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NEW APPROACH TO SYMMETRIZATION OF THREE–PHASE NETWORK

Daniel Mayer — Petr Kropík *

The presented method of synthesis of a symmetrization circuit makes it possible to characterize its physical structure with all particulars. A factor of nonsymmetry of a three-phase system of load currents was introduced. Computation of parameters of elements of the symmetrization circuit is then formulated as an optimization problem of determining such parameters with which the factor of nonsymmetry reaches its minimum. The described method also allows investigating surroundings of this minimum, which allows respecting the production tolerances of symmetrization elements. Numerical calculation was carried out by means of program system MATLAB 7.0.1 SP1, particularly of the Optimization Toolbox module.

K e y w o r d s: MATLAB, nonsymmetry of a three-phase system, objective function, optimization, symmetrization circuit

1 INTRODUCTION

Nonsymmetries in three-phase networks result in various undesirable phenomena such as a lower or higher voltage at the terminals of electrical appliances than its rated value, higher temperature rise of electromotors, oversaturation of transformers, interference effects *etc.* Nonsymmetry in a three-phase network can be eliminated or at least reduced by means of the Steinmetz circuit containing a symmetrization capacitor and symmetrization choke [1–4].

Further we will consider a limit case of the nonsymmetrical load, which is the one-phase load. Typical examples of high-power one-phase loads are electrothermal appliances, for example induction melting surfaces. A one-phase load may be connected to a three-phase network by means of the Steinmetz circuit in star or delta (see Fig. 1). As the one-phase load is purely resistive, we can easily determine (see, for instance, [3]) the parameters of the symmetrization elements. If the power factor of the load should be equal to one, for the star connection we obtain

$$L = \frac{R}{\sqrt{3}\omega}\,, \quad C = \frac{\sqrt{3}}{\omega R} \tag{1}$$

and for the delta connection

$$L = \sqrt{3}\frac{R}{\omega}\,, \quad C = \frac{1}{\sqrt{3}\omega R}\,. \tag{2}$$

During operation of some electrical appliances their output continuously varies and in such cases it is necessary to ensure their symmetrization in real time by means of contactless switching of symmetrization elements (see, for example, [5]).

Design of the Steinmetz symmetrization circuit with elements L and C calculated from (1) or (2) generally does not lead to reaching such currents that would form a symmetric three-phase system. The reason is that in the circuits in Fig. 1 we neglect certain effects. The first one is the resistance of the symmetrization choke and, moreover, the choke may be nonlinear. Often the load is not of purely resistive type and in some cases it may also be nonlinear. The considered circuit is strongly sensitive to the change of parameters so that the mentioned simplifications can lead to highly inaccurate results. In papers [6–8] we dealt with the solution of problems associated with symmetrization based on an improved model of the Steinmetz circuit. In this paper we will show that symmetrization parameters L and C may be determined as a solution of the optimization problem. At the same time we will investigate the behaviour of the symmetrization circuit in surroundings of the optimal values. This is important because the parameters of real symmetrization elements cannot be guaranteed quite accurately (they lie in certain tolerance limits).

2 SYNTHESIS OF THE STEINMETZ CIRCUIT WITH A ONE–PHASE LOAD AS AN OPTIMIZATION PROBLEM

Let the Steinmetz circuit (in star) be connected to a three-phase line (Fig. 2). Symbols R_2 and L_2 denote the parameters of the load, C_1 is the capacitance of the symmetrization capacitor, R_3 and L_3 are the parameters of the symmetrization choke. Let its reactance be a p-multiple of its resistance, *ie* ,

$$L_3 = R_3\frac{\omega}{p}\,. \tag{3}$$

2.1 Harmonic steady state

The case occurs when the voltage in the network is harmonic and the Steinmetz circuit is linear. Using the method of loop currents we find the phasors of currents in the line

$$\boldsymbol{I}_1 = \widetilde{\boldsymbol{I}}_1\,,\ \boldsymbol{I}_2 = -\widetilde{\boldsymbol{I}}_1 + \widetilde{\boldsymbol{I}}_2\,,\ \boldsymbol{I}_3 = -\widetilde{\boldsymbol{I}}_2 \tag{4}$$

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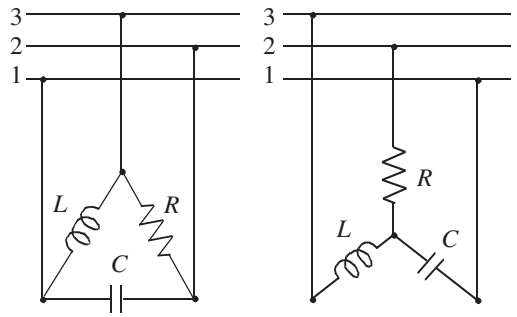

Fig. 1. The Steinmetz circuit – in delta and star connections.

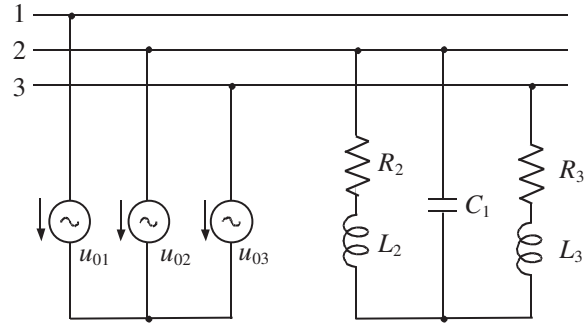

Fig. 2. A three-phase line with the Steinmetz circuit in star.

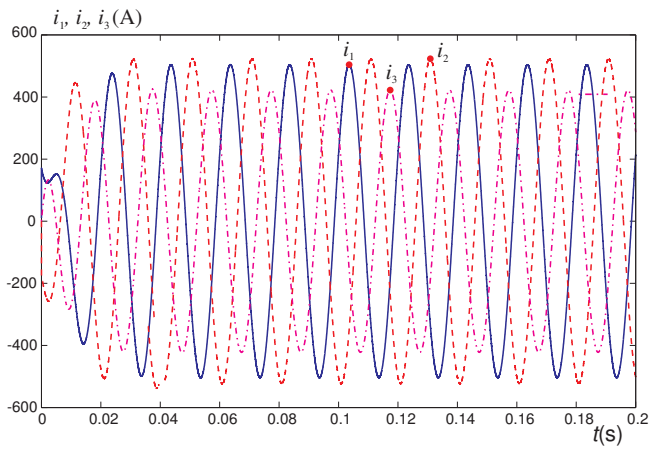

Fig. 3. Time dependences of currents i_1, i_2, i_3 before optimization.

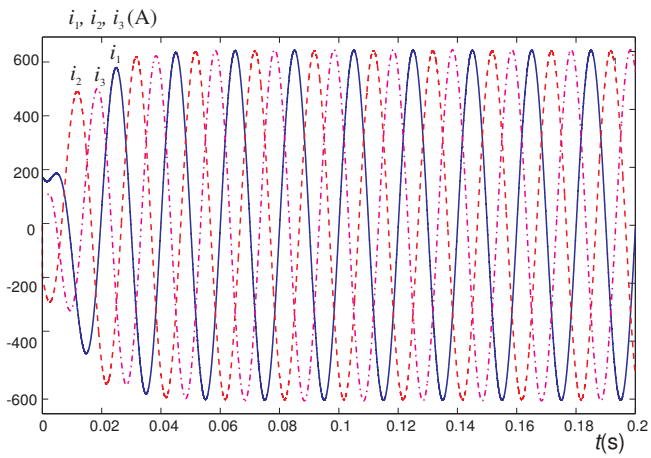

Fig. 4. Time dependences of currents i_1, i_2, i_3 after optimization.

where the currents in independent loops are determined from equations

$$\begin{bmatrix} R_2 + j\left(\omega L_2 - \frac{1}{\omega L}\right) & -R_2 - j\omega L_2 \\ -R_2 - j\omega L_2 & R_2 + R_3 + j\omega(L_2 + L_3) \end{bmatrix} \begin{bmatrix} \widetilde{\boldsymbol{I}}_1 \\ \widetilde{\boldsymbol{I}}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{U}_{01} - \boldsymbol{U}_{02} \\ \boldsymbol{U}_{02} - \boldsymbol{U}_{03} \end{bmatrix} \tag{5}$$

In order to evaluate the level of nonsymmetry we introduce an objective function (factor of nonsymmetry)

$$\rho(L_3, C_1) = \frac{\boldsymbol{I}_1 + \mathbf{a}^2\boldsymbol{I}_2 + \mathbf{a}\boldsymbol{I}_3}{\boldsymbol{I}_1 + \mathbf{a}\vec{I}_2 + \mathbf{a}^2\boldsymbol{I}_3} \tag{6}$$

where $\mathbf{a} = e^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$ and $\mathbf{a}^2 = e^{j\frac{4\pi}{3}} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$. The numerator in (6) contains a negative sequence of currents $(\boldsymbol{I}_1, \boldsymbol{I}_2, \boldsymbol{I}_3)$ while the denominator their positive sequence. For a symmetric system evidently $\rho = 0$. The optimization problem is now formulated as follows: we look for such values of parameters L_3 and C_1 for which function $\rho(L_3, C_1)$ reaches its minimum.

2.2 Periodical (nonharmonic) steady state

If the network contains higher harmonics of voltage or if the Steinmetz circuit contains nonlinear elements, it is necessary to start from the differential equations describing the circuit. For example, for a nonlinear load resistance $R_2 = R_2(i_2)$ in the circuit in Fig. 2, a system of equations holds

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} u_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 & -1/C_1 & -1/C_1 \\ 1/L_2 & -R_2(i_2)/L_2 & 0 \\ -1/L_3 & 0 & -R_3/L_3 \end{bmatrix} \begin{bmatrix} u_1 \\ i_2 \\ i_3 \end{bmatrix} + \begin{bmatrix} 0 \\ (u_{02} - u_{01})/L_2 \\ (u_{03} - u_{01})/L_3 \end{bmatrix} \tag{7}$$

and current $i_1(t)$ can be calculated from relation

$$i_1 = C_1 \frac{\mathrm{d}u_1}{\mathrm{d}t}\,. \tag{8}$$

As currents and voltages in the circuit are generally no longer harmonic, we cannot use their complex representation. So we cannot introduce the objective function by equation (6) and the concept "symmetric three-phase system" loses its sense. We will, therefore, look for such values of parameters L_3 and C_1 that the system of currents i_1, i_2, i_3 approaches to the symmetric system as much as possible. As far as the nonlinearities are not so strong, we can assume that they produce only small harmonics. In such a case the system of currents i_1, i_2, i_3 can be substituted by their basic harmonics and we can continue utilizing the objective function according to (6).

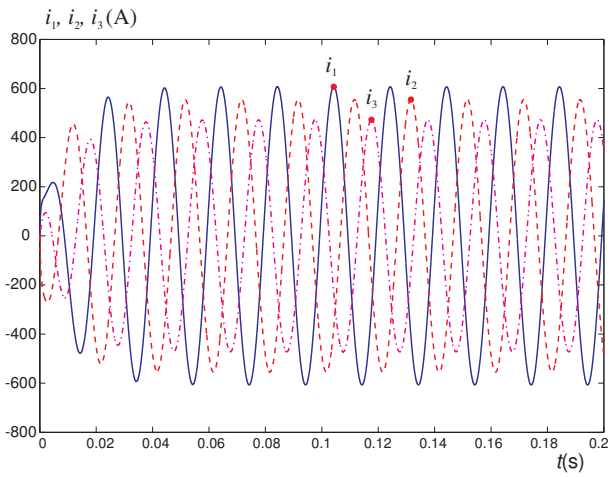

Fig. 5. Time dependences of currents i_1, i_2, i_3 before optimization.

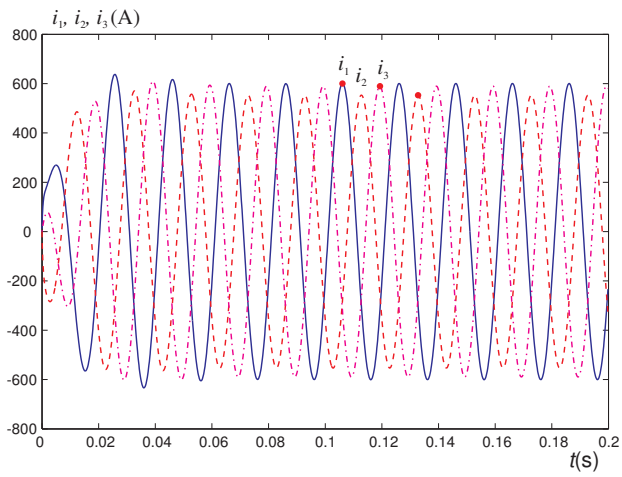

Fig. 6. Time dependencies of currents i_1, i_2, i_3 — first approximation.

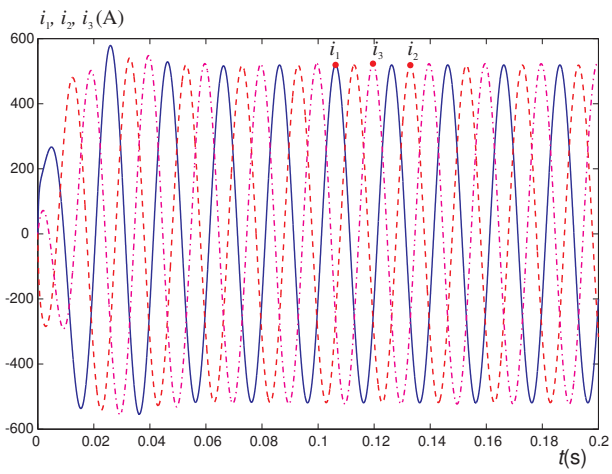

Fig. 7. Time dependencies of currents i_1, i_2, i_3 after optimization.

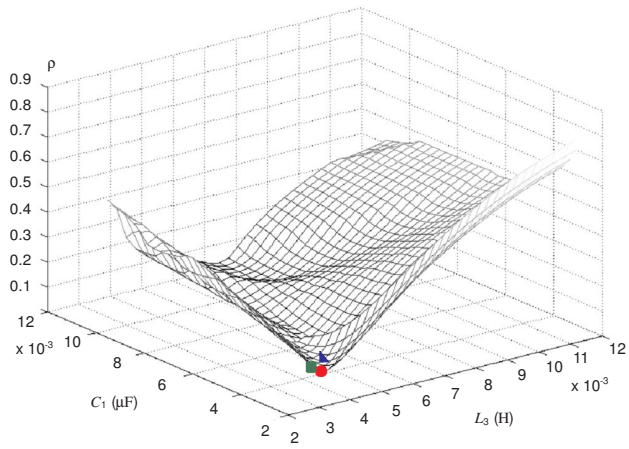

Fig. 8. Distribution of the criterial function near its minimum.

3 NUMERICAL EXAMPLES

Example 1. First we determine parameters L_3 and C_1 for the Steinmetz circuit in Fig. 2 where

$$u_{01} = U_{01}\sin(\omega t)\,, \qquad \omega = 2\pi f\,,$$
$$u_{02} = U_{02}\sin(\omega t - 120^\circ)\,, \qquad f = 50\,,\ R_2 = 2\Omega\,,$$
$$u_{03} = U_{03}\sin(\omega t + 120^\circ)\,, \qquad L_2 = 0\,,\ R_3 = 0\,.$$
$$U_{01} = U_{02} = U_{03} = 400\,\mathrm{V},$$

The starting values: $C_1 = 2.2 \times 10^{-3}$ F, $L_3 = 3.2 \times 10^{-3}$ H. Now we perform their optimization. The final values are $C_1 = 2.7838 \times 10^{-3}$ F, $L_3 = 3.688 \times 10^{-3}$ H. These values are in accordance with the results obtained from formulas (1), *ie* $C_1 = 2.7580 \times 10^{-3}$ F, $L_3 = 3.677 \times 10^{-3}$ H. The time dependences before and after optimization are depicted in Figs. 3 and 4.

Example 2. Now we determine parameters L_3 and C_1 for the Steinmetz circuit in Fig. 2 where all parameters are the same as in the previous example except for L_2 and R_3 for which $L_2 = 0.1R_2/\omega = 0.2/100\pi = 0.000637$ H and $R_3 = 0.1\omega L_3$.

The starting values are calculated from (1) (see the previous example) $C_1 = 2.758 \times 10^{-3}$ F, $L_3 = 3.677 \times 10^{-3}$ H. Current amplitudes are: $I_1 = 607.245$ A, $I_2 = 555.33$ A, $I_3 = 472.489$ A, phasors are $\boldsymbol{I}_1 = 590.619 + 141.122j$, $\boldsymbol{I}_2 = -266.594 - 486.815j$, $\boldsymbol{I}_3 = -322.865 + 344.969$. Criterial function value is $\rho = 0.1427$.

The first approximation values (see bellow): $C_1 = 3.95\times10^{-3}$ F, $L_3 = 4.2\times10^{-3}$ H. Current amplitudes are: $I_1 = 600.622$ A, $I_2 = 553.331$ A, $I_3 = 589.762$ A, phasors are $\boldsymbol{I}_1 = 560.255 - 216.475j$, $\boldsymbol{I}_2 = -434.368 - 342.782j$, $\boldsymbol{I}_3 = -111.464 + 579.133j$. Criterial function value is $\rho = 0.0373$.

After optimisation, $C_1 = 4.1009 \times 10^{-3}$ F, $I_3 = 4.5776\times10^{-3}$ H. Current amplitudes are: $I_1 = 519.200$ A, $I_2 = 518.612$ A, $I_3 = 523.005$ A, phasors are $\boldsymbol{I}_1 = 479.365 - 199.441j$, $\boldsymbol{I}_2 = -414.689 - 311.434j$, $\boldsymbol{I}_3 = -64.5427 + 519.007j$. Criterial function value is $\rho = 0.0001$.

A simple method was used to calculate the first approximation. Selected intervals of L_3 and C_1 were split, using a predefined step (for example 1/60 of each interval). The value of criterial function was obtained in each step and the minimal value was selected.

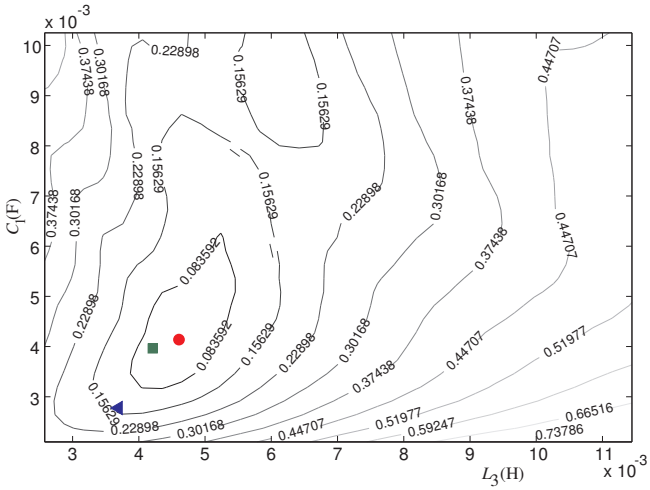

Fig. 9. Surroundings of the minimum of the criterial function.

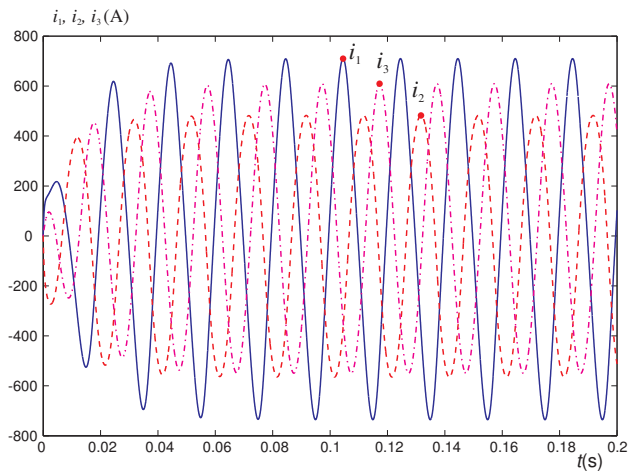

Fig. 10. Time dependencies of currents i_1, i_2, i_3 before optimization.

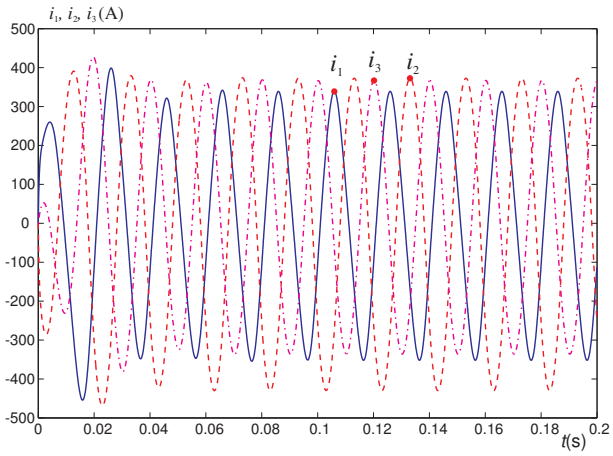

Fig. 11. Time dependences of currents i_1, i_2, i_3 — first approximation.

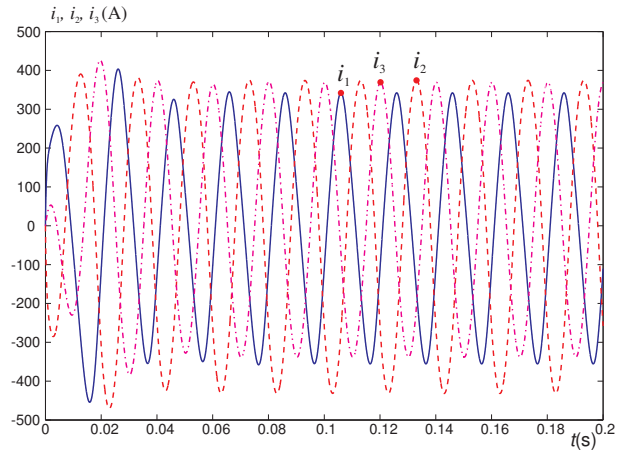

Fig. 12. Time dependences of currents i_1, i_2, i_3 after optimization.

The time dependences of currents before optimization are depicted in Fig. 5 and the first approximation in Fig. 6.

The time dependences of currents after optimization and the corresponding criterial function are depicted in Figs. 7, 8 and 9.

Example 3. In this example all parameters are the same as in the previous example, except for R_2 and L_2 for which $R_2 = 2.0 + 5 \times 10^{-4} i_2 + 1.5 \times 10^{-6} i_2^2\,\Omega$, $L_2 = 0.1 R_2/\omega$ H, *ie* R_2 and L_2 are non-linear.

The starting values are calculated from (1): $C_1 = 2.758 \times 10^{-3}$ F, $L_3 = 3.677 \times 10^{-3}$ H. Current amplitudes are $I_1 = 722.835$ A, $I_2 = 523.644$ A, $I_3 = 579.64$ A, phasors are $\boldsymbol{I}_1 = 722.203 - 30.214j$, $\boldsymbol{I}_2 = -418.192 - 315.149j$, $\boldsymbol{I}_3 = -605.526 + 518.071j$. Criterial function value is $\rho = 0.3021$.

The first approximation values (see previous example for used method): $C_1 = 4.40 \times 10^{-3}$ F, $L_3 = 6.27 \times 10^{-3}$ H. Current amplitudes are $I_1 = 345.533$ A, $I_2 = 401.318$ A, $I_3 = 351.486$ A, phasors are $\boldsymbol{I}_1 = 302.691 - 166.689j$, $\boldsymbol{I}_2 = -390.982 - 90.494j$, $\boldsymbol{I}_3 = 24.6684 + 471.591j$. Criterial function value is $\rho = 0.0024$.

After optimization $C_1 = 4.3371 \times 10^{-3}$ F, $L_3 = 6.2279 \times 10^{-3}$ H. Current amplitudes are $I_1 = 349.049$ A, $I_2 = 403.108$ A, $I_3 = 353.853$ A, phasors are $\boldsymbol{I}_1 = 304.286 - 171.013j$, $\boldsymbol{I}_2 = -393.412 - 87.883j$, $\boldsymbol{I}_3 = 27.430 + 474.776j$. Criterial function value is $\rho = 5.8297 \times 10^{-15}$.

The time dependences of currents before optimization are depicted in Fig. 10 and the first approximation in Fig. 11.

The time dependences of currents after optimization and the corresponding criterial function are depicted in Figs. 12, 13 and 14.

4 REMARKS TO THE OPTIMIZATION PROCEDURE

All presented computations were implemented using the programming language of system MATLAB (version 7.0.1 SP1) and its toolboxes Optimization Toolbox and Symbolic Math Toolbox. First we solve numerically the

system of differential equations (7). For solving, functions from the family of functions ODExx were used, for example

```
[t,res_soust] =
ode45(@proudy_soust,[t_0,t_final],[u1_0,i2_0,i3_0],
options,U01,U02,U03,R2,L2,L3,R3,C1,omg);
```

The problem is sensitive to accuracy of computations of quantities u_1 (or i_1), i_2 and i_3. The criterial function in the neighbourhood of the minimum is flat. That is why we used different variants of the starting set of functions ODE, particularly setting of the maximum step of computation, *ie*

```
options =
odeset('MaxStep',1e-6,'InitialStep',1e-6);
```

For the same reason we used different variants of the ODE functions, particularly

```
[t,res_soust] =
ode15s(@proudy_soust,[t_0,t_final],[u1_0,i2_0,i3_0],
options,U01,U02,U03,R2,L2,L3,R3,C1,omg);
[t,res_soust] =
ode23t(@proudy_soust,[t_0,t_final],[u1_0,i2_0,i3_0],
options,U01,U02,U03,R2,L2,L3,R3,C1,omg);
```

The knowledge of voltage u_1 is then used for computation of i_1 (8) by means of function `diff`

```
i1 = C1 .* (diff(res_soust(:,1)) ./ diff(t));
```

Now we calculate the amplitudes of particular currents, for example

```
I1 = abs((max(i1)-min(i1))./2);
```

Then it is necessary to find the maxima of currents important for computation of phase shifts $\varphi_1, \varphi_2, \varphi_3$. The accuracy of this computation is decisive for the result of the whole optimization because the problem exhibits high sensitivity in this respect. So we must determine a suitable step for searching these maxima.

Now we can calculate $\varphi_1, \varphi_2, \varphi_3$

```
fi1 = (asin(i1(poloha_max_i1)./I1)-omg.*t_pro_fi1);
fi2 = (asin(i2(poloha_max_i2)./I2)-omg.*t_pro_fi2);
fi3 = (asin(i3(poloha_max_i3)./I3)-omg.*t_pro_fi3);
```

just after the phasors of particular currents

```
faz_I1 = I1.*exp(j.*fi1);
faz_I2 = I2.*exp(j.*fi2);
faz_I3 = I3.*exp(j.*fi3);
```

quantities

```
a = (-1/2)+j*(sqrt(3)/2);
a2 = (-1/2)-j*(sqrt(3)/2);
```

and finally the objective function using (6)

```
ro=abs((faz_I1+a2.*faz_I2+a.*faz_I3)./(faz_I1+a.*faz_I2
+a2.*faz_I3));
```

Optimization of the problem is carried out by means of the standard functions from the Optimization Toolbox of MATLAB. It is realized in the following form

```
options = optimset('fminsearch');
options.TolFun=1e-15;
options.TolX=1e-15;
options.MaxFunEvals=1000;
[min,fval,exitflag,output] =
fminsearch(@objective_f,input,options);
```

Variable options represents a set of initialization parameters used for the optimization process.

Function `fminsearch` uses an algorithm based on the Nelder-Mead nonlinear simplex method. This algorithm requires neither numerical nor analytical computation of the gradient (unlike function `fminunc` — see below). If is dimension of the variable, the simplex in the n-dimensional space is described by $n+1$ vectors that form its vertices. In 2D the simplex is represented by a triangle, while in 3D it is a prism. In every iteration a new point is found inside or in the neighbourhood of the simplex. Consequently we compare the value of the objective function at the newly found point with its values at the vertices of the simplex and one of the vertices of the simplex is substituted by the new point. So we obtain a new simplex. This step is repeated until the diameter of the simplex is smaller than the required accuracy of calculation. For highly discontinuous problems the function `fminsearch` is more suitable than function `fminunc`.

The function `fminunc` is implemented as a nonlinear optimization of function of more variables without constraint. It is used in the following form

```
options = optimset('fminunc');
options.TolFun=1e-15;
options.TolX=1e-15;
options.MaxFunEvals=1200;
options.GradObj='on';
[min,fval,exitflg,output,grad,hessian] =
fminunc(@objective_f,input,options);
```

The variable `options` represents a set of initialization parameters, similarly as in the case of function `fminsearch`. Most parameters have an analogous function. We used one specific parameter `GradObj` set on value `'on'`.

If `GradObj` is set on `'on'`, function for computation of the gradient at a given point is required at each step.

Function is based on the BFGS (Broyden, Fletcher, Goldfarb, Shanno) quasi-Newton method with combined quadratic and cubic searching (for problems of medium extent). This function uses the BFGS method for actualization of the approximation of the Hessian. The method (Davidson, Fletcher, Powell) for approximating the inverse Hessian may be used for setting the parameter `HessUpdate` on value `'dfp'` (and parameter `LargeScale` on value `'off'`).

In the cases requiring elimination of unsuitable values of variables (for instance a negative value of capacitance)

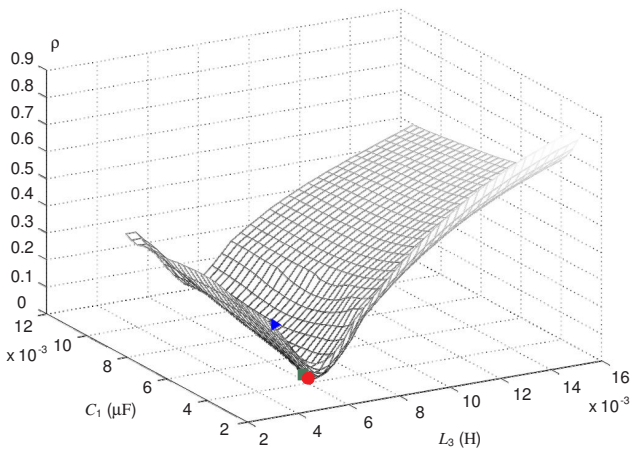

Fig. 13. Distribution of the criterial function near its minimum.

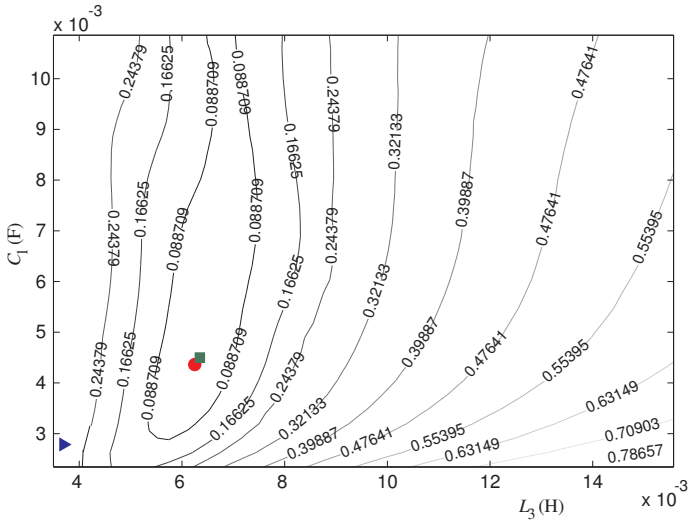

Fig. 14. Surroundings of the minimum of the criterial function.

we can use the function `fmincon`. This function is implemented as a nonlinear optimization of a function of more variables with constraints. The constraints are given as a system of nonequations or a matrix of coefficients of the system and vector of right sides.

5 CONCLUSION

The aim of this work was to show that during the synthesis of a symmetrizing circuit it is necessary to consider also the inductance component of the load, resistance component of the symmetrizing coil, or non-linear character of the load. If we neglect these "secondary" influences, we can make a serious mistake that lead to a symmetrizing circuit of very bad quality.

Acknowledgements

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