{7}$$
or a structurally constrained state feedback [6]
$$u = (K+L)x \tag{8}$$
so that the closed loop system
$$\dot{x} = (A+BFC)x + (\delta A + \delta BFC)x \tag{9}$$
or
$$\dot{x} = (A+B(K+L))x + (\delta A + \delta B(K+L))x \tag{10}$$
is stable for all admissible uncertainties described by (2)–(6). A cost function associated with system (1) is
$$J = \int_0^\infty (x^\top Qx + u^\top Ru)\mathrm{d}t \tag{11}$$
where $Q = Q^\top \ge 0$, and $R = R^\top > 0$ are matrices of compatible dimensions.

Definition. Consider the uncertain system (1). If there exist a control law u^*, a positive scalar J^* such that for all admissible uncertainties the closed loop system is stable, and cost function (11) satisfies $J \le J^*$, then J^* is said to be guaranteed cost and u^* is said to be a guaranteed cost control law for uncertain system (1).

The nominal model of system (1) is given by
$$\dot{x} = Ax + Bu\,, \quad y = Cx\,. \tag{12}$$
Let us recall some commonly used notions for continuous time systems. Matrix $D \in R^{n\times n}$ is called stable when all its eigenvalues lie in the left half complex plane, $\mathrm{Re}\{\lambda_i(D)\} < 0$ for $i = 1,2,\ldots,n$. The system (12) with a stable A is called a stable system. System (12) is called to be output feedback stabilizable if there exists a real output feedback gain matrix F such that $A+BFC$ is a stable matrix. The pair (A,C) is called detectable if

there exists a real matrix X such that $A+XC$ is stable. The following lemma is well known [15].

LEMMA 1. *Suppose P to be a solution to the following Lyapunov matrix equation*

$$A^\top P + PA + Q = 0\,. \qquad (13)$$

Then A is stable iff $P > 0$ and $Q > 0$.

If there exists such a P, matrix A is said to be quadratically stable. A linear time invariant system is stable if and only if it is quadratically stable. It is possible, however, for example for polytopic linear systems to be stable without being quadratically stable [3]. In the next development we will consider exclusively quadratically stable systems. In the next development we employ the following Lemma.

LEMMA 2. *Let two matrices $C \in R^{l\times n}$ and $K \in R^{m\times n}$, $l \le n, m \le n$ be given. If $FC \neq 0$, there exist two matrices $F \in R^{m\times l}$ and $L \in R^{m\times n}$ such that the following equality holds*

$$FC = K + L\,. \qquad (14)$$

P r o o f . This is a standard fact.

Due to Lemma 2 it is possible to recalculate the results obtained from static structurally constrained state feedback to the static output feedback. Note that the structure of the gain matrix F can be determined by the designer.

2 DESIGN OF ROBUST CONTROLLERS

3.1 Structurally constrained state feedback design

In this section the idea of a structurally constrained state feedback optimal control [6] has been used for the design of robust controllers. The main results of this section are summarized in the following theorem.

THEOREM 1. *Consider a linear time invariant continuous system (1) and a static structurally constrained state feedback (8). Then, the following statements are equivalent.*

1. *The system is robust structurally constrained state feedback stabilizable.*
2. *There exists a symmetric and positive definite matrix $P > 0$ and matrices K and L satisfying the following matrix inequality*

$$(A + B(K + L))^\top P + P(A + B(K + L)) + Q_L < 0 \qquad (15)$$

where

$$Q_L = (\delta A + \delta B(K + L))^\top P + P(\delta A + \delta B(K + L))\,.$$

3. *There exist $P = P^\top > 0$, $R = R^\top > 0$ and matrices K and L satisfying the following matrix inequalities*

$$A^\top P + PA - PB(R^{-1} + I)B^\top P + Q_L - K^\top RK - L^\top L + (L + B^\top P)^\top (L + B^\top P) < 0 \qquad (16)$$

$$(RK + B^\top P)^\top \phi_L^{-1}(RK + B^\top P) - R < 0 \qquad (17)$$

where

$$\phi_L = -\big(A^\top P + PA - PB(R^{-1} + I)B^\top P + Q_L - K^\top RK - L^\top L + (L + B^\top P)^\top (L + B^\top P)\big)$$

P r o o f . The proof that the first and second statements are equivalent is given by Lemma 1. To prove the third statement recall that, for any matrices E_{11}, E_{12} and E_{22}, where E_{11} and E_{22} are symmetric, the following statements are equivalent

- $$\begin{bmatrix} E_{11} & E_{12} \\ E_{12}^\top & E_{22} \end{bmatrix} > 0$$
- $$E_{22} > 0\,,\ E_{11} - E_{12}E_{22}^{-1}E_{12}^\top > 0 \qquad (18)$$
- $$E_{11} > 0\,,\ E_{22} - E_{12}^\top E_{11}^{-1}E_{12} > 0\,.$$

Using the Schur complement formula (18), inequality (15) can be rewritten as follows

$$-[(A + B(K + L))^\top P + P(A + B(K + L)) + Q_L] > 0$$

and for $R > 0$, it is equivalent to the following inequality

$$\begin{bmatrix} R & RK + B^\top P \\ (RK + B^\top P)^\top & \phi_L \end{bmatrix} > 0\,.$$

From (18) both the equivalence of (16) and (17) and the above inequality result directly, which proves the equivalence of second and third statement.

COROLLARY 1. *Let the pair (P, K) solve the following two matrix equations for the nominal system (12)*

$$A^\top P + PA - PB(I + R^{-1})B^\top P + Q_{cn} = 0 \qquad (19)$$

where

$$Q_{cn} = (L + B^\top P)^\top (L + B^\top P) - L^\top L - K^\top RK + Q$$

and

$$K = -R^{-1}B^\top P\,. \qquad (20)$$

Then the necessary and sufficient conditions for system stabilizability hold.

The algorithm to handle the robust LQR suboptimal problem with uncertain system via static structurally constrained state feedback can be summarized as follows.

Algorithm A.

1. Set $i = 1, L_0 = 0, P_0 = 0, R_c = (I + R^{-1})^{-1}, Q_{10} = 0$.
2. Compute
$$Q_{ci} = Q + (L_{i-1} + B^\top P_{i-1})^\top (L_{i-1} + B^\top P_{i-1}) + Q_{1i} - L_{i-1}^\top L_{i-1} - K_{i-1}^\top RK_{i-1}\,.$$

3. Compute $P_i = P_i^\top > 0$ from the ARE
$$A^\top P_i + P_i A - P_i B R_c^{-1} B^\top P_i + Q_{ci} = 0\,.$$
4. Compute the gain matrix K
$$K_i = -R^{-1} B^\top P_i\,.$$
5. For given matrices K_i, C and using the Lemma 2 compute the matrices F_i and L_i.
6. Compute Q_{1i}
$$Q_{1i} = \|Q_{Li}\| I = I \sum_{l=1}^{p} \epsilon_{lm} \big\|(A_l + B_l(L_i + K_i))^\top P_i + P_i(A_l + B_l(L_i + K_i))\big\| \qquad (21)$$
where $\epsilon_{lm} = \max|\epsilon_l|$, $l = 1, 2, \ldots, p$
7. Compute $er = \|L_i - L_{i-1}\|$, if $er \leq error$ stop, else $i = i + 1$ and go to Step 2.
8. If there is no solution, change Q, R or decrease ϵ_{lm} and go to Step 2.
9. System (1) with (8) may not be robust stabilizable.

If the sequence $L_0, L_1, \ldots$ converges, say to L, the gain matrix K is given by (20) and the uncertain system is robustly stable for given uncertainties (21). Although the convergence of the *Algorithm A* has not been formally proven, it has converged for all tests performed in connection with this research. Typically, the number of iterations for convergence varies from 20 to 50 depending on the type of uncertainty, values of ϵ_{lm} and entries of matrices Q and R. Note that if a robust controller with guaranteed cost is to be designed,
$$\int_0^\infty x^\top [Q + (K + L)^\top R(K + L)] x \mathrm{d}t \leq x_0^\top P x_0$$
the above matrix Q_c (*Algorithm A*) has to be changed as follows
$$Qc = Q + (K + L)^\top R(K + L) + (L + B^\top P)^\top (L + B^\top P) - L^\top L - K^\top R K + Q_1\,.$$
For proof see Theorem 3. For different types of uncertainties the matrix Q_{1i} (21) may be slightly changed. For example, for the matrix bounded uncertainties (5) the corresponding term in (15) can be bounded as follows
$$\delta A^\top P + P\delta A \leq \eta P + \frac{1}{\eta}\lambda_M(P) q_a Q_a = \sqrt{\gamma_a} Q_a^{\frac{1}{2}} (P + \lambda_M(P) I)$$
where $\eta > 0$ is some positive constant.

3.2 Robust Output Feedback Controller Design

In this paragraph we will present a new procedure for the design of a static output feedback controller for the continuous time system (1). Note that it is well known [19] that the fixed order dynamic output feedback of order less or equal n is a special case of the static output feedback problem. The main results are summarized in the following theorem.

THEOREM 2. *Consider the linear uncertain continuous time system* (1)*. Then, the following statements are equivalent.*

- *The system* (1) *is robust static output feedback stabilizable.*
- *There exist a positive definite matrix $P = P^\top > 0$ and a matrix F satisfying the following matrix inequality*
$$(A + BFC)^\top P + P(A + BFC) + Q_o < 0 \qquad (22)$$
where
$$Q_o = (\delta A + \delta BFC)^\top P + P(\delta A + \delta BFC)\,.$$
- *There exist positive definite matrices $P > 0$ and $R > 0$ and a matrix F satisfying the following matrix inequalities*
$$A^\top P + PA - C^\top F^\top RFC - PBR^{-1}B^\top P + Q_o < 0 \qquad (23)$$
$$(B^\top P + RFC)\phi_o^{-1}(B^\top P + RFC)^\top - R < 0 \qquad (24)$$
where
$$\phi_o = -(A^\top P + PA - C^\top F^\top RFC - PBR^{-1}B^\top P + Q_o)\,.$$

P r o o f . Proof of this theorem goes the same way as for Theorem 1.

COROLLARY 2. *For the nominal model* (12) *an approximate solution* (P, F) *to the inequalities* (23) *and* (24) *is given as a solution of the two matrix equalities.*
$$A^\top P + PA - PBR^{-1}B^\top P + Q = 0 \qquad (25)$$
$$F = -R^{-1}B^\top P C^\top (CC^\top)^{-1}\,. \qquad (26)$$
Equations (23) *and* (24) *give impulse for the following algorithm for computation of the gain matrix* F.

Algorithm B.

1. Set $i = 1, P_0 = 0, F_0 = 0, Q_{20} = 0$.
2. Solve the ARE
$$A^\top P_i + P_i A + Q_{si} - P_i B R^{-1} B^\top P_i = 0$$
with respect to $P_i = P_i^\top > 0$
where $Q_{si} = Q - C^\top F_{i-1}^\top R F_{i-1} C + Q_{2i-1}$.
3. Compute the gain matrix F
$$F_i = -R^{-1} B^\top P_i C^\top (CC^\top)^{-1}\,.$$
4. Calculate $Q_{2i} = \|Q_{0i}\| I$. For the uncertainties (4) the matrix Q_{2i} is
$$Q_{2i} = I\Big(\sum_{l=1}^{p} \epsilon_{lm} \|(A_l + B_l F_i C)^\top P_i + P_i(A_l + B_l F_i C)\|\Big)\,.$$
5. Compute $er = \|P_i - P_{i-1}\|$. If $er \geq error$, increase i by one and go to Step 2, else Step 6.
6. Check the condition (24)
$$B^\top P_i(I - C^\top (CC^\top)^{-1} C) Q^{-1} (B^\top P_i (I - C(CC^\top)^{-1} C))^\top - R < 0\,.$$
If it holds stop, else change Q, R or decrease ϵ_{lm} and go to Step 2.

7. The system (1) with (7) may not be robust stabilizable.

If the sequence $P_0, P_1, \dots$ converges, say to P, F is given by (26). We would like to note that if the cost

$$\int_0^\infty x^\top [Q + C^\top F^\top RFC] x \mathrm{d}t \le x_0^\top P x_0$$

is to be guaranteed with control algorithm

$$u = KCx$$

the above matrix Q_s (Algorithm B) has to be changed as follows

$$Q_s = Q + Q_2 .$$

For proof see Theorem 3.

3.3 Static Output Feedback Simultaneous Stabilization

In general, a polytope description of uncertainties results in a less conservative controller design than other characterizations of uncertainty [3]. However, as the number of uncertain parameters increases, the number of vertices increases exponentially, and the design time increases exponentially, too [2]. For a system with a large number of uncertain parameters, robust controller design based on LMI algorithm may become impractical. Let the system be represented by the state realization (1) with uncertainties (4)

$$\begin{aligned} \dot{x} &= (A + \sum_{i=1}^{p} \epsilon_i A_i) x + (B + \sum_{i=1}^{p} \epsilon_i B_i) u \\ y &= (C + \sum_{i=1}^{p} \epsilon_i C_i) x . \end{aligned} \tag{27}$$

The system represented by (27) is a polytope of linear systems. The linear matrix inequality approach requires that system (27) be described by a list of its vertices, *ie*, in the form

$$\{(A_{v1}, B_{v1}, C_{v1}), (A_{v2}, B_{v2}, C_{v2}), \dots, (A_{vN}, B_{vN}, C_{vN})\} \tag{28}$$

where $N = 2^p$.

The system represented by (28) is quadratically stable if and only if there is a Lyapunov matrix $P > 0$ such that [3]

$$A_{vi}^\top P + P A_{vi} < 0 , \quad i = 1, 2, \dots, N . \tag{29}$$

Consequently, the system (28) is static output feedback quadratically stabilizable if and only if there is a Lyapunov matrix $P > 0$ and a feedback matrix F such that

$$\begin{aligned} (A_{vi} + B_{vi} F C_{vi})^\top P + P (A_{vi} + B_{vi} F C_{vi}) < 0 , \\ i = 1, 2, \dots, N \end{aligned} \tag{30}$$

If (30) holds for $P > 0$ and some F, then the vertices of the polytope (28) are said to be simultaneously quadratically stabilized by F. It is well known [1] that if P is a common Lyapunov matrix for the vertices of the polytope (28), it serves as a common Lyapunov function for the uncertain system (27) for all admissible uncertainties $\epsilon_i \in \langle \underline{\epsilon_i}, \overline{\epsilon_i} \rangle$, $i = 1, 2, \dots, p$. In (28), each vertex is computed for a different permutation of the p variables ϵ_i, alternatively taken at maximum and minimum values.

THEOREM 3. *Consider the system* (28)*. Then the following statements are equivalent.*

- *The system* (28) *is static output feedback simultaneously stabilizable with a guaranteed cost*
$$\int_0^\infty (x^\top Q x + u^\top R u) dt \le x_0^\top P x_0 = J^* \tag{31}$$
and $P > 0$.
- *There exist matrices* $P > 0$, $R > 0$, $Q > 0$ *and a matrix* F *such that the following inequality holds*
$$\begin{aligned} (A_{vi} + B_{vi} F C_{vi})^\top P + P (A_{vi} + B_{vi} F C_{vi}) + Q \\ + C_{vi}^\top F^\top R F C_{vi} \le 0 \end{aligned} \tag{32}$$
for $i = 1, 2, \dots, N$.
- *There exist matrices* $P > 0$, $R > 0$, $Q > 0$ *and a matrix* F *that the following inequalities hold*
$$A_{vi}^\top P + P A_{vi} - P B_{vi} R^{-1} B_{vi}^\top P + Q \le 0 \tag{33}$$
$$(B_{vi}^\top P + R F C_{vi}) \phi_{ui}^{-1} (B_{vi}^\top P + R F C_{vi})^\top - R \le 0 \tag{34}$$

where

$$\phi_{ui} = -(A_{vi}^\top P + P A_{vi} - P B_{vi} R^{-1} B_{vi}^\top P + Q)$$

for $i = 1, 2, \dots, N$.

P r o o f . Consider the control algorithm with output feedback to have the form

$$u = Fy = FC_{vi} x$$

then for the closed loop system results

$$\dot{x} = (A_{vi} + B_{vi} F C_{vi}) x, \quad i = 1, 2, \dots, N .$$

For $V = x^\top P x$, the time derivative of V along the system (28) is

$$\frac{\mathrm{d}V}{\mathrm{d}t} = x^\top [(A_{vi} + B_{vi} F C_{vi})^\top P + P (A_{vi} + B_{vi} F C_{vi})] x .$$

If the inequality (32) holds then there exist matrices $P > 0$, $R > 0$, $Q > 0$ and F such that

$$\frac{\mathrm{d}V}{\mathrm{d}t} \le -x^\top (Q + C_{vi}^\top F^\top R F C_{vi}) x < 0$$

for $i = 1, 2, \dots, N$. Therefore the closed loop system is asymptotically stable. Furthermore, by integrating both sides of the inequality from 0 to T and using the initial condition x_0, we obtain

$$V(0) - V(T) \ge \int_0^\top x^\top (Q + C_{vi}^\top F^\top R F C_{vi}) x \mathrm{d}t .$$

As the closed loop system is asymptotically stable when $T \to \infty$ then

$$x(T)^\top P x(T) \to 0 ,$$

Hence, we get

$$\int_0^\infty x^\top (Q + C_{vi}^\top F^\top R F C_{vi}) x \mathrm{d}t \le x_0^\top P x_0 \tag{35}$$

and the control algorithm $u = Fy$ is a guaranteed cost control law and

$$J^* = x_0^\top P x_0$$

is a guaranteed cost value for uncertain closed loop system. The equivalence of the second and the third statement is proved in Theorem 2.

Define $S = P^{-1}$. Using the Schur complement formula the inequality (33) is equivalent to the following linear matrix inequalities

$$\begin{bmatrix} SA_{vi}^\top + A_{vi}S - B_{vi}R^{-1}B_{vi}^\top & S\sqrt{Q} \\ \sqrt{Q}S & -I \end{bmatrix} < 0$$
$$\gamma I < S\,, \quad i = 1, 2, \dots, N \tag{36}$$

where $\gamma \geq 0$ is some non-negative constant. With $P = S^{-1}$, inequality (34) can be rewritten as follows

$$\begin{bmatrix} -R & B_{vi}^\top P + RFC_{vi} \\ (B_{vi}^\top P + RFC_{vi})^\top & -\phi_{ui} \end{bmatrix} < 0$$
$$i = 1, 2, \dots, N\,. \tag{37}$$

The algorithm for static output feedback simultaneous stabilization for the system(28) with a guaranteed cost (35) using the non-iterative LMI approach is given as follows.

Algorithm C.

1. Using the LMI based algorithm calculate S from the inequalities (36). $P = S^{-1}$.
2. Via the LMI based algorithm compute F from inequalities (37).
3. If the solution (36) is not feasible, the polytope system (28) is not simultaneously stabilizable and if (37) is not feasible (the closed loop system (28) is not stable) change Q and R or decrease $|\epsilon_i|$ $i = 1, 2, \dots, p$.

If the solutions (36) and (37) are feasible with respect to S and F then the uncertain system (27) is quadratically stable with a guaranteed cost control algorithm

$$u = Fy \qquad \text{and} \qquad J^* = x_0^\top P x_0$$

is the guaranteed cost for the uncertain closed loop system. Theorem 2 implies the following Corollary.

COROLLARY 3. *For the system* (28) *the following statements are equivalent.*

- *The system* (28) *is static output feedback simultaneously stabilizable.*
- *There exist positive definite matrices* $Q > 0$, $R > 0$, $P > 0$ *and a matrix* F *satisfying the following matrix inequalities.*

$$A_{vi}^\top P + PA_{vi} - PB_{vi}R^{-1}B_{vi}^\top P - C_{vi}^\top F^\top RFC_{vi} + Q \leq 0 \tag{38}$$

$$(B_{vi}^\top P + RFC_{vi})\Phi_{si}^{-1}(B_{vi}^\top P + RFC_{vi})^\top - R \leq 0 \tag{39}$$

where

$$\Phi_{si} = -(A_{vi}^\top P + PA_{vi} - PB_{vi}R^{-1}B_{vi}^\top P - C_{vi}^\top F^\top RFC_{vi} + Q)$$

$i = 1, 2, \dots N$.

From Corollary 3 results the following design procedure for a static output feedback simultaneously stabilization of system (28) based on the V-K LMI iterative algorithm.

Algorithm D.

1. $j = 1, F_0 = 0$.
2. Using the LMI based algorithm calculate $S_j = P_j^{-1} > 0$ from the inequality (38).
3. Using LMI based algorithm compute the gain matrix F_j from the inequality (39).
4. Compute $er = \|F_j - F_{j-1}\|$, if $er \leq error$ stop else $j = j + 1$ and go to Step 2.
5. If there is no solution change, Q and R or decrease $|\epsilon_i| i = 1, 2, ..., p$ and compute the new system (27).

The philosophy of Algorithm D is very closely related to the *V-K iteration* algorithm proposed in [4]. The V-K iteration algorithm is based on the alternative solution of two convex LMI optimization problems, obtained by either fixing the matrix P or the gain F. This algorithm is *guaranteed to converge*, but not necessarily, to the global optimum of the problem depending on the starting condition for the matrix F. If Algorithm D is feasible for an uncertain closed loop system, the following cost is guaranteed

$$\int_0^\infty x^\top Q x \mathrm{d}t \leq x_0^\top P x_0\,.$$

4 EXAMPLES

The first example has been borrowed from [2] to demonstrate the use of Algorithm A. Note that in [2] the LMI based algorithm has been used. It is known that the presented system is static output feedback stabilizable. However, the result presented in [2] is with a typographical error in the second row of matrix F. Let (A, B, C) in (1) be defined as

$$A = \begin{bmatrix} -0.036 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.010 & 0.0024 & -4.0208 \\ 0.1002 & q_1(t) & -0.707 & q_2(t) \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.4422 & 0.1761 \\ q_3(t) & -7.59222 \\ -5.520 & 4.490 \\ 0 & 0 \end{bmatrix} \qquad C = [0 \quad 1 \quad 0 \quad 0]$$

with parameters bounds $-0.6319 \leq q_1(t) \leq 1.3681$, $1.22 \leq q_2(t) \leq 1.420$, and $2.7446 \leq q_3(t) \leq 4.3446$. Find a stabilizing output feedback matrix F. The nominal model of (A, B) is given by the above matrices when we substitute for the entries $A(3, 2) = 0.3681$, $A(3, 4) = 1.32$ and $B(2, 1) = 3.5446$. The structured model uncertainty (4) $(A1, A2, B1)$ are matrices with the following entries $A1(3, 2) = 1$, $A2(3, 4) = 0.1$ and $B1(2, 1) = 0.8$ with $\epsilon_i \in \langle -1, 1\rangle$, $i = 1, 2$ and $\gamma_1 \in \langle -1, 1\rangle$. Other entries of the above uncertain matrices are equal to zero. The nominal model is unstable with eigenvalues:

$$eig\{-2.0516, 0.2529 \pm 0.3247i, -0.2078\}\,.$$

Let the structure of F be defined as

$$F^\top = [F(1,1) \quad F(2,1)]\,.$$

Then (14) implies $FC = K + L$, one of the solutions to L and F may be

$$\begin{aligned} L(1,1) &= -K(1,1)\,, & L(2,1) &= -K(2,1)\,,\\ L(1,2) &= 0\,, & L(2,2) &= 0\,,\\ L(1,3) &= -K(1,3)\,, & L(2,3) &= -K(2,3)\,,\\ L(1,4) &= -K(1,4)\,, & L(2,4) &= -K(2,4)\,,\\ F(1,1) &= K(1,2)\,, & F(2,1) &= K(2,2)\,. \end{aligned}$$

For $Q = I$, $R = I$, $\epsilon_m = 1$ and $\gamma_m = 1$ the gain matrix F is

$$F^\top = [0.0649 \quad -1.1613]\,.$$

The nominal model closed loop eigenvalues are

$$eig\,CL\{-22.4761, -0.0988, -0.2648 \pm 1.0169i\}\,.$$

Example 2. In this example, we consider the ship-steering problem [23] to demonstrate the Algorithm B. In [23] the state feedback has been designed for simultaneous stabilization of two systems. For system (1) we obtain [23]

$$A = \begin{bmatrix} -0.363 & -0.309 & 0\\ -3.6545 & -0.892 & 0\\ 0 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0.133\\ -0.892\\ 0 \end{bmatrix}.$$

The structure of uncertainty matrices (4) is given as $(A1, A2, A3, A4, B1, B2)$ where each matrix (A_l, B_k) has only one nonzero element, the position of which is that of the $\delta A(i,j)$ and $\delta B(v,u)$.

$$\delta A = \begin{bmatrix} 0.0325 & 0.015 & 0\\ -0.3578 & 0.0595 & 0\\ 0 & 0 & 0 \end{bmatrix} \qquad \delta B = \begin{bmatrix} -0.0085\\ 0.0595\\ 0 \end{bmatrix}.$$

We assume that each element of matrices δA and δB changes independently from others. $\epsilon_i \in \langle -1, 1\rangle$, $i = 1,2,3,4$ and $\gamma_j \in \langle -1, 1\rangle$, $j = 1,2$ can arbitrarily vary in time, provided that each element is within the given boundaries. The computation results are as follows. For

$$C = [1 \quad 1 \quad 1]$$

the gain matrix is $F = 4.0105$ and the closed loop eigenvalues are

$$eig\,CL\{-0.5305 \pm 0.8495i\,,\ -3.238\}\,.$$

and $\epsilon_m = \gamma_m = 1$, $Q = I$, and $R = 0.1$. For

$$C = [0 \quad 1 \quad 1]$$

and $\epsilon_m = \gamma_m = 1$, $Q = 2.1$, $R = .1$ the gain matrix is $F = 4.0591$ and the closed loop eigenvalues are

$$eig\,CL\{-0.7652 \pm .6301i\,,\ -3.3452\}\,.$$

Example 3. In this example, to demonstrate the LMI based *Algorithm C* we consider the linear model of helicopter in a vertical plane [12]. The system model modified for the purpose of illustrating the application of our results, is given by (28), where

$$A_{v1} = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -1.7555\\ 0.0482 & -1.01 & 0.0024 & -4.0208\\ 0.1002 & -1.0145 & -0.707 & 1.3229\\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_{v1}^\top = [0.4422 \quad 3.5446 \quad 0.0188 \quad 0.8445]$$

$$A_{v2} = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & 0.8445\\ 0.0482 & -1.01 & 0.0024 & -4.0208\\ 0.1002 & 1.5855 & -0.707 & 1.3229\\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_{v2} = B_{v1}\,.$$

For

$$C_{v1} = C_{v2} = \begin{bmatrix} 0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0 \end{bmatrix}$$

and $R = 0.0000001 * I, Q = 0.0001 * I$ the gain matrix and the corresponding closed loop eigenvalues for the first and second systems are

$$F = [-15.72018 \quad -17.02213]$$

$$eig\,CL1\{-64.2684\,, -0.629\,, -0.053519 \pm 2.2608i\}$$

$$eig\,CL2\{-64.2127\,, -0.2387\,, -0.275554 \pm 1.69637i\}$$

and for $R = 0.000001, Q = I$

$$F = [-1224.208 \quad -1474.8829]$$

$$eig\,CL1\{-4992.406\,, -0.658606\,, -0.10508 \pm 2.63i\}$$

$$eig\,CL2\{-4992.405, -0.290413, -0.2896137 \pm 1.62099i\}\,.$$

By trial and error the value of γ in (36) has been found $\gamma = 50$.

Example 4. In this example, we consider the linear model of two cooperating DC motors. The problem is to design two PI controllers for a laboratory MIMO system which will guarantee stability of closed loop uncertain system. In this example, we have considered Algorithm A, the V-K iterative LMI algorithm based on Algorithm D for simultaneous stabilization of a polytope system and finally a non-iterative LMI algorithm based on Algorithm C. The system model is given by (1) with a matrix polytope type uncertain structure (4), where

$$A = \begin{bmatrix} 0 & -0.2148 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 1 & -1.0142 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -0.2605 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & -0.9107 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & -0.1639 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & -0.8137 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.2279 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -0.8251 & 0 & 0\\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B^\top = \begin{bmatrix} 0.3148 & 0.0478 & 0 & 0 & -0.0841 & -0.0287 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.1028 & -0.0091 & 0 & 0 & 0.3676 & 0.2448 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and the model uncertainties (4) are

$$A1 = \begin{bmatrix} 0 & -0.025 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1395 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.0938 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2911 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.01888 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0208 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0333 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.1173 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A2 = \begin{bmatrix} 0 & 0.0125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0594 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0116 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0308 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0156 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0565 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0434 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1258 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B1^\top = \begin{bmatrix} 0.0625 & -0.0798 & 0 & 0 & 0.0016 & 0.0072 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0462 & 0.0449 & 0 & 0 & 0.077 & -0.005 & 0 & 0 \end{bmatrix}$$

$$B2^\top = \begin{bmatrix} -.0094 & 0.0151 & 0 & 0 & -0.121 & -0.003 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0019 & -0.003 & 0 & 0 & -0.064 & 0.0189 & 0 & 0 \end{bmatrix}$$

$$\epsilon_i \in \langle -1, 1 \rangle\,, \quad i = 1, 2\,.$$

The number of polytope systems is 4 and the polytope vertices are computed for different permutations of the two variables ϵ_i, $i = 1, 2$ alternatively taken at their maximum $\overline{\epsilon_i}$ and minimum $\underline{\epsilon_i}$, $i = 1, 2$,*ie*

$$\begin{aligned} A_{v1} &= A + \underline{\epsilon_1} A1 + \underline{\epsilon_2} A2 \quad A_{v2} = A + \underline{\epsilon_1} A1 + \overline{\epsilon_2} A2 \\ A_{v3} &= A + \overline{\epsilon_1} A1 + \underline{\epsilon_2} A2 \quad A_{v4} = A + \overline{\epsilon_1} A1 + \overline{\epsilon_2} A2\,. \end{aligned} \tag{40}$$

In the same way the solution to B_{v1} to B_{v4} can be found. The mathematical model of the polytope system considers a list of its vertices given as follows

$$\dot{x} = A_{vi} x + B_{vi} u \quad i = 1, 2, 3, 4\,. \tag{41}$$

The decentralized control structure for the two PI controllers can be obtained by the choice of the static output feedback gain matrix F structure which is given as follows

$$F = \begin{bmatrix} F_{11} & 0 & F_{13} & 0 \\ 0 & F_{22} & 0 & F_{24} \end{bmatrix}. \tag{42}$$

The result of computation of a static output feedback gain matrix F for Algorithm A and the given decentralized control structure is as follows

$$F = \begin{bmatrix} -0.3897 & 0 & -0.5007 & 0 \\ 0 & -0.3905 & 0 & -0.5007 \end{bmatrix}$$

$$|\epsilon_1| = |\epsilon_2| = .8\,,\ R = 1000 * I\,,\ Q = 3 * I$$

and the closed loop nominal model eigenvalues are

$$eig\,CL\{-0.1383 \pm 0.547i\,,\ -0.1281 \pm 0.3919\,,$$
$$-\,0.8269\,,\ -.6153\,,\ -0.3912 \pm 0.0421i\,,\ -.4602 \pm .2321i\}$$

The result of computation of the static output feedback decentralized control structure gain matrix F via Algorithm D which simultaneously stabilizes the four polytope systems (41) is as follows

$$F = \begin{bmatrix} -0.0247 & 0 & -0.0065 & 0 \\ 0 & -0.0055 & 0 & -0.0075 \end{bmatrix}.$$

The above result have been obtained for $Q = .0002 * I$, $R = 100 * I$, $|\epsilon_1| = |\epsilon_2| = 1$ and $\gamma = .01$ (36). All eigenvalues of the four polytope systems are situated in the left half-plane of the complex plane.

The result of calculation of the static output feedback gain matrix F with the prescribed decentralized structure (42) using the LMI non iterative based algorithm described by Algorithm C is as follows

$$F = \begin{bmatrix} -0.0128 & 0 & -0.0065 & 0 \\ 0 & -0.0035 & 0 & -0.0066 \end{bmatrix}.$$

The above result has been obtained for $Q = .0001 * I$, $R = 100 * I$, $|\epsilon_1| = |\epsilon_2| = 1$, $\gamma = .01$ whereby for the guaranteed cost we obtain $\lambda_M(P)\|x_0\|^2 = 96.7077\|x_0\|^2$. The maximal eigenvalue for the four polytopic systems is $maxeig = -.0067$. Note that the closed loop system is quadratically stable with the guaranteed cost also for $\|\epsilon_1\| = \|\epsilon_2\| = 1.1$. Practically the same robust stability boundary has been obtained via Algorithm C and Algorithm D. These results have been obtained for $R = 1 * I$, $Q = .00000001 * I$, $\gamma = 0$

Algorithm C: $|\epsilon_1| = |\epsilon_2| = 2.3$, $maxeig = -0.0004$ and

Algorithm D: $|\epsilon_1| = |\epsilon_2| = 2.3$, $maxeig = -0.0003$.

As the closed loop system is quadratically stable in all vertices, the robust stability of the uncertain system (27) with the above designed decentralized static output feedback gain matrix F and uncertainties (4) is proved.

5 CONCLUSIONS

The main aim of this paper has been to propose new methods for solving the design of robust controllers via static output feedback for linear continuous time systems. New necessary and sufficient conditions for static output feedback stabilizability for continuous time systems have been proposed. The derived conditions underlay the design procedure of robust controllers using the iterative, LMI based V-K iterative and non iterative algorithms with a guaranteed cost. The LMI based algorithms give considerably less conservative results than the classical iterative approach. Four numerical experiments are given to test the performance of the proposed algorithm.

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