NEW APPROACH TO SYMMETRIZATION OF THREE–PHASE NETWORK

Daniel Mayer — Petr Kropík *

The presented method of synthesis of a symmetrization circuit makes it possible to characterize its physical structure with all particulars. A factor of nonsymmetry of a three-phase system of load currents was introduced. Computation of parameters of elements of the symmetrization circuit is then formulated as an optimization problem of determining such parameters with which the factor of nonsymmetry reaches its minimum. The described method also allows investigating surroundings of this minimum, which allows respecting the production tolerances of symmetrization elements. Numerical calculation was carried out by means of program system MATLAB 7.0.1 SP1, particularly of the Optimization Toolbox module.

K e y w o r d s: MATLAB, nonsymmetry of a three-phase system, objective function, optimization, symmetrization circuit

1 INTRODUCTION

Nonsymmetries in three-phase networks result in various undesirable phenomena such as a lower or higher voltage at the terminals of electrical appliances than its rated value, higher temperature rise of electromotors, oversaturation of transformers, interference effects etc. Nonsymmetry in a three-phase network can be eliminated or at least reduced by means of the Steinmetz circuit containing a symmetrization capacitor and symmetrization choke [1–4].

Further we will consider a limit case of the nonsymmetrical load, which is the one-phase load. Typical examples of high-power one-phase loads are electrothermal appliances, for example induction melting surfaces. A one-phase load may be connected to a three-phase network by means of the Steinmetz circuit in star or delta (see Fig. 1). As the one-phase load is purely resistive, we can easily determine (see, for instance, [3]) the parameters of the symmetrization elements. If the power factor of the load should be equal to one, for the star connection we obtain

\[ L = \frac{R}{\sqrt{3}\omega}, \quad C = \frac{1}{\sqrt{3}\omega R} \tag{1} \]

and for the delta connection

\[ L = \frac{\sqrt{3}R}{\omega}, \quad C = \frac{1}{\sqrt{3}\omega R} \tag{2} \]

During operation of some electrical appliances their output continuously varies and in such cases it is necessary to ensure their symmetrization in real time by means of contactless switching of symmetrization elements (see, for example, [5]).

Design of the Steinmetz symmetrization circuit with elements \( L \) and \( C \) calculated from (1) or (2) generally does not lead to reaching such currents that would form a symmetric three-phase system. The reason is that in the circuits in Fig. 1 we neglect certain effects. The first one is the resistance of the symmetrization choke and, moreover, the choke may be nonlinear. Often the load is not purely resistive type and in some cases it may also be nonlinear. The considered circuit is strongly sensitive to the change of parameters so that the mentioned simplifications can lead to highly inaccurate results. In papers [6–8] we dealt with the solution of problems associated with symmetrization based on an improved model of the Steinmetz circuit. In this paper we will show that symmetrization parameters \( L \) and \( C \) may be determined as a solution of the optimization problem. At the same time we will investigate the behaviour of the symmetrization circuit in surroundings of the optimal values. This is important because the parameters of real symmetrization elements cannot be guaranteed quite accurately (they lie in certain tolerance limits).

2 SYNTHESIS OF THE STEINMETZ CIRCUIT WITH A ONE–PHASE LOAD AS AN OPTIMIZATION PROBLEM

Let the Steinmetz circuit (in star) be connected to a three-phase line (Fig. 2). Symbols \( R_2 \) and \( L_2 \) denote the parameters of the load, \( C_1 \) is the capacitance of the symmetrization capacitor, \( R_3 \) and \( L_3 \) are the parameters of the symmetrization choke. Let its reactance be a \( p \)-multiple of its resistance, \( i.e., \):

\[ L_3 = R_3 \frac{\omega}{p}. \tag{3} \]

2.1 Harmonic steady state

The case occurs when the voltage in the network is harmonic and the Steinmetz circuit is linear. Using the method of loop currents we find the phasors of currents in the line

\[ I_1 = \bar{I}_1, \quad I_2 = -\bar{I}_1 + \bar{I}_2, \quad I_3 = -\bar{I}_2 \tag{4} \]
where the currents in independent loops are determined from equations

\[
\begin{bmatrix}
R_2 + j(\omega L_2 - \frac{1}{\pi T}) & -R_2 - j\omega L_2 & \frac{I_1}{L_2} \\
-R_2 - j\omega L_2 & R_2 + j(\omega (L_2 + L_3))& \frac{I_0}{L_2} \\
\end{bmatrix}
= \begin{bmatrix}
U_{01} - U_{02} \\
U_{02} - U_{03}
\end{bmatrix}
\]

(5)

In order to evaluate the level of nonsymmetry we introduce an objective function (factor of nonsymmetry)

\[
\rho(L_3, C_1) = \frac{I_1 + a^2 I_2 + a I_3}{I_1 + a I_2 + a^2 I_3}
\]

(6)

where \(a = e^{j\frac{2\pi}{3}} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}\) and \(a^2 = e^{j\frac{4\pi}{3}} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}\).

The numerator in (6) contains a negative sequence of currents \((I_1, I_2, I_3)\) while the denominator their positive sequence. For a symmetric system evidently \(\rho = 0\). The optimization problem is now formulated as follows: we look for such values of parameters \(L_3\) and \(C_1\) for which function \(\rho(L_3, C_1)\) reaches its minimum.

### 2.2 Periodical (nonharmonic) steady state

If the network contains higher harmonics of voltage or if the Steinmetz circuit contains nonlinear elements, it is necessary to start from the differential equations describing the circuit. For example, for a nonlinear load resistance \(R_2 = R_2(i_2)\) in the circuit in Fig. 2, a system of equations holds

\[
\frac{d}{dt} \begin{bmatrix} u_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 & -1/C_1 & -1/C_1 \\ 1/L_2 & -R_2(i_2)/L_2 & 0 \\ -1/L_3 & 0 & -R_3/L_3 \end{bmatrix} \begin{bmatrix} u_1 \\ i_2 \\ i_3 \end{bmatrix} + \begin{bmatrix} 0 \\ (u_{02} - u_{01})/L_2 \\ (u_{03} - u_{01})/L_3 \end{bmatrix}
\]

(7)

and current \(i_1(t)\) can be calculated from relation

\[
i_1 = C_1 \frac{du_1}{dt}.
\]

(8)

As currents and voltages in the circuit are generally no longer harmonic, we cannot use their complex representation. So we cannot introduce the objective function by equation (6) and the concept “symmetric three-phase system” loses its sense. We will, therefore, look for such values of parameters \(L_3\) and \(C_1\) that the system of currents \(i_1, i_2, i_3\) approaches to the symmetric system as much as possible. As far as the nonlinearities are not so strong, we can assume that they produce only small harmonics. In such a case the system of currents \(i_1, i_2, i_3\) can be substituted by their basic harmonics and we can continue utilizing the objective function according to (6).
Example 1. First we determine parameters $L_3$ and $C_1$ for the Steinmetz circuit in Fig. 2 where

\[
\begin{align*}
    u_{01} &= U_{01} \sin(\omega t), \\
    u_{02} &= U_{02} \sin(\omega t - 120^\circ), \\
    u_{03} &= U_{03} \sin(\omega t + 120^\circ), \\
    U_0 &= U_{02} = U_{03} = 400 \text{ V},
\end{align*}
\]

The starting values: $C_1 = 2.2 \times 10^{-3} \text{ F}, L_3 = 3.2 \times 10^{-3} \text{ H}$. Now we perform their optimization. The final values are $C_1 = 2.7583 \times 10^{-3} \text{ F}, L_3 = 3.688 \times 10^{-3} \text{ H}$. These values are in accordance with the results obtained from formulas (1), i.e. $C_1 = 2.7580 \times 10^{-3} \text{ F}, L_3 = 3.677 \times 10^{-3} \text{ H}$. The time dependences before and after optimization are depicted in Figs. 3 and 4.

Example 2. Now we determine parameters $L_3$ and $C_1$ for the Steinmetz circuit in Fig. 2 where all parameters are the same as in the previous example except for $L_2$ and $R_3$ for which $L_2 = 0.1R_2/\omega = 0.2/100\pi = 0.000637 \text{ H}$ and $R_3 = 0.1\omega L_3$.

The starting values are calculated from (1) (see the previous example) $C_1 = 2.758 \times 10^{-3} \text{ F}, L_3 = 3.677 \times 10^{-3} \text{ H}$. Current amplitudes are: $I_1 = 607.245 \text{ A}, I_2 = 555.33 \text{ A}, I_3 = 472.489 \text{ A}$, phasors are $\mathbf{I}_1 = 590.619 + 141.122j, \mathbf{I}_2 = -266.594 - 486.815j, \mathbf{I}_3 = -322.865 + 344.969$. Criterial function value is $\rho = 0.1427$.

The first approximation values (see bellow): $C_1 = 3.95 \times 10^{-3} \text{ F}, L_3 = 4.2 \times 10^{-3} \text{ H}$. Current amplitudes are: $I_1 = 600.622 \text{ A}, I_2 = 553.331 \text{ A}, I_3 = 589.762 \text{ A}$, phasors are $\mathbf{I}_1 = 560.255 - 216.475j, \mathbf{I}_2 = -434.368 - 342.782j, \mathbf{I}_3 = -111.464 + 579.133j$. Criterial function value is $\rho = 0.0373$.

After optimisation, $C_1 = 4.1009 \times 10^{-3} \text{ F}, I_3 = 4.5776 \times 10^{-3} \text{ H}$. Current amplitudes are: $I_1 = 519.200 \text{ A}, I_2 = 518.612 \text{ A}, I_3 = 523.005 \text{ A}$, phasors are $\mathbf{I}_1 = 479.365 - 199.441j, \mathbf{I}_2 = -414.689 - 311.434j, \mathbf{I}_3 = -64.5427 + 519.007j$. Criterial function value is $\rho = 0.0001$.

A simple method was used to calculate the first approximation. Selected intervals of $L_3$ and $C_1$ were split, using a predefined step (for example 1/60 of each interval). The value of criterial function was obtained in each step and the minimal value was selected.
The time dependences of currents before optimization are depicted in Fig. 5 and the first approximation in Fig. 6.

The time dependences of currents after optimization and the corresponding criterial function are depicted in Figs. 7, 8 and 9.

**Example 3.** In this example all parameters are the same as in the previous example, except for \( R_2 \) and \( L_2 \) for which \( R_2 = 2.0 \times 10^{-4} \) \( \Omega \), \( L_2 = 0.1 R_2 / \omega \) \( \Omega \), i.e. \( R_2 \) and \( L_2 \) are non-linear.

The starting values are calculated from (1): \( C_1 \) = 2.758 \( \times 10^{-3} \) \( \text{F} \), \( L_3 \) = 3.677 \( \times 10^{-3} \) \( \text{H} \). Current amplitudes are \( I_1 \) = 722.835 \( \text{A} \), \( I_2 \) = 523.644 \( \text{A} \), \( I_3 \) = 579.64 \( \text{A} \), phasors are \( I_1 \) = 722.203 – 30.214 \( \text{j} \), \( I_2 \) = -418.192 – 315.149 \( \text{j} \), \( I_3 \) = -605.526 + 518.071 \( \text{j} \). Criterial function value is \( \rho = 0.3021 \).

The first approximation values (see previous example for used method): \( C_1 \) = 4.40 \( \times 10^{-3} \) \( \text{F} \), \( L_3 \) = 6.27 \( \times 10^{-3} \) \( \text{H} \). Current amplitudes are \( I_1 \) = 345.533 \( \text{A} \), \( I_2 \) = 401.318 \( \text{A} \), \( I_3 \) = 351.486 \( \text{A} \), phasors are \( I_1 \) = 302.691 – 166.689 \( \text{j} \), \( I_2 \) = -390.982 – 90.494 \( \text{j} \), \( I_3 \) = 24.6684 + 471.591 \( \text{j} \). Criterial function value is \( \rho = 0.0024 \).

After optimization \( C_1 \) = 4.3371 \( \times 10^{-3} \) \( \text{F} \), \( L_3 \) = 6.2279 \( \times 10^{-3} \) \( \text{H} \). Current amplitudes are \( I_1 \) = 349.049 \( \text{A} \), \( I_2 \) = 403.108 \( \text{A} \), \( I_3 \) = 353.853 \( \text{A} \), phasors are \( I_1 \) = 304.286 – 171.013 \( \text{j} \), \( I_2 \) = -393.412 – 87.883 \( \text{j} \), \( I_3 \) = 27.430 + 474.776 \( \text{j} \). Criterial function value is \( \rho = 5.8297 \times 10^{-15} \).

The time dependences of currents before optimization are depicted in Fig. 10 and the first approximation in Fig. 11.

The time dependences of currents after optimization and the corresponding criterial function are depicted in Figs. 12, 13 and 14.

4 Remarks to the Optimization Procedure

All presented computations were implemented using the programming language of system MATLAB (version 7.0.1 SP1) and its toolboxes Optimization Toolbox and Symbolic Math Toolbox. First we solve numerically the
System of differential equations (7). For solving, functions from the family of functions ODEpole were used, for example

\[
[t, res, sout] = \\
\text{ode45}(@proudy, sout, [t_0, t_final], [u_1, u_2, u_3], \\
\text{options}, C_1, C_2, C_3, \omega_0, \text{omg})
\]

The problem is sensitive to accuracy of computations of quantities \( u_1 \) (or \( i_1 \)), \( i_2 \) and \( i_3 \). The critical function in the neighbourhood of the minimum is flat. That is why we used different variants of the starting set of functions ODE, particularly setting of the maximum step of computation, \( \text{ie} \)

\[
\text{options} = \\
\text{oderset}('\text{MaxStep}', 1e-6, '\text{InitialStep}', 1e-6);
\]

For the same reason we used different variants of the ODE functions, particularly

\[
[t, res, sout] = \\
\text{ode15s}(@proudy, sout, [t_0, t_final], [u_1, u_2, u_3], \\
\text{options}, C_1, C_2, C_3, \omega_0, \text{omg})
\]

\[
[t, res, sout] = \\
\text{ode23t}(@proudy, sout, [t_0, t_final], [u_1, u_2, u_3], \\
\text{options}, C_1, C_2, C_3, \omega_0, \text{omg})
\]

The knowledge of voltage \( u_1 \) is then used for computation of \( i_1 \) (8) by means of function \( \text{diff} \)

\[
i_1 = C_1 \cdot (\text{diff}(\text{soust}(i_1))/\text{diff}(t));
\]

Now we calculate the amplitudes of particular currents, for example

\[
i_1 = \text{abs}((\text{max}(i_1)-\text{min}(i_1))/2);
\]

Then it is necessary to find the maxima of currents important for computation of phase shifts \( \varphi_1, \varphi_2, \varphi_3 \). The accuracy of this computation is decisive for the result of the whole optimization because the problem exhibits high sensitivity in this respect. So we must determine a suitable step for searching these maxima.

Now we can calculate \( \varphi_1, \varphi_2, \varphi_3 \)

\[
\varphi_1 = (\text{asin}(i_1(\text{poloha}_1)./i_1)-\text{omg} \cdot t \cdot \text{pro} \cdot \text{fi}_1);
\]
\[
\varphi_2 = (\text{asin}(i_2(\text{poloha}_2)./i_2)-\text{omg} \cdot t \cdot \text{pro} \cdot \text{fi}_2);
\]
\[
\varphi_3 = (\text{asin}(i_3(\text{poloha}_3)./i_3)-\text{omg} \cdot t \cdot \text{pro} \cdot \text{fi}_3);
\]

just after the phasors of particular currents

\[
faz_1 = i_1 \cdot \exp(j \cdot fi_1);
\]
\[
faz_2 = i_2 \cdot \exp(j \cdot fi_2);
\]
\[
faz_3 = i_3 \cdot \exp(j \cdot fi_3);
\]

quantities

\[
a_1 = (-1/2)j \cdot (\sqrt{3}/2);
\]
\[
a_2 = (-1/2)j \cdot (\sqrt{3}/2);
\]

and finally the objective function using (6)

\[
\text{ro} = \text{abs}((\text{faz}_1 \ast a_2 + \text{faz}_2 \ast a_1 \ast \text{faz}_3)./(\text{faz}_1 \ast a_1 \ast \text{faz}_2 \\
\ast a_2 \ast \text{faz}_3))
\]

Optimization of the problem is carried out by means of the standard functions from the Optimization Toolbox of MATLAB. It is realized in the following form

\[
\text{options} = \text{optimset}('\text{fminsearch}');
\]
\[
\text{options.TolFun}=1e-15;
\]
\[
\text{options.TolX}=1e-15;
\]
\[
\text{options.MaxFunEvals}=1000;
\]
\[
\text{options.MaxIter}=1000;
\]
\[
\text{options.GradObj}='\text{on}';
\]
\[
\text{[min,fval,exitflag,output,grad,hessian]} = \text{fminsearch}(\text{Objective_f,input,}\text{options});
\]

Variable options represents a set of initialization parameters used for the optimization process.

Function \( \text{fminsearch} \) uses an algorithm based on the Nelder-Mead nonlinear simplex method. This algorithm requires neither numerical nor analytical computation of the gradient (unlike function \( \text{fminunc} \) — see below). If is dimension of the variable, the simplex in the \( n \)-dimensional space is described by \( n + 1 \) vectors that form its vertices. In 2D the simplex is represented by a triangle, while in 3D it is a prism. In every iteration a new point is found inside or in the neighbourhood of the simplex. Consequently we compare the value of the objective function at the newly found point with its values at the vertices of the simplex and one of the vertices of the simplex is substituted by the new point. So we obtain a new simplex. This step is repeated until the diameter of the simplex is smaller than the required accuracy of calculation. For highly discontinuous problems the function \( \text{fminsearch} \) is more suitable than function \( \text{fminunc} \).

The function \( \text{fminunc} \) is implemented as a nonlinear optimization of function of more variables without constraint. It is used in the following form

\[
\text{options} = \text{optimset}('\text{fminunc}');
\]
\[
\text{options.TolFun}=1e-15;
\]
\[
\text{options.TolX}=1e-15;
\]
\[
\text{options.MaxFunEvals}=1200;
\]
\[
\text{options.GradObj}='\text{on}';
\]
\[
\text{[min,fval,exitflag,output,grad,hessian]} = \text{fminunc}(\text{Objective_f,input,}\text{options});
\]

The variable options represents a set of initialization parameters, similarly as in the case of function \( \text{fminsearch} \). Most parameters have an analogous function. We used one specific parameter \( \text{GradObj} \) set on value ‘on’.

If \( \text{GradObj} \) is set on ‘on’, function for computation of the gradient at a given point is required at each step.

Function is based on the BFGS (Broyden, Fletcher, Goldfarb, Shanno) quasi-Newton method with combined quadratic and cubic searching (for problems of medium extent). This function uses the BFGS method for actualization of the approximation of the Hessian. The method (Davidson, Fletcher, Powell) for approximating the inverse Hessian may be used for setting the parameter \( \text{HessUpdate} \) on value ‘dfp’ (and parameter \( \text{LargeScale} \) on value ‘off’).

In the cases requiring elimination of unsuitable values of variables (for instance a negative value of capacitance)
we can use the function \texttt{fmincon}. This function is implemented as a nonlinear optimization of a function of more variables with constraints. The constraints are given as a system of nonequations or a matrix of coefficients of the system and vector of right sides.

5 CONCLUSION

The aim of this work was to show that during the synthesis of a symmetrizing circuit it is necessary to consider also the inductance component of the load, resistance component of the symmetrizing coil, or non-linear character of the load. If we neglect these “secondary” influences, we can make a serious mistake that lead to a symmetrizing circuit of very bad quality.

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Daniel Mayer, Prof, Ing, DrSc (1930). Received the Ing, PhD and DrSc degrees in electrical engineering from the Technical University in Prague in 1952, 1958 and 1979, respectively. In 1959 Associate Professor at the University of West Bohemia in Pilsen, in 1968 Full Professor of the Theory of Electrical Engineering. Many years head of the Department of the Theory of Electrical Engineering. Research interests: circuit theory, electromagnetic field theory, electrical machines and apparatus, history of electrical engineering. He published 6 books, more than 220 scientific papers and 10 patents. He is a Fellow of the IEE, member ICS, ISTET and UICEE, member of editorial boards of several international journals and leader of many grant projects (mayer@kte.zcu.cz).

Petr Kropík, Ing (1971). He graduated from the Faculty of Applied Sciences (University of West Bohemia Pilsen) in 1995 in branch computer science. He was employed in SPT Telecom a. s. (in branch of computer networks, clientserver database applications) and after three years he is an s Assistant at the Department of the Theory of Electrical Engineering, Faculty of Electrical Engineering (University of West Bohemia, Pilsen). He teaches some subjects from computer science. His professional interest: optimisation methods, UNIX/LINUX operating systems, SQL databases and computer networks. Since 2001 he has been an external student for PhD (pkropik@kte.zcu.cz).