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ROOT ANALYSIS OF LUM SMOOTHING TECHNIQUES

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The nonlinear merit of rank-order based filters precludes the use of frequency analysis and impulse response to characterise the filter properties. Therefore, in nonlinear filtering domain the filter analysis is performed through deterministic properties such as analysis of root signals, *ie* signals invariant to the additional filtering passes. The statistical properties are derived in the form of output distributions, which are used to study the noise attenuation. This paper focuses on the root signals of LUM (lower-upper-middle) smoothers, where the traditional forms and the adaptive threshold forms are considered.

K e y w o r d s: LUM smoother, weighted median, convergence property, root signal.

1 INTRODUCTION

Unlike linear filters where the filter properties are derived through superposition-based methods such as the impulse response and frequency analysis, in the case of nonlinear filters, *eg*, rank-order based filters, these methods are not suitable. As an example, for overwhelming majority of rank-order based filters the impulse response is zero and thus the characteristic does not describe the filter behaviour. On that account, except evaluating the filter performance, statistical and deterministic properties [1], [15], [16], [19] belong to important measures of nonlinear filters.

In the case of statistical analysis, the determination of the output distribution and selection probabilities is used most frequently. Basically, by using the above- mentioned statistical characteristics it is possible to determine the filter robustness.

However, deterministic properties are described through convergence properties or root signals, *ie*, signals that are invariant to further filtering. Thus, the root signals [19] are investigated in dependence on local signal structures. In this paper the deterministic analysis of LUM smoothers is presented.

2 LUM SMOOTHER

Lower-upper-middle (LUM) smoothers [12],[13] belong to a class of LUM filters [6] that are based on the comparison of a lower- and an upper- order statistic with the central sample from a filter window W. Thus, in dependence on the filter parameter, a wide range of detail preserving and smoothing characteristics can be obtained. In practice, two variants of LUM smoothing are used. The first one is a set of the traditional LUM smoothers [6], [11] with a fixed level of smoothing, whereas the second one is represented by its adaptive forms [8], [9], [10].

2.1 Traditional LUM Smoothers

The output of the traditional LUM smoother [6], [11] is determined in dependence on the control parameter k for smoothing. Thus,

$$y_k = \text{med}\{x_{(k)}, x^*, x_{(N-k+1)}\} \tag{1}$$

where med is a median operator, N is the window size and k is a filter parameter for smoothing. In the case of definition (1), $x_{(k)}$ denotes a lower-order statistic, $x_{(N-k+1)}$ is an upper-order statistic and x^* is the central sample of filter window.

Varying the control parameter k changes the level of smoothing from no smoothing (*ie*, identity filter for $k = 1$, where $y_1 = x^*$) to the maximum amount of smoothing (*ie*, median, $k = (N+1)/2$). Thus, the LUM smoother can achieve the best balance between the noise smoothing and the signal-detail preservation. If x^* lies in a range formed by these order statistics, it is not modified. If x^* lies outside of this range, it is replaced with a sample that lies closer to the median.

Some convergence properties of LUM smoothers were derived [15], [16] in dependence on the signal length, filter length and the set of weights. However, it was necessary to express the LUM smoother equivalently to center-weighted median (CWM) [18], *ie*, WM filter, where except the central weight w_δ the other weights are equal to 1. Note that the central weight is odd and can have a range from 1 to N.

$$y = \text{med}\Big\{W \bigcup \{\underbrace{x^*, x^*, \dots, x^*}_{w_\delta - 1}\}\Big\}. \tag{2}$$

The relationship between parameter w_δ in the CWM and parameter k in the LUM smoother is given as follows

$$w_\delta = N - 2k + 2\,. \tag{3}$$

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However, in the term of the filter implementation, equation (1) requires fewer operations than that of (2), since fewer elements must be sorted.

For 2-D signals such as an image, the number of iterations necessary to obtain a root signal cannot be analytically expressed. In the case of 2-D signals, it is possible to express whether the filter possesses the convergence property, only.

2.2 Adaptive LUM Smoothers

There exist some possibilities of an adaptive LUM smoother. Namely, in this paper the adaptive LUM smoother based on the set of threshold [8], [9], [12] is used, however, another approaches [10] such as a neural LUM smoother and a combination of LUM smoothers, neural network and impulse detector [7] were developed, too.

The output of an adaptive LUM smoother controlled by the threshold system is given by

$$y = y_s \,, \tag{4}$$

where s, bounded by $1 \leq s \leq (N+1)/2$, is a sum of additional parameters c_k defined by

$$\begin{aligned} \text{IF } |x^* - y_k| \geq Tol_k \ \text{ THEN } c_k = 1 \\ \text{ELSE } \ c_k = 0 \end{aligned} \tag{5}$$

ie, the simple comparison between the associated threshold Tol_k and the absolute difference of the central sample x^* and the LUM smoother output y_k. Thus, in every location of a running window the output of the adaptive LUM smoother is determined by the most appropriate smoothing level k corresponding to sum s of parameters c_k.

The above-presented adaptive solution can be realised in two ways. In the first approach there is utilised a sub-optimal set of fixed threshold (method is called LUM FTC) [8], [9], [12] and the second approach is controlled by the set of adaptive thresholds (LUM ATC) [9] that are computed on the basis of standard deviation. Note that for 3×3 LUM FTC the following sub-optimal thresholds were obtained: $\{0, 10, 15, 30, 30\}$ [8].

3 DETERMINISTIC ANALYSIS

Consider a 3×3 filter window. Thus the window size is $N = 9$ and $(N+1)/2 = 5$ traditional LUM smoothers are possible (for $k = 1, 2, \ldots, 5$). Now we present the convergence property for some special cases of LUM smoothers.

It is evident that for $k = 1$ the output signal is equal to the filter input and the input is a root since LUM smoothers work as an identity filter.

To derive the convergence properties for LUM smoothers with control parameter $k = 2$, the CWM (2) expression is necessary. Thus, for the central weight $w_\delta = 7$, the resulting filter will filter any finite-length signal to a root with the only one filter passed, *ie*, it is idempotent [15].

From [8] it can be proved that the LUM smoother with $k = 3$ will converge to a root with some filter passes. For the window size $N = 2k + 1$ and the central weight $w_s = 2m - 1$ (for $M = 1, 2, \ldots, K$) the convergence to a root occurs, if the following condition is satisfied:

$$3M \geq K + 4 \,. \tag{6}$$

In the case of LUM smoothers with a 3×3 filter window, condition (6) is valid for $k = 1$ and $k = 2$.

In the case of a median filter [11], [14], *ie*, LUM smoother with $k = 5$, too, these filters possesses the convergence property [15], [16], [18]. Although the LUM smoothers with $k = 4$ and $k = 3$ do not satisfy (6), it can converge to a root, too.

Now we perform a new analysis. As the test images serve the well-known image models Lena (Fig. 1a) and Bridge and their corrupted equivalents (Fig. 1d shows image Lena corrupted by 10 % impulse noise). Likewise, this analysis will be obtained for the traditional LUM smoothers with $k = 3, 4, 5$ and the adaptive LUM FTC and LUM ATC. Thus, it will be possible to compare (Table 1 and Table 3) the number of needful iterations to possess the convergence property. On the other hand, Table 2 and Table 4 show the evaluating of root signals through the well-known mean absolute error (MAE) and mean square error (MSE).

Table 1. The number of iterations needed to possess a root signal

	Lena	Lena I10
$k = 3$	27	21
$k = 4$	19	16
$k = 5$	97	93
FTC	**8**	**9**
ATC	13	**9**

Table 2. Quality of root signals

	Lena		Lena I10	
	MAE	MSE	MAE	MSE
$k = 3$	1.563	23.2	2.479	59.4
$k = 4$	3.171	57.5	3.779	74.8
$k = 5$	5.940	136.8	6.436	149.1
FTC	**0.557**	22.7	**1.395**	**44.3**
ATC	0.915	**16.1**	1.778	47.0

Table 3. The number of iterations needed to possess a root signal

	Bridge	Bridge I10
$k = 3$	30	12
$k = 4$	15	13
$k = 5$	82	58
FTC	27	8
ATC	**14**	**8**

Fig. 1. (a) original image Lena (b) root signal on image Lena obtained by median filter (LUM $k = 5$) (c) root signal on image Lena obtained by LUM FTC (d) image Lena corrupted by 10 % impulse noise (e) root signal on image Lena I10 by LUM $k = 3$ (f) root signal on image Lena I10 by LUM ATC.

Table 4. Quality of root signals

	Bridge		Bridge I10	
	MAE	*MSE*	*MAE*	*MSE*
$k=3$	2.520	40.8	3.964	105.6
$k=4$	5.136	96.8	6.132	133.8
$k=5$	10.341	262.2	10.860	282.0
FTC	**1.018**	34.3	**2.435**	**81.5**
ATC	1.535	**30.3**	2.993	90.0

From Table 1 and Table 3 it is evident that all proposed methods converge to a root after a finite number of passes. In the case of median filter (LUM smoother with $k = 5$), it is necessary to perform more than 80 iterations. In addition, the obtained root signal (Fig. 1b) is very blurred; the high frequency elements (image details and edges) are removed.

In the case of traditional LUM smoothers with $k = 3$ and $k = 4$, these filters converge to a root after 15-30 passes approximately with simultaneous preservation of image details.

However, excellent results were obtained by adaptive methods that provide minimal deviation from the original image and minimal passes to achieve root signals. These properties should represent an adequate motivation for the use of adaptive LUM smoothers in the practice.

4 CONCLUSION

In this paper, the convergence properties of 3×3 traditional LUM smoothers and 3×3 adaptive threshold LUM smoothers were presented. By theoretical analysis and experimental results it was proved that in the case of a 3×3 filter window each form of LUM smoother converges to a root after a finite number of passes. Similar to our previous works [9–13], the best results and the excellent similarity with the original were obtained by adaptive threshold LUM smoothers.

Besides significant theoretical aspect of the deterministic analysis, the importance of root signal lies in the possibility of their use in modern communication and multimedia systems [3], [4], [17] based on the coding and cryptography methods [2], [5] that are easier to apply on the smoothed image than the randy image. The acquisition of the proposed threshold LUM smoothers (FTC and ATC approaches) is related to excellent fast convergence to root signals. In addition, the visual quality of root signals produced by threshold LUM is near to original.

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