Journal of ELECTRICAL ENGINEERING, VOL. 53, NO. 5-6, 2002, 126–131

A DETERMINISTIC LQ TRACKING PROBLEM: PARAMETRIZATION OF THE CONTROLLER AND THE PLANT

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The paper treats the design of an LQ optimal controller via the Youla-Kučera parametrization. It is assumed that the nominal plant and controller descriptions are known and that the modified plant can be described via the dual Youla-Kučera parametrization. The main aim of this paper is to design the LQ controller based on this modified plant.

K e y w o r d s: LQ control, Youla-Kučera parametrization

1 INTRODUCTION

Optimal control design, based on LQ performance criterion has been derived historically first in terms of the state space approach. By this method Riccati equations have to be solved [1]. An alternative algebraic approach involving the input-output plant description and leading to the solution of Diophantine and spectral factorization equations has been pioneered by [2], [3].

Various approaches are possible with the algebraic approach when dealing with the LQ design. While the first solutions directly provide the controller based on the plant information [4], attention of the researchers has recently been focused on the parametrized solutions. Nice surveys dealing with the use of the Youla-Kučera parametrization of the controller have been presented by [5], [6].

Recently, the dual Youla-Kučera parametrization has been introduced and used in the closed-loop identification [7]. Here, the dual problem is solved where it is assumed that a nominal controller is known and parametrization of all possible plants is identified that lead to stable closed-loop systems.

The main aim of this paper is to present the LQ control design involving both the controller and plant Youla-Kučera parametrizations. This problem arises when the plant Youla-Kučera parameter is identified and the controller Youla-Kučera parameter is sought [6]. The choice of the LQ cost follows the ideas presented in [8] where penalization of the control signal derivative rather than the control signal itself is assumed. This choice of the LQ cost reflects more closely the practical needs of process control.

The paper is organized as follows. In Section 2 the closed-loop system description is presented. The design of a deterministic LQ controller and a comparison of the proposed approach with the classical approach are presented in Section 3. An illustrative example is given in Section 4. Finally, Section 5 offers the conclusions.

1.1 Notation

All systems in this work are assumed to be SISO and continuous-time. The systems are described by means of fractions of polynomials in complex argument s, used in $\mathcal{L}$-transform. $\mathcal{RH}_\infty$ denotes the set of stable proper rational transfer function and S denote the set of stable polynomials.

For simplicity, the arguments of polynomials are omitted whenever possible — a polynomial $X(s)$ is denoted by X. We denote $X^*(s) = X(-s)$ for any rational function $X(s)$.

2 CLOSED–LOOP SYSTEM

2.1 System description

Consider the closed-loop system illustrated in Fig. 1. A continuous-time linear time-invariant input-output nominal representation of the plant to be controlled is considered

$$Ay = Bu\,, \tag{1}$$

where y, u are process output and controller output, respectively. A and B are polynomials that describe the input-output properties of the plant.

We assume that the condition $\deg B \le \deg A$ holds (*ie*, the transfer function of the plant is proper) and A and B are coprime polynomials.

The reference w is considered to be from a class of functions expressed as

$$Fw = H\,, \tag{2}$$

where H, F are coprime polynomials and $\deg H \le \deg F$.

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ISSN 1335-3632

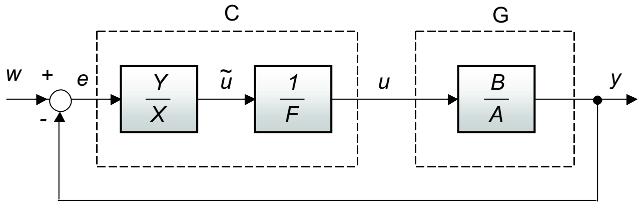

Fig. 1. Block diagram of the nominal closed-loop system

The controller that gives a stable closed-loop polynomial $DM_1, M-2$ and internally stabilizes the closed-loop system is described by the equations

$$X\tilde{u} = Ye\,, \quad Fu = \tilde{u}\,, \tag{3}$$

where X, Y are coprime polynomials and $X(0) \neq 0$. The second equation assures that the controller tracks the class of references specified by (2).

NOTE 1. *When considering the most common case of references – step changes, then $H = 1, F = s$ in (2) and the precompensator is given as $1/s$. However, if the controlled plant has a pole on the stability boundary ($s = 0$), then the precompensator can be removed. In general, the precompensator is not necessary if F divides A, which is unfortunately not true for the majority of the plants.*

Consider the nominal plant and the nominal controller transfer functions in the fractional representations

$$G = \frac{N_G}{D_G}\,, \qquad C = \frac{N_C}{D_C}\,, \tag{4}$$

where

$$N_G = \frac{B}{M_1}\,, \qquad D_G = \frac{A}{M_1}\,, \tag{5}$$

$$N_C = \frac{Y}{M_2}\,, \qquad D_C = \frac{FX}{M_2} \tag{6}$$

and $M_1, M_2 \in \mathcal{S}$ with degrees $\deg M_1 = \deg A$ and $\deg M_2 = \deg FX, D_G, N_G, D_C$ and $N_C \in \mathcal{RH}_\infty$.

A stabilizing controller is then given by solution of Diophantine equation

$$D_G D_C + N_G N_C = 1\,. \tag{7}$$

Substituting equations (5) and (6) into (7), the condition of stability in $\mathcal{S}$ takes the form

$$AFX + BY = M_1 M_2\,. \tag{8}$$

Let us now assume that, based on new data, the new plant estimate is specified by the dual Youla-Kučera parameter Q given as

$$Q = \frac{Q_n}{Q_d}\,. \tag{9}$$

The resulting modified plant transfer function is then defined by the known relation which is summarized in the following theorem.

THEOREM 1. *Let the nominal model plant $G = N_G/D_G$, with N_G and D_G coprime over $\mathcal{RH}_\infty$, be stabilized by controller $C = N_C/D_C$, with N_C and D_C coprime over $\mathcal{RH}_\infty$. Then the set of all plants stabilized by the controller C is given by*

$$G(Q) = \frac{N_q}{D_q} = \frac{N_G + D_C Q}{D_G + N_C Q}, \tag{10}$$

where

$$Q \in \mathcal{RH}_\infty\,. \tag{11}$$

P r o o f . Dual to that of [9].

Since our method involves polynomials rather than polynomial fractions, we present a short overview of the corresponding transformation between both descriptions.

COROLLARY 1. *Let the nominal model plant $G = N_G/D_G$, with N_G, D_G, B and A defined by (5), be stabilized by a controller $C = N_C/D_C = Y/FX$, with N_C, D_C, Y and FX defined by (6). Then the set of all plants stabilized by controller C is given by*

$$G(Q) = \frac{B_q}{A_q} = \frac{B_m Q_d + FX_m Q_n}{A_m Q_d - Y_m Q_n}, \tag{12}$$

where

$$\begin{gathered} Q = \frac{Q_n}{Q_d} \in \mathcal{RH}_\infty\,, \quad A_m = AM_2\,,\ B_m = BM_2\,, \\ X_m = XM_1\,, \quad Y_m = YM_1\,. \end{gathered} \tag{13}$$

3 LQ TRACKING PROBLEM

In this section two approaches to the LQ tracking problem will be presented:

- The first one is the classical one and it is based on the determination of optimal closed-loop poles that minimize the The LQ cost function. The controlled plant is considered of the form (10).
- The second approach follows more modern ideas of the Youla-Kučera parametrization of all stabilizing controllers. In this approach, the design of the controller is based on the change of the system – Youla-Kučera parameter Q.

The general conditions required to govern the control system properties are:

- stability of the control system,
- asymptotic tracking of the reference.

The goal of optimal deterministic LQ tracking is to design a controller that enables the control system to satisfy the above basic requirements and in addition the control law that minimizes the cost function

$$J = \int_0^\infty \{\varphi\tilde{u}^2(t) + \psi e^2(t)\}\mathrm{d}t\,, \tag{14}$$

where $e = w - y$ denotes the control error and $\varphi > 0$, $\psi \geq 0$ are weighting coefficients. The cost function (14) can be rewritten using Parseval's theorem to obtain an expression in the complex domain

$$J = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \{\tilde{u}^*(s)\varphi\tilde{u}(s) + e^*(s)\psi e(s)\}\mathrm{d}s\,. \tag{15}$$

3.1 Classical LQ problem

The design of a deterministic LQ controller for the modified system (10) was described in [8]. The results of this work are summarized in the following theorem.

THEOREM 2. *Define stable polynomials D_c and D_f resulting from spectral factorizations*

$$D_c^* D_c = \varphi A_q^* F^* A_q F + \psi B_q^* B_q \,, \tag{16}$$

$$D_f^* D_f = A_q^* A_q H^* H \,, \tag{17}$$

then the internal stability and solution of the deterministic LQ problem (14) is given by the controller polynomials X_c, Y_c calculated from a pair of Diophantine equations. The solution exists if $A_q F$ and B have no unstable common factors and is unique.

The controller is given by the solution of the coupled bilateral Diophantine equations:

$$\psi B_q^* D_f - A_q F V^* = D_c^* Y_c \,, \tag{18}$$

$$\varphi A_q^* F^* D_f + B_q V^* = D_c^* X_c \,. \tag{19}$$

Proof. See [8].

COROLLARY 2. *If polynomials $A_q F$ and B_q are coprime, then pair of Diophantine equations (16), (17) is reduced to the implied Diophantine equation*

$$A_q F X_c + B_q Y_c = D_c D_f \,. \tag{20}$$

Proof. See [2], [10].

3.2 LQ problem: Youla-Kučera parametrization

The main aim of this paper is to derive alternative expressions for the controller based on the Youla-Kučera parametrization of both the plant and the controller. Let us at first summarize the known results dealing with parametrized systems:

THEOREM 3. *Let the nominal model plant $G = N_G/D_G$, with N_G and D_G coprime over $\mathcal{RH}_\infty$ be stabilized by controller $C = N_C/D_C$, with N_C and D_C coprime over $\mathcal{RH}_\infty$. Then the set of all stabilizing controllers for the the plant G is given by*

$$C(S) = \frac{N_s}{D_s} = \frac{N_C + D_G S}{D_C - N_G S}, \tag{21}$$

where

$$S \in \mathcal{RH}_\infty \,. \tag{22}$$

Proof. [9].

COROLLARY 3. *Let the nominal model plant $G = N_G/D_G = B/A$, with N_G, D_G, B and A defined by (5), be stabilized by controller $G = N_G/D_G = Y/FX$, with N_C, D_C, Y and FX defined by (6). Then the set of all plants stabilizing controllers for the plant G is given by*

$$C(S) = \frac{Y_s}{FX_s} = \frac{Y_m S_d + A_m F S_n}{F X_m S_d - B_m F S_n} = \frac{Y_m S_d + A_m F S_n}{X_m S_d - B_m S_n} \frac{1}{F} \,, \tag{23}$$

where

$$S = \frac{F S_n}{S_d} \in \mathcal{RH}_\infty \,, \quad A_m = A M_2 \,, \ B_m = B M_2 \,, \quad X_m = X M_1 \,, \quad Y_m = Y M_1 \,. \tag{24}$$

NOTE 2. *The presence of F in the numerator of S assures asymptotic tracking of the reference.*

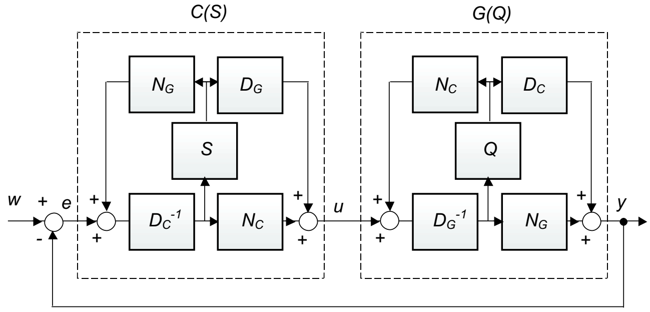

Fig. 2. Block diagram of the closed-loop system

THEOREM 4. *Consider the closed-loop system with the configuration in Fig. 2 defined by $G(Q)$ and $C(S)$ where $Q = Q_n/Q_d$ and $S = F S_n/S_d$ are stable proper rational functions. The closed-loop system is stable if and only if Q and FS together define a stable loop.*

Proof. [11]

We now present the solution of the deterministic LQ controller design within the Youla-Kučera parametrization framework assuming given the plant model $G(Q)$ and the controller $C(S)$. The aim is to derive a procedure to compute S that minimizes (14) with the plant update characterized by Q.

THEOREM 5. *Consider the minimization of the cost function (14) with respect to the Youla-Kučera parameter S that is specified as a transfer function. Assume that the nominal system $G = B/A$ is stabilized by nominal controller $C = Y/FX$ and that a stable transfer function Q is known. Solve modified spectral factorization equations (16), (17) for stable D_c, D_f and the coupled bilateral Diophantine equations for S_n and S_d*

$$-\varphi D_f A_q^* F^* Y_m + \psi D_f B_q^* X_m - Q_d D V^* = D_c^* S_n \,, \tag{25}$$

$$\varphi D_f A_q^* F^* A_m F + \psi D_f B_q^* B_m + Q_n F D V^* = D_c^* S_d \,. \tag{26}$$

The optimal Youla-Kučera parameter is then given as

$$S = \frac{Fs_n}{s_d} = \frac{FS_n}{S_d}\,, \text{ where } s_n = \frac{S_n}{D_cD_f} \text{ and } s_d = \frac{S_d}{D_cD_f}\,. \tag{27}$$

P r o o f . To begin the proof, the two signals ($\tilde{u}$, e) used in the cost function (14) are derived using equations (12), (23) describing the closed-loop system (so that the desired signals are functions of only external signal w)

$$\tilde{u} = \frac{(A_mQ_d - Y_mQ_n)(Y_mS_d + A_mFS_n)F}{(A_mFX_m + B_mY_m)(Q_dS_d + Q_nFS_n)}w\,, \tag{28}$$

$$e = \frac{(A_mQ_d - Y_mQ_n)(X_mS_d - B_mS_n)F}{(A_mFX_m + B_mY_m)(Q_dS_d + Q_nFS_n)}w\,. \tag{29}$$

Substituting for w from (2) yields

$$\tilde{u} = \frac{(A_mQ_d - Y_mQ_n)H}{M_1M_2(AFX + BY}\Big(\frac{Y_mS_d + A_mFS_n}{Q_dS_d + Q_nFS_n}\Big), \tag{30}$$

$$e = \frac{(A_mQ_d - Y_mQ_n)H}{M_1M_2(AFX + BY}\Big(\frac{X_mS_d - B_mFS_n}{Q_dS_d + Q_nFS_n}\Big). \tag{31}$$

Because the pair Q_nF, Q_d can have only stable common factors, we can write

$$1 = Q_ds_d + Q_nFs_n, \tag{32}$$

where s_n, s_d are stable rational functions

$$s_d = \frac{S_d}{Q_dS_d + Q_nFS_n}\,, \quad s_n = \frac{S_n}{Q_dS_d + Q_nFS_n}\,. \tag{33}$$

Stability of s_n and s_d then implies stability of the term $S_dQ_d + S_nFQ_n$ that appears as a factor in the denominators. We can then write

$$\begin{aligned}\tilde{u} &= \frac{A_qH}{M_1M_2M_1M_2}(Y_ms_d + A_mFs_n)\\ &= \frac{A_qH}{D}\Big(Y_m\frac{1 - Q_nFs_n}{Q_d} + A_mFs_n\Big)\\ &= \frac{A_qH}{DQ_d}(Y_m + A_qFs_n)\end{aligned} \tag{34}$$

$$\begin{aligned}e &= \frac{A_qH}{M_1M_2M_1M_2}(X_ms_d - B_ms_n)\\ &= \frac{A_qH}{D}\Big(X_m\frac{1 - Q_nFs_n}{Q_d} - B_ms_n\Big)\\ &= \frac{A_qH}{DQ_d}(X_m - B_qs_n)\end{aligned} \tag{35}$$

where $D = M_1M_2M_1M_2$.

Minimizing equation (14) with respect to all stable S represents minimization of the following cost function in the complex domain

$$\begin{aligned}J &= \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\{\tilde{u}^*(s)\varphi\tilde{u}(s) + e^*(s)\psi e(s)\}\mathrm{d}s\\ &= \frac{1}{2\pi j}\int_{-j\infty}^{j\infty}(\varphi S_{\tilde{u}} + \psi S_e)\,\mathrm{d}s,\end{aligned} \tag{36}$$

where $S_{\tilde{u}}$ and S_e are spectral functions of the form and denoting $P = (\frac{A_qH}{DQ_d})$ we have

$$\begin{aligned}S_{\tilde{u}} = \tilde{u}^*\tilde{u} &= \Big(P(Y_m + A_qFs_n)\Big)^*\Big(P(Y_m + A_qFs_n)\Big) =\\ &P^*P(A_q^*A_qF^*Fs_n^*s_n + Y_mA_q^*F^*s_n^* + Y_m^*A_qFs_n + Y_m^*Y_m)\,,\end{aligned} \tag{37}$$

$$\begin{aligned}S_e = e^*e &= \Big(P(X_m - B_qs_n)\Big)^*\Big(P(X_m - B_qs_n)\Big) =\\ &P^*P(B_q^*B_qs_n^*s_n + X_mB_q^*s_n^* + X_m^*B_qs_n + X_m^*X_m)\,.\end{aligned} \tag{38}$$

Completing to squares [2] yields

$$\begin{aligned}\varphi S_{\tilde{u}} + \psi S_e = P^*P\Big(&(\varphi A_q^*A_qF^*F + \psi B_q^*B)s_n^*s_n +\\ &+ (\varphi Y_mA_q^*F^* - \psi X_mB_q^*)s_n^* + \varphi Y_m^*A_qF - \psi X_m^*B_q)s_n\\ &+ (\varphi Y_m^*Y_m + \psi X_m^*X_m)\Big).\end{aligned} \tag{39}$$

Let us now consider first the term containing $s_n^*s_n$

$$\begin{aligned}S_1 &= P^*P(\varphi A_q^*A_qF^*F + \psi B_q^*B_q)s_n^*s_n\\ &= \frac{(D_fD_c)^*(D_fD_c)}{D^*DQ_d^*Q_d}\,,\end{aligned}$$

where the stable polynomials D_c, D_f are defined from two spectral factorization equations (16), (17). The cost is thus given as

$$\varphi S_{\tilde{u}} + \psi S_e = R^*R + y_d,$$

with

$$R = \Big(\frac{D_fD_c}{DQ_d}s_n + \frac{\varphi D_fA_q^*F^*Y_m}{DQ_dD_c^*} - \frac{\psi D_fB_q^*X_m}{DQ_dD_c^*}\Big)$$

where y_d is the rest that is independent of s_n. We can manipulate the second and third terms in brackets. These can be separated into

$$-\frac{\varphi D_fA_q^*F^*Y_m}{DQ_dD_c^*} + \frac{\psi D_fB_q^*X_m}{DQ_dD_c^*} = \frac{S_n}{DQ_d} + \frac{V^*}{D_c^*}\,. \tag{40}$$

Multiplication with $DQ_dD_c^*$ yields (25). The first term S_n/DQ_d is stable and the second one V^*/D_c^* is unstable. Because the second term is unstable and s_n is required be stable, it vanishes in the cost. The brace now reads

$$\Big(\frac{D_fD_c}{DQ_d}s_n - \frac{S_n}{DQ_d}\Big)\,. \tag{41}$$

Clearly, the best what can be done is to set the brace to zero, hence

$$s_n = \frac{S_n}{D_f D_c}\,. \tag{42}$$

Because the denominator is stable, so is s_n as well. The denominator s_d is derived from (32), substituting s_n from (42), and S_n from (40)

$$\begin{aligned} s_d &= \frac{1}{Q_d} - \frac{Q_n F}{Q_d} s_n = \frac{1}{Q_d} \\ &- \frac{Q_n F(\psi D_f B_q^* X_m - \varphi D_f A_q^* F^* Y_m - D Q_d V^*)}{Q_d D_f D_c D_c^*} \\ &= \frac{\varphi D_f A_q^* F^* A_q F + \psi D_f B_q^* B_m + Q_n F D V^*}{D_f D_c D_c^*} \end{aligned} \tag{43}$$

In order to have s_d stable, the denominator must also be stable. This can only be assured if the unstable D_c^* cancels with the numerator and results in (16) and (17). s_d is thus given as

$$s_d = \frac{S_d}{D_f D_c}\,. \tag{44}$$

COROLLARY 4. *If polynomials Q_d and $Q_n F$ are coprime, then the pair of Diophantine equations (16), (17) can be reduced to the implied Diophantine equation*

$$Q_d S_d + Q_n F S_n = D_f D_c. \tag{45}$$

P r o o f . Multiplying (25) by $Q_n F$ and (26) by Q_d gives

$$\begin{aligned} D_c^* Q_n F S_n &= -\varphi D_f A_q F^* Y_m F Q_n \\ &\quad + \psi D_f B_q^* F X_m Q_n - Q_d Q_n F D V^* \end{aligned} \tag{46}$$

$$\begin{aligned} D_c^* Q_d S_d &= \varphi D_f A_q F^* A_m F Q_d \\ &\quad + \psi D_f B_q^* B_m Q_d + Q_d Q_n F D V^*\,. \end{aligned} \tag{47}$$

Summation of the previous equations gives

$$\begin{aligned} &D_c^*(Q_d S_d + Q_n F S_n) = \\ =&\varphi D_f A_q^* A_q F^* F + \psi D_f B_q^* B_q = D_f D_c D_c^* \end{aligned} \tag{48}$$

and (45) follows directly.

Comparison of two approaches to the LQ tracking problem is summarized by the following corollary.

COROLLARY 5. *The classical LQ controller (Y_c, X_c) obtained from* (18), (19) *and the parametrized LQ controller (Y_s, X_s) obtained from* (23), (27) *with stable polynomials D_c and D_f calculated from* (16) *and* (17) *are identical.*

P r o o f . It is not difficult to check that

$$\frac{Y_c}{X_c} = \frac{Y_s}{X_s}\,.$$

The transfer function of the classical controller (Y_c, X_c) can be obtained from equations (18) and (19)

$$\frac{Y_c}{X_c} = \frac{\psi B_q^* D_f + A_q F V^*}{\varphi A_q^* D_f + B_q V^*}\,.$$

Using equations (25), (26) and (27), we can rewrite the Youla-Kučera parameter S as follows

$$\frac{S_n}{S_d} = \frac{-\varphi D_f A_q^* F^* Y_m + \psi D_f B_q^* X_m - Q_d D V^*}{\varphi D_f A_q^* F^* A_m F + \psi D_f B_q^* B_m + Q_n F D V^*}\,. \tag{50}$$

Putting (50) into (23)we have

$$\begin{aligned} \frac{Y_s}{X_s} = \frac{Y_m S_d + A_m F S_n}{X_m S_d - B_m S_n} &= \frac{\psi B_q^* D_f D + A_q F V^* D}{\varphi A_q^* D_f D + B_q V^* D} \\ &= \frac{\psi B_q^* D_f A_q F V^*}{\varphi A_q^* D_f + B_q V^*}\,. \end{aligned} \tag{51}$$

4 EXAMPLE

In this section, an example is presented to show all steps of the calculation in both cases of LQ design. Let us consider a nominal controlled system described by the transfer function

$$G = \frac{B}{A} = \frac{5}{3s+1}\,.$$

The reference has been chosen as a step change $w(t) = 1(t)$. The proper transfer function of the nominal controller without the precompensator is given as

$$C = \frac{Y}{X} = \frac{0.5111s + 0.2}{0.6667s + 1.444}$$

and yields the closed-loop pole polynomial of the form

$$M_1 M_2\,, \quad \text{where} \quad M_1 = M_2 = s + 1$$

Let us now assume that, based on new data, the new plant estimate is specified by the dual Youla-Kučera parameter Q given as

$$Q = \frac{Q_n}{Q_d} = \frac{0.5}{5s+1}\,.$$

The resulting modified plant transfer function is then (cf. (12))

$$G(Q) = \frac{B_q}{A_q} = \frac{25.17s + 5}{15s^2 + 7.833s + 0.9}\,.$$

The weighting coefficients φ and ψ in the cost function (14) have been selected as $\varphi = 0.7$, $\psi = 0.8$. Both stable polynomials D_c and D_f obtained from spectral factorizations (16), (17) are of the form

$$D_c = 12.55s^3 + 26.63s^2 + 27.3s + 4.472\,,$$
$$D_f = 15s^2 + 7.833s + 0.9\,.$$

Finally, calculation of the classical LQ controller C_c for system $G(Q)$ gives

$$C_c = \frac{Y_c}{FX_c} = \frac{13.42s^2 + 7.006s + 0.805}{12.55s^3 + 26.63s^2 + 4.794s}$$

Alternatively, calculating the optimal Youla-Kučera parameter S of the controller from (25) and (26)

$$S = \frac{FS_n}{S_d} = \frac{-4.183s^4 - 6.916s^3 - 4.589s^2 - 0.690s}{37.65s^4 + 92.45s^3 + 108.1s^2 + 39.83s + 4.025}$$

gives the same controller

$$C(S) = \frac{Y_s}{FX_s} = \frac{YM_1S_d + AM_2FS_n}{FXM_1S_d - BM_2FS_n} = \frac{13.42s^2 + 7.006s + 0.805}{12.55s^3 + 26.63s^2 + 4.794s}\,.$$

NOTE 3. *Although the Youla-Kučera parameter S is a transfer function with the degree of the denominator equal to four, the resulting controller polynomials Y_s, X_s are only of the degree equal to two. This controller degree reduction is caused by the fact that both its numerator and denominator are divisible by D as it is shown in* (51).

5 CONCLUSIONS

In this paper, deterministic LQ controller has been presented. The controller has been derived based on assumption that a nominal stabilizing controller for a nominal plant has changed. This is useful when updating the controller on the basis of new data collected on the plant. The relation between the controller and the plant update characterized by the Youla-Kučera parametrizations of all stabilizing systems is given by a Diophantine equation.

Acknowledgement

The authors are pleased to acknowledge the financial support of the Scientific Grant Agency of the Slovak Republic under grants No. 1/7337/20 and 1/8108/01.

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Received 26 November 2001

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