

# OCCURRENCE AND ADVERSE CONSEQUENCES OF 3N<sup>th</sup> ORDER HARMONICS IN 4-WIRE POWER LINES

Ivan Bojna — Mojmír Kollár \*

In this paper, the occurrence and analysis of higher harmonic components of currents in the neutral wire and its influence on the quality of electric power supplied is presented. This is of practical interest namely in line systems with highly non-linear loads nowadays spreading more and more widely.

**Key words:** 4-wire power lines, electric power quality, non-linear circuits, neutral wire overloading

## 1 INTRODUCTION

Together with the ever-growing number of non-linear electric devices being used nowadays in consumer electric networks (TV sets, flicker of fluorescent lighting equipment, PCs with their monitors, dimer systems, audio equipment and even the heating plates and other so-called low-cost electrical appliances), more and more frequently the question rises of the supplied electric power quality. One of the most important quality indicators of the supplied power is the (unwanted) content of higher order harmonics currents and the voltage drops caused by them. Of particular interest are the harmonics of 3N-order (N is an integer) which, beside undesirable distortion of the voltage shape, may cause overloading of the so-called *neutral* in 4-wire power lines. In the case of (in general anticipated) linear and symmetric load (*ie* equal impedances in all three lines of 4-wire circuit) the current in the *neutral wire* is zero. However, if the 3-phase load is symmetric but not a linear one, under certain circumstances the current through the neutral wire can be even substantially higher than the nominal line current, causing thus the neutral wire to be overloaded. Since in 4-wire systems there is a clear tendency to distribute the load (at least approximately) equally to all three lines, the neutral wire cross-section is usually designed to be less than that of the line wires. Therefore, overloading of the neutral wire can be expected already at smaller currents than the nominal line currents.

## 2 THEORY

Let us consider a 4-wire (3-lines: R, S, T) network with line currents  $i_R(t)$ ,  $i_S(t)$ ,  $i_T(t)$  being represented by their phasors  $\mathcal{J}_R$ ,  $\mathcal{J}_S$ ,  $\mathcal{J}_T$ , and with the current in the neutral wire  $i_N(t)$ , and  $\mathcal{J}_N$  respectively. The line is powered by a symmetric 3-phase voltage source. In the case of symmetric 3-phase load ( $\mathcal{Z}_R = \mathcal{Z}_S = \mathcal{Z}_T = \mathcal{Z}$ ) the line

currents are symmetric, too. In other words, the phasors representing the currents of individual lines are

$$\begin{aligned}\mathcal{J}_R &= \mathcal{J} \\ \mathcal{J}_S &= \mathcal{J}e^{-j\frac{2\pi}{3}} \\ \mathcal{J}_T &= \mathcal{J}e^{j\frac{2\pi}{3}}\end{aligned}\quad (1)$$

and the phasor of the current in the neutral wire, (see Fig.1) is easily shown to be zero

$$\mathcal{J}_N = -(\mathcal{J}_R + \mathcal{J}_S + \mathcal{J}_T) = \mathcal{J} \left(1 + e^{-j\frac{2\pi}{3}} + e^{j\frac{2\pi}{3}}\right) = 0.$$

In the case of a symmetric but non-linear load, all the currents can be described by a sum of harmonic components

$$\begin{aligned}\mathcal{J}_R(\omega, t) &= \sum_{\nu=1}^{\infty} \mathcal{J}_\nu e^{j\nu\omega t} \\ \mathcal{J}_S(\omega, t) &= \sum_{\nu=1}^{\infty} \mathcal{J}_\nu e^{-j\nu\frac{2\pi}{3}} e^{j\nu\omega t} \\ \mathcal{J}_T(\omega, t) &= \sum_{\nu=1}^{\infty} \mathcal{J}_\nu e^{j\nu\frac{2\pi}{3}} e^{j\nu\omega t} \\ \mathcal{J}_N(\omega, t) &= -\sum_{\nu=1}^{\infty} \mathcal{J}_\nu \left(1 + e^{-j\frac{2\pi}{3}} + e^{j\frac{2\pi}{3}}\right) e^{j\nu\omega t}\end{aligned}\quad (2)$$

While for the basic harmonic component ( $\nu = 1$ ) and analogously for all higher components ( $\nu \neq 3k$ ,  $k = 1, 2, 3, \dots$ ) of those line currents components, ordinal numbers of which are not divisible by 3 without remainder, or for:  $\text{mod}(\nu, 3) = 0$ , as before  $\mathcal{J}_N(\omega t) \equiv 0$ . In cases where  $\text{mod}(\nu, 3) \neq 0$ , or for ( $n = 3k$ ,  $k = 1, 2, 3, \dots$ ) the term in braces will give

$$1 + e^{-j3k\frac{2\pi}{3}} + e^{j3k\frac{2\pi}{3}} = 1 + e^{-jk2\pi} + e^{jk2\pi} = 3,$$

\* Department of Electromagnetic Theory, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology, Ilkovičova 3, 812 19 Bratislava, Slovakia, email: mkollar@elf.stuba.sk

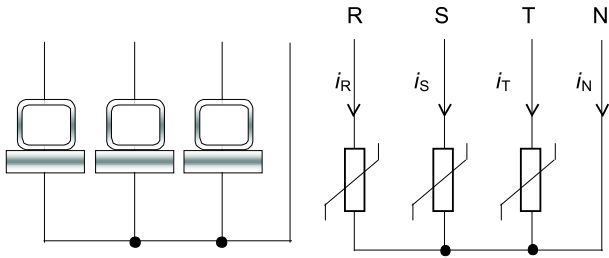


Fig. 1. Non-linear symmetric load of three equal PCs and three equal monitors

giving

$$\mathcal{J}_N(\omega, t) = -3 \sum_{\nu=1}^{\infty} \mathcal{J}_\nu e^{j\nu\omega t}$$

$$|\mathcal{J}_N(\omega, t)| = 3 |\mathcal{J}_R(\omega, t)|,$$

$$I_N = 3I_R \quad \text{or} \quad I_N = 3I.$$

Clearly, in the neutral wire, the magnitude (or RMS values) of the harmonic components with ordinal numbers divisible by 3 without remainder are three times the magnitude (or RMS value) of the (equal) line currents. The higher harmonic components with ordinal numbers  $\nu$  for which  $\text{mod}(\nu, 3) \neq 0$ , are algebraically added in the neutral wire.

### 3 EXAMPLE AND DISCUSSION

The above discussion is well illustrated on a simple 4-wire 3-phase network powered by a symmetrical source with line voltages of 230/400 V and with three equal PCs together with three equal monitors (Fig. 1) as a non-linear but symmetric load. Recorded time dependences of the line currents and the neutral wire current are shown in Fig. 2

The amplitudes of the basic and higher components according to the Fourier analysis of the waveforms in Fig. 2, as well as appropriate RMS values of the shown non-harmonic time-dependences are listed in Tab. 1. As revealed, the RMS of the neutral-wire current ( $I_N$ ) in this case is almost twice the value of  $I$  (the equal line currents:  $I_R, I_S, I_T$ ).

It may be interesting to notice that not only the higher harmonic components of the line currents are quite developed but also the 13-th predominates the 11-th due to impulsive nature of the switched sources involved (see the leftmost columns in Tab. 1). This relation is even more pronounced in the case of the neutral wire current with a considerably augmented 15-th component (middle column). On the other hand, the situation is quite different for the square of individual harmonics (right-most column) in the neutral wire, the magnitude of which is directly related to the power dissipated by these components. It can be seen that from the heat generation point of view crucial (and critical) is the 3-rd component, and those of higher orders do not contribute substantially. Compare around 5% attributed to the 15-th component, if the value of the 3-rd (in Tab. 1) is set to 100%.

Among the problems caused by higher harmonics and effects more or less responsible for the deterioration of the "power quality" are:

- resonance in networks at certain harmonics, which - due to over-voltage - may cause the enhanced stress of the network elements,
- overloading and consecutive overheating of the network components (cables, transformers),
- malfunction of the protection (eg transformers) due to the current growth,

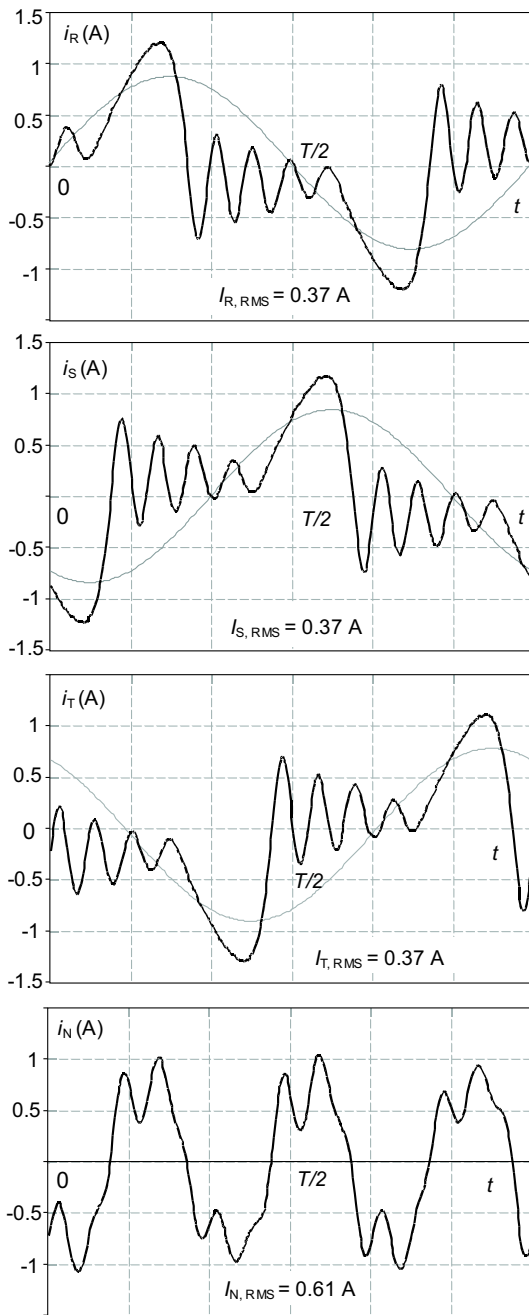


Fig. 2. Time dependences of the line currents and the neutral wire current in circuit from Fig.1

- disturbances caused by voltages induced in the telecommunication and data lines,
- incorrect functioning of the electronic control circuits.

Therefore, elimination of higher harmonics to prevent their adverse effects may include:

- reconstruction of the power network (enlarging the neutral wire cross-section, creation of separate circuits for non-linear appliances feeding having the neutral wires with enlarged cross-sections),
- filtration of the DC component caused by the third harmonic, using the pertinent connection of the feeding transformers (*eg* transformer connection star/delta 0,4/0,4 kV),
- installment of passive filters for the higher harmonics,
- installment of active filters for the higher harmonics (generation of the harmonic load current using electronic means).

**Table 1.** Harmonic components — amplitudes of the line and neutral wire currents, and the square of the neutral wire current component (proportional to the heat dissipation). Percentage in first two left columns is referenced to the first harmonic component, and asterisk (\*) is to indicate the expected zero values (due to anticipated symmetry). The percentage in the right-most column is related to the value of the power dissipated by the third harmonic component.

$\nu$	Line current		Current in neutral wire		Square of current in neutral wire	
	(A)	%	(A)	%	(A <sup>2</sup> )	%
0	0.0132	3.6*	0.0396	10.8*		-
1	0.3670	100	0.0138	3.8	0.0002	0.0
2	0.0133	3.6*	0.0010	0.3*		-
3	0.2705	73.7	0.8104	220.8	0.6567	100.0
4	0.0069	1.9*	0.0010	0.3*		-
5	0.1551	42.3	0.0285	7.8	0.0008	0.1
6	0.0016	0.4*	0.0047	1.3*		-
7	0.0744	20.3	0.0199	5.4	0.0004	0.1
8	0.0033	0.9*	0.0010	0.3*		-
9	0.0767	20.9	0.2271	61.9	0.0516	7.9
10	0.0020	0.5*	0.0008	0.2*		-
11	0.0992	27.0	0.0391	10.7	0.0015	0.2
12	0.0026	0.7*	0.0076	2.1*		-
13	0.1328	36.2	0.0669	18.2	0.0045	0.7
14	0.0006	0.2*	0.0003	0.1*		-
15	0.0601	16.4	0.1740	47.4	0.0302	4.6
16	0.0012	0.3*	0.0008	0.2*		-
17	0.0202	5.5	0.0119	3.2	0.0001	0.0
18	0.0009	0.2*	0.0025	0.7*		-
19	0.0070	1.9	0.0052	1.4	0.0000	0.0
20	0.0006	0.2*	0.0004	0.1*		-
21	0.0055	1.5	0.0155	4.2	0.00024	0.0

## 5 CONCLUSIONS

The content of higher harmonic components, without remainder divisible by 3, in consumer power lines is still

growing. Beside other effects (*eg* causing the EMC incompatibility due to high-frequency radiation) this may lead also to overheating of the neutral wire in 4-wire 3-phase systems. Particularly in power-line networks with a greater number of non-linear appliances (*eg* computing centers buildings with a high number of compact fluoride lamps *etc*), where the power consumption of the non-linear appliances represents an essential part of the net power consumption, it is necessary to expect the aforementioned harmful consequences and perhaps to take appropriate measures for their elimination (*eg* installment of filters for suppression of higher harmonic currents).

Since the contribution of the 9-th, 15-th and so forth harmonics from the power dissipation point of view is less important, the most harmful is the 3-rd harmonic component. Therefore the switched devices should be designed to work in pace with suppressed 3-rd harmonics - not only to produce less disturbing field in the surroundings but also to prevent the 3-phase 4-wire system from possible overheating by the overloaded neutral wire. Nowadays, the importance of the power quality is pointed out more frequently and quality contracts should exist between the transmission system operators, distribution companies and their customers.

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**Ivan Bojna** (Doc, Ing, PhD), born in 1942, is Associate Professor at the Department of Electromagnetic Theory, Slovak University of Technology, Bratislava, Slovakia. He studied electrical engineering at the Slovak Technical University in Bratislava, and received his PhD degree from the same university, in 1986. The main field of his scientific activities is investigation and measuring of quantities in circuits with non-sinusoidal currents and voltages in electric power systems.

**Mojmír Kollár** (Ing, PhD) was born in Ružomberok, Slovakia, in 1945. He graduated from the Faculty of Electrical Engineering, Slovak Technical University, Bratislava, in solid state physics, in 1968. In 1985 he received the PhD degree in Electromagnetic Theory from Alma Mater, where he at present works as a lecturer at the Department of Electromagnetic Theory. The main fields of his research and teaching activities are the circuit and electromagnetic field theory.