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COMMUNICATIONS

EASY TUNING OF PID CONTROLLER

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This note presents new tuning rules for PID controllers that can be viewed as extensions of the Aström and Hägglund's generalized Ziegler-Nichols rule [1]. Compared to other many tuning techniques given in references, where each of them claims to be best suited for a particular problem, in this case the designer may to influence the closed-loop performance.

K e y w o r d s: PID controllers, frequency tuning method, ultimate point

1 INTRODUCTION

The tuning of PID controllers in the frequency domain is a topic of great interest in the industry field. The widespread use of simple methods of the Ziegler-Nichols type [12], [1], [2], [6], [10], [11] clearly indicates the need for simple methods that use minimal process information but give the required closed-loop performance. It has been stated that more than 95 percent of the controllers are of PID [1]. The PI or PID controller implementation has been recommended for the control of processes of low to medium order with rather small time delay [6]. The tuning rules of PID controllers can be divided in to the following groups:

- Tuning rules based on a measured step response.
- Tuning rules based on minimizing an appropriate performance criterion.
- Tuning rules that give a specified closed-loop response.
- Robust tuning rules

By [9], there are 154 tuning rules for PI controllers and 258 tuning rules for PID controllers. Most of them are suited for a particular problem or give the prespecified closed-loop performance. In this paper the proposed tuning rules are based on both ultimate frequency, ultimate gain and specified closed-loop response given by partially shaping of the open-loop frequency response. We pursue the idea of [1] and [8]. The obtained results are illustrated on benchmark examples [3].

2 PRELIMINARIES

Aström and Hägglund [1] have proposed a method for tuning PID controllers in the frequency domain which is a generalization of the Ziegler-Nichols tuning rules. For PID noninteractive algorithm described by the transfer function

$$R(s) = k(1 + \frac{1}{T_i s} + T_d s) \tag{1}$$

the determination of its parameters is summarized in the following expressions

$$k = \frac{r_b \cos(\phi_b - \phi_a)}{r_a} \tag{2}$$

$$T_i = \frac{1}{2\alpha\omega_c}(\tan(\phi_b - \phi_a) + \sqrt{4\alpha + \tan^2(\phi_b - \phi_a)}) \tag{3}$$

$$T_d = T_i \gamma \tag{4}$$

where ω_c is the ultimate frequency; r_a and ϕ_a are the module and phase of point A on the frequency response of the investigated plant; r_b and ϕ_b are the module and the phase of point B on the open-loop frequency characteristic, and γ is the ratio between the derivative T_d and the integral T_i time constant. The parameters of PID controller designed by (1)–(4) guarantee that the ultimate point A will be moved to point B. There are some particular cases. When a phase margin P_m is specified the following conditions are imposed to point B: $r_b = 1, \phi_b = P_m$ and the starting point is on the real axis as an ultimate point with $r_a = 1/k_u$ and $\phi_a = 0$. k_u is ultimate gain of controller. When a gain margin G_m is specified, the following conditions are imposed to point B: $r_b = 1/G_m$ $\phi_b = 0$ and the starting point A is ultimate point of open loop with P controller.

The combined tuning by the phase and gain margin leads to moving two specified points of plant's Nyquist diagram to positions: $r_1 = 1, \phi_1 = P_m$ and $r_2 = 1/G_m, \phi_2 = 0$. Even if both specifications have been reached with the same control parameters, the design procedure becomes rather more complicated in comparison with classical Ziegler-Nichols tuning. For more detail the reader can consult [8].

In this paper the ultimate point A can be moved from real axis to another point of the complex plain and closed-loop stability and performance are guaranteed by a corresponding choice of the co-ordinates of point B.

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3 PID CONTROLLER TUNING

Let the co-ordinates of ultimate point A for some transfer function $F_R(s)$ and plant $G(s)$ be given by the following expression

$$1 + k_u F_R(\omega_c i) G(\omega_c i) = 0 \quad i = \sqrt{-1}$$

or

$$G(\omega_c i) = -\frac{1}{k_u F_R(\omega_c i)}. \tag{5}$$

In this paper we assume that $F_R(s)$ is given as follows

- $F_R(s) = k_R(1 + \frac{1}{T_a s})$
- $F_R(s) = k_R$ (6)
- $F_R(s) = k_R(1 + T_b s)$

For the first case of (6) the following results are obtained. When co-ordinates of point B are given as $(-\eta + i\rho)$, for PID controller one obtains

$$\frac{T_a \omega_c}{k_u k_R (i - T_a \omega_c)} k(1 + \frac{1}{i T_i \omega_c} + i T_d \omega_c) = -\eta + i\rho \tag{7}$$

From (7) one obtains

$$k = k_u k_R \alpha \quad \alpha = \frac{\eta T_a \omega_c - \rho}{T_a \omega_c}$$

$$T_i = \frac{\alpha}{\frac{\eta}{T_a} + \rho \omega_c + \alpha T_d (\omega_c)^2} \tag{8}$$

when the derivative time constant is given or if $T_d = T_i \gamma$

$$T_i = \frac{1}{\omega_c \gamma}(-\varepsilon + \sqrt{\varepsilon^2 + \gamma} \tag{9}$$

where

$$\varepsilon = \frac{\eta + T_a \omega_c \rho}{2(\eta T_a \omega_c - \rho)}$$

provided that

$$\eta T_a \omega_c - \rho > 0 \tag{10}$$

Note that PI controller tuning parameters can be obtained from (8) when $T_d = 0$ and provided that

$$-\frac{1}{T_a \omega_c} < \frac{\rho}{\eta} < T_a \omega_c \tag{11}$$

Finally, the PD controller tuning parameters are as follows

$$k = k_u k_R \alpha \quad T_d = -\frac{\eta + \rho T_a \omega_c}{\omega_c (\eta T_a \omega_c - \rho)} \tag{12}$$

provided that

$$-\frac{\rho}{\eta} > \frac{1}{T_a \omega_c}$$

When the third case in (6) holds, that is

$$F_R(s) = k_R(1 + T_b s)$$

for PID controller parameters the following results are obtained

$$k = k_u k_R \beta \quad \beta = T_b \omega_c \rho + \eta$$

$$T_i = 0.5(\delta + \sqrt{(}\delta^2 + \frac{4}{\gamma \omega_c^2}) \tag{13}$$

$$T_d = T_i \gamma \quad \delta = \frac{\eta T_b \omega_c - \rho}{\beta \gamma \omega_c}$$

Finally, when the ultimate point A is chosen at the real axis, that is $F_R(s) = k_R$ for PID controller and point $B(-\eta + i\rho)$ the following results are obtained

$$k = k_u k_R \eta \quad T_d = T_i \gamma$$

$$T_i = \frac{1}{\omega_c \gamma}(-\frac{\rho}{2\eta} + \sqrt{(\frac{\rho}{2\eta})^2 + \gamma}) \tag{14}$$

When the derivative time constant T_d is fixed for T_i one obtains

$$T_i = \frac{\eta}{\omega_c} \frac{1}{\rho + \eta T_d \omega_c} \tag{15}$$

provided that

$$\frac{\rho}{\eta} > -T_d \omega_c$$

There are many different quality criteria of the systems evolution. The fact that improvement of the controller design in one respect will very often bring deterioration in another, is well known. In [7] the following evolution criteria have been proposed. Two classical measures are common to characterize the mid frequency part of the system, the phase margin and gain margin. However, in recent years a restriction of the maximum sensitivity function

$$M_s = \max_\omega |S(s)| = \max_\omega |\frac{1}{1 + G(s)R(s)}| \tag{16}$$

has been more and more excepted as an exclusive quality and robustness measure [2]. When further damping of the step response or increased phase margin is required, a restriction on the maximum complementary sensitivity function

$$M_t = \max_\omega |T(s)| = \max_\omega |\frac{G(s)R(s)}{1 + G(s)R(s)}| \tag{17}$$

should be added, especially for plants with integral action. The default values are $M_s = 1.7, \quad M_t = 1.3$. The above two criteria can be expressed as restrictions on the Nyquist curve of the open loop transfer function. The constraint that sensitive function $S(s)$ is less than a given value M_s implies that the open loop should be outside a circle with radius $1/M_s$ and center -1. The constraint that $T(s)$ is less than a given value of M_t also implies that the open loop transfer function is outside of the Hall circle [5]. In [4] it is shown that a constraint on both M_s and M_t can be replaced by a slightly more conservative constraint that implies that the Nyquist curve avoids the circle with radius R and center $-C$, where

$$C = \frac{2M_s M_t - M_s - M_t + 1}{2M_s(M_t - 1)} \tag{18}$$

$$R = \frac{M_s + M_t - 1}{2M_s(M_t - 1)} \tag{19}$$

Changing the transfer function and parameters of $F_R(s)$ one can measure out different ultimate points A, or different points of the plant Nyquist diagram. The closed loop stability and performance design can be by removing suitable point of plant Nyquist diagram to point B with co-ordinates $(-\eta + i\rho)$ outside of restricted area defined by circle $(R, -C)$.

4 EXAMPLES

Aström and Hägglund [3] have proposed 11 benchmark systems for evaluating the PID controllers. The first five systems are standards that are well suited to parametric studies. The remaining benchmark systems are more specialized. They illustrate systems with various difficulties of control. PID control is not well suited for all of them. In this paper four examples have been taken from the "remaining" part of benchmark systems.
As a first example we show the system with fast and slow modes. The transfer function of this system is given as follows

$$G(s) = \frac{100}{(s+2)^2}\left(\frac{1}{s+1} + \frac{0.5}{s+0.05}\right)$$

The above system has been stabilized for

$$R(s) = \frac{0.1608s^2 + 0.2618s + 0.8066}{2.456s}$$

with

$$T_a = 10\,\mathrm{s} \quad \eta = 0.3 \quad \rho = -0.5 \quad \gamma = 0.25 \quad k_R = 0.05$$

The co-ordinates of ultimate point A are $(-2.9817 - 0.1086i)$. The performance of the closed-loop system can be characterized by the following parameters

$$G_m = \text{inf dB}$$
$$P_m = 77.943\,\text{degree}$$
$$\omega_P = 1.7207\,\text{rad/s}$$

The second example belongs to the class of conditionally stable systems. The transfer function of this system is

$$G(s) = \frac{(s+6)^2}{s(s+1)^2(s+36)}$$

The obtained results for PID controller are as follows

$$k = 1.51 \quad T_i = 4.3543\,\mathrm{s} \quad T_d = 1.0886\,\mathrm{s}$$

The above results have been obtained for

$$T_a = 13\,\mathrm{s} \quad \eta = 0.3 \quad \rho = -0.5 \quad k_R = 1$$

Co-ordinates of ultimate point A are $(-0.2149 - 0.011i)$ with $\gamma = 0.25$ Performance of the closed-loop system can be characterized by the following parameters

$$G_m = -303.88\,\text{dB}$$
$$P_m = 58.047\,\text{degree}$$
$$\omega_P = 1.0176\,\text{rad/s}$$

The third example belongs to the class of oscillatory systems with transfer function

$$G(s) = \frac{\omega_o^2}{(s+1)(s^2 + 2\zeta\omega_o s + \omega_o^2)}$$

for $\omega_o = 1 \quad \zeta = 0.1$. The obtained results for PID controller and PI filter are as follows

$$R(s) = \frac{1.311s^2 + 0.839s + 0.1342}{6.251s}$$

The above PID controller parameters have been obtained for

$$T_a = 30\ \mathrm{s} \quad \eta = 0.3 \quad \rho = -0.5 \quad \gamma = 0.25 \quad k_R = 0.1$$

Co-ordinates of ultimate point A are $(-2.3471 - 0.078i)$. Performance of the closed-loop system is given by following parameters

$$G_m = \text{inf dB}$$
$$P_m = 96.232\,\text{degree}$$
$$\omega_P = 0.02157\,\text{rad/s}$$

and ultimate parameters are $\omega_c = 0.0808\text{rad/s} \quad k_u k_R = 0.42566$.
The last example belongs to the class of unstable systems. The modified transfer function is

$$G(s) = \frac{1}{(s-1)^2}$$

The designed PID controller parameters are

$$k = 0.75 \quad T_i = 16.165\,\mathrm{s} \quad T_d = 4.04\,\mathrm{s}$$

The above results are obtained for

$$T_b = 4\,\mathrm{s} \quad \eta = 3 \quad \rho = 0 \quad \gamma = 0.25$$

Co-ordinates of ultimate point A are $(-0.16 + 0.7838i)$. The obtained gain margin and phase margin are

$$G_m = -9.5424\ \text{dB}$$
$$P_m = 68.339\,\text{degree}$$
$$\omega_P = 5.89\,\text{rad/s}$$

5 CONCLUSION

The ability of PI and PID controllers to control most of practical industrial processes has led to their wide acceptance in industrial applications. In this note modified design procedure of Aström and Hägglund' [3] generalized Ziegler-Nichols tuning rules are proposed. The proposed approach allows to stabilize such systems which are not good candidates for PID control.

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References

[1] ASTRÖM, K. J.—HÄGGLUND, T.: New Tuning Methods For PID Controllers, In: Proceedings of 3rd ECC, Roma, 1995, 2456–2462.

[2] ASTRÖM, K. J.—HÄGGLUND, T.: PID Controllers: Theory, Design and Tuning, 2nd Edition, Research Triangle Park, NC: Instrument Society of America.

[3] ASTRÖM, K. J.—HÄGGLUND, T.: Benchmark Systems For PID Control, In: IFAC Workshop PID'00, Terrasa 2000, 181–182.

[4] ASTRÖM, K. J.—PANAPOPOULOS, J. H.—HÄGLUND, T.: Design of PI Controllers Based on Non-Convex Optimization, Automatica **34** No. 5 (1998), 585–601.

[5] HALL, A. C.: The Analysis and Synthesis of Linear Servomechanisms, Technology Press, Cambridge, Mass, 1943.

[6] ISERMAN, R.: Digital Control Systems, Volume 1. Fundamentals Deterministic Control, 2nd Revised Edition, Springer-Verlag, 1989.

[7] KRISTIANSSON, B.—LENNARTSON, B.: Convenient Almost Optimal and Robust Tuning of PI and PID Controllers, In: IFAC 15th Triennial World Congress, Barcelona, 2002 CD.

[8] MORILLA, F.—DORMIDO, S.: Methodologies for the Tuning of PID Controllers in the Frequency Domain, In: IFAC Workshop PID'00 Terrasa, 2000, 155–160.

[9] O'DWYER, A.: A Summary of PI and PID Controller Tuning Rules for Processes with Time Delay, In: IFAC Workshop PID'00, Terrasa, 2000, Part 1: 175–180, Part 2: 242–247.

[10] VITEČKOVA, M.—VITEČEK, A.—SMUTNÝ, L.: Controller Tuning for Controlled Plants with Time Delay, In: IFAC Workshop PID'00, Terrasa, 2000, 283–288.

[11] VITEČKOVA, M.—VITEČEK, A.—SMUTNÝ, L.: Simple PI and PID Controllers Tuning for Monotone Self-Regulating Plants, In: IFAC Workshop PID'00, Terrasa, 2000, 289–294.

[12] ZIEGLER, J. G.—NICHOLS, N. B.: Optimum Settings for Automatic Controllers, Trans. ASME **64** (1942), 759–768.

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