ELECTRIC FIELD NEAR BUNDLE CONDUCTORS

Daniel Mayer — Vít Veselý *

The paper deals with computation of electric field distribution along the surfaces of a system of parallel conductors with various potentials. The method starts from integral equations and the elaborated algorithm is applied to hv and uhv overhead lines with bundle conductors. The results allow evaluating the danger of giving rise to corona (even with respecting the influence of a rough surface of the cable) with all its interference effects. The algorithm was compared with other already published methods with remarkable agreement.

Keywords: hv and uhv overhead lines, bundle conductors, corona, method of integral equations.

1 INTRODUCTION

Hv and uhv overhead lines are mostly realised by bundle conductors. Electric field strength on their surfaces depends on the potential of particular conductors, their radii, mutual distances and quality of their surface. Exceeding the critical value $E_{\text{crit}} \approx 21$ kV/cm gives rise to a corona. One of the advantages of using bundle conductors is reduction of this discharge that, as it is well known, increases losses along the line, unfavourably affects telecommunication devices and produces acoustic noise. A surprising amount of papers were published on the topic, see the bibliographic list [6] compiled already in 1964.

Various methods have been used for determining the electric field near the bundle. Older papers, see, for instance, [2], [15] started from an analytical solution of the electric field produced by one conductor and the resultant electric field of a bundle was calculated using superposition. The results obtained in this way are, however, only approximate. Other authors [3], [4] and [8] replace the particular conductors in the bundle by a system of suitably located current filaments and solve their electric field. Conformal mapping was used in [13]. Simulated charge method was successfully applied in [11] and [12]. Numerical calculation based on standard finite element techniques (and realised by means of professional codes) would be comfortable, nevertheless, problems with geometrical incommensurability (small cross-sections of the conductors versus their large mutual distances and domain containing air) would lead to using a strongly non-uniform mesh and enormous number of equations. The task represents, moreover, an open boundary problem, which is often a source of further errors.

That is why the method of integral equations (see [5]) may appear advantageous for solving this case; its fundamentals can also be found in [6] or [14]. The unknown quantity is now the charge density $\sigma$ along the surface of particular conductors. We formulate the corresponding equation for $\sigma$ and solve it numerically. We are, however, mainly interested in the vector of the electric field strength on the surface of particular conductors. It has only the normal component $E_n$ that is given by a simple (Coulomb) formula

$$E_n = \frac{\sigma}{\varepsilon_0}.$$ 

The principal advantage of this approach consists in the fact that the numerical computation of the charge density is much simpler than computation of the distribution of the potential near the conductors (which would, of course, also provide the values of $E_n$). While the distribution of the potential in the solved arrangement represents a 2D problem, solution of the charge density is only a 1D problem. Reduction of the dimension by 1 leads to a significant simplification of the task. Unlike in the open boundary problem associated with computation of the potential, the charge is distributed along surfaces of the conductors, i.e. in a spatially bounded domain.

2 MATHEMATICAL MODEL OF THE PROBLEM

Considered is homogeneous, linear and isotropic dielectric (air) of permittivity $\varepsilon_0$ with a set of n direct parallel conductors with constant potentials $\varphi_k$, $k = 1, \ldots, n$. The electric charge on their surfaces is supposed to be distributed continuously, with charge density $\sigma_k$ that does not change along their lengths. The electric field near the conductors is obviously two-dimensional. Potential at a general point B of this field is given by expression [7]

$$\varphi(r_B) = \frac{1}{2\pi\varepsilon_0} \sum_{k=1}^n \sigma_k \int \varphi_k(r^{(k)}) \ln \frac{1}{|r^{(k)} - r_B|} dl^{(k)},$$

(1)

$k = 1, \ldots, n$

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where $l^{(k)}$ is a simply connected curve in which the cross-section $S^{(k)}$ intersects the plane of the field perpendicular to the conductors, $r^{(k)}$ is the radiusvector of the element $d(l^{(k)})$, $r_B$ is the radiusvector of point B and $|r^{(k)} - r_B|$ is the distance of a variable point A of the planar curve $l^{(k)}$ from point B outside the conductors (see Fig. 1). We put, moreover, that

$$\lim_{|r_B| \to \infty} \varphi(r_B) = 0.$$ 

Let point $B \in l^{(k)}$. Now (1) transforms into the first-kind Fredholm integral equation with an unknown distribution of the charge density $\sigma_k$.

$$2\pi \varepsilon_0 \varphi_k = \sum_{k=1}^{n} \int_{l^{(k)}} \sigma_k(r^{(k)}) \ln \frac{1}{|r^{(k)} - r_B|} d(l^{(k)}) ,$$

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$$\lim_{|r_B| \to \infty} \varphi(r_B) = 0.$$
The system can be rewritten into a matrix form

\[ \mathbf{A} \sigma = 2\pi\varepsilon_0 \varphi \]  

(9)

where

\[ \mathbf{A}(m, n) = [a_{pq}], \quad p, q = 1, \ldots, m, \]

\[ a_{pq} = \begin{cases} \Delta l_p \ln|\mathbf{r}_p - \mathbf{r}_q|, & \text{for } p \neq q \\ \Delta l_p \left(1 - \ln \frac{\Delta l_p}{2}\right), & \text{for } p = q. \end{cases} \]

Here \( \sigma(m, 1) \) is the column vector of charge densities in the particular segments and \( \varphi(m, l) \) their potentials.

The presented relations respect the influence of all conductors in the system (all bundle conductors in all phases) and possibly the influence of the earth.

### 3 Illustrative Examples

The next examples solve, however, only the case of conductors in a one phase bundle (ie we consider neither the influence of other phases nor of the earth). Determined is the distribution of the electric field strength on the surface of one conductor in the bundle.

**a. A bundle conductor \( n = 4 \)**

For the conductors placed in vertices of a regular square (see Fig. 3) we calculated the distribution of the normal component of \( E_n \) along the surface of one conductor. All conductors in the bundle have the same potential \( \varphi = 100 \text{ kV} \). Perimeters of all conductors were discretised.
Fig. 7. A massive conductor and equivalent transmission cable for \( n = 8 \).

Fig. 8. Distribution of the normal component of the electric field strength along the surface of one conductor of the transmission cable.

Table 1. Values of \( E_n \) for various arrangements of conductors.

<table>
<thead>
<tr>
<th>arrangement</th>
<th>( E_n ) (kV/cm)</th>
<th>( E_n/E_n^{(1)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>bundle conductor ( n = 8 ) (Fig. 5)</td>
<td>( E_{n,\text{max}} = 9.858 )</td>
<td>0.339</td>
</tr>
<tr>
<td></td>
<td>( E_{n,\text{min}} = 4.837 )</td>
<td>0.166</td>
</tr>
<tr>
<td>one equivalent conductor</td>
<td>( E_{n,1} = 29.072 )</td>
<td>—</td>
</tr>
<tr>
<td>bundle conductor, large distance among particular conductors</td>
<td>( E_n = 7.268 )</td>
<td>0.250</td>
</tr>
</tbody>
</table>

into \( N^{(1)} = \cdots = N^{(4)} = 50 \) line segments. The results are depicted in the polar co-ordinates in Fig. 4. The maximum and minimum values of the electric field strength are \( E_{n,\text{max}} = 14.30 \text{ kV/cm}, \) \( E_{n,\text{min}} = 10.58 \text{ kV/cm}. \) The total electric charge per unit length of one conductor of the bundle is

\[
Q = \int_S \sigma dS \equiv \sum_{i=1}^{N} \sigma_i \Delta l_i = 6.8783 \times 10^{-7} \text{ C/m}.
\]

If we replace the bundle conductor by a single conductor of radius \( r = 2r_0 \) (its cross-section being the same as the total cross-section of the bundle conductor), the electric field strength \( E_n \) would be distributed uniformly and its value would be

\[
E_n = \frac{nQ}{2\pi \varepsilon_0 r} = \frac{4 \cdot 6.8783 \times 10^{-7}}{2\pi \varepsilon_0} \frac{1}{0.02} = 24.73 \text{ kV/m}.
\]

If we use a bundle conductor where the distance between individual conductors is very large (their mutual electrostatic interaction is negligible), the electric field strength \( E_n \) will again be uniform and its value would be

\[
E_n = \frac{Q}{2\pi \varepsilon_0 r_i} = \frac{6.8783 \times 10^{-7}}{2\pi \varepsilon_0} \frac{1}{0.01} = 12.36 \text{ kV/m}.
\]

b. A bundle conductor \( n = 8 \)

We calculated \( E_n \) on the surface of one conductor arranged at vertices of a regular octagon (Fig. 5). Its distribution is in Fig. 6. Other results analogous to the previous case are summarised in Tab. 1.

c. Influence of rough surface of the transmission cable

The transmission (for instance aluminium-steel) cable may be taken as an extreme case of the bundle conductor, whose partial conductors are placed very close one to another. While on the surface of a massive conductor the normal component \( E_n \) is distributed uniformly, in the case of the transmission cable this component changes from zero on the internal surface (influence of shielding) to its maximum \( E_{n,\text{max}} \) on the external surface. Obviously \( E_{n,\text{max}} > E_n \). Figure 7 depicts both the cross-section of the massive conductor and cross-section of the cable with 8 conductors. Figure 8 contains a graph in polar co-ordinates showing the distribution of the electric field strength on the surface of one cable conductor. Calculations were performed for potential \( \varphi = 100 \text{ kV} \), perimeter of each conductor being divided into \( N = 50 \) line segments. Analogously to previous cases we obtain

- for a single conductor of the cable
  \( E_{n,\text{max}} = 11.103 \text{ kV/cm}, \) \( E_{n,\text{min}} \approx 0, \)
  \( Q \equiv \sum_{i=1}^{N} \sigma_i \Delta l_i = 2.089 \times 10^{-7} \text{ C/m}, \)

- for an equivalent bundle conductor
  \( E_n = \frac{nQ}{2\pi \varepsilon_0 \sqrt{8r_i}} = 10.077 \text{ kV/cm}. \)

On the surface of the cable (in comparison with the equivalent massive conductor with smooth surface) the dielectric stress is higher. The measure of this increase may be given by the ratio between the maximal value of its normal component to the magnetic field strength of the massive conductor: \( E_{n,\text{max}}/E_n = 1.102. \)

Let us remark that microscopic roughness of the surface caused, for example, by corrosion or small water drops gives also rise to corona.
4 CONCLUSION

The presented algorithm for computation of dielectric stress on the surface of a conductor in a bundle was compared with results obtained by other methods. Table 2 contains some results obtained by different algorithms for a bundle containing \( n = 4, 8 \) and \( 12 \) conductors characterised by \( d/r = 26.099 \) (\( d \) being the distance of axes of two neighbouring conductors in the bundle and \( r \) their radius). The perimeter of each conductor was then divided into \( N = 32 \) line segments. The agreement between particular methods is outstanding.

Table 2. Comparison of results.

<table>
<thead>
<tr>
<th>( E_{\text{min}}/E_{\text{max}} )</th>
<th>( n = 4 )</th>
<th>( n = 8 )</th>
<th>( n = 12 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>the described method</td>
<td>0.7217</td>
<td>0.6024</td>
<td>0.6454</td>
</tr>
<tr>
<td>by Cahen [3]</td>
<td>0.7203</td>
<td>0.6593</td>
<td>0.6418</td>
</tr>
<tr>
<td>by Timascheff [13]</td>
<td>0.7222</td>
<td>0.6632</td>
<td>0.6463</td>
</tr>
</tbody>
</table>

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References


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