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COMMUNICATIONS

COMPLEX PERMITTIVITY MEASUREMENTS USING SCALAR NETWORK ANALYSER

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A novel experimental technique is presented for the X band measurements of the complex permittivity components. Scalar Automatic Network Analyser (SANA) is used for a broadband waveguide measurement of the reflection coefficient, and the experimental data are processed by a simple analysis. The application of the technique for low and/or high permittivity materials with different losses is discussed.

K e y w o r d s: complex permittivity, dielectric constant, loss factor, microwave measurements, permittivity measurements, reflection method

1 INTRODUCTION

Microwave measurements of the complex permittivity have been a research issue for many decades. The review of existing techniques can be found in [1-3]. The older and widely used techniques were based on measurement of the standing wave ratio. To ensure desirable precision the sample must have a well-defined geometry and preferably should fill the whole waveguide cross section. The analysis of data leads to a solution of transcendental equation $tgh(x)/x - y = 0$ resulting in a discrete set of complex roots from which only one can be appropriate. The uncertainty in determination of the correct root (x) could be removed by measuring at least two, or more, samples of different thickness. The measurement and its evaluation, in such a case more complex, are somewhat simplified if the thickness of the second sample is chosen to be twice the thickness of the first one, [4]. A review of newer methods of the permittivity measurements is in [3]. Two types of measurements can be distinguished: (i) off-resonance two port scattering measurement of the waveguide or coaxial line, (ii) off-resonance single-port measurement of short-circuited transmission line. The off-resonance means that the sample length is not a multiple of the line wavelength so the resonance effects does not occur.

Our approach is based on "multi-point" or "correlated-point technique", [4] and the permittivity value is obtained from the least square fit of the measured and theoretically expected data in the vicinity of resonance, detected as a measured minimum of the reflection coefficient magnitude.

2 MEASUREMENT

In this paper we describe a scalar network analyser measurements of the permittivity. The dielectric sample

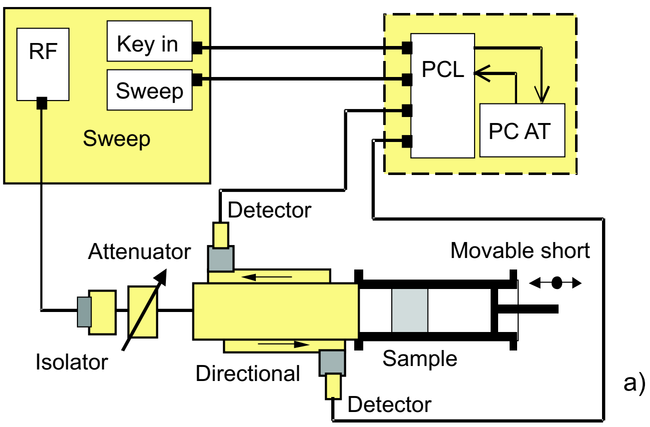

Fig. 1a. The experimental set-up

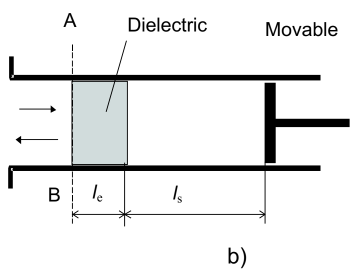

Fig. 1b. Detail of the sample and short positioning

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is in a shape of parallelepiped length l_e, and fills the cross-section of a rectangular waveguide. A movable short is placed at the distance l_s behind the sample. The experimental set-up is schematically shown in Fig. 1a, and the detail of the sample and short positioning is in Fig. 1b. In experiments the frequency dependence of the reflection coefficient magnitude $|\rho|$ in reference plane AB (the left edge of the dielectric sample) is measured for different values of l_s.

Because the sample together with the shorted part of the empty waveguide form a resonant system, by measuring the frequency dependence of the reflection coefficient magnitude $|\rho(f)|$, one or more resonances (displayed as a drop of its value) can be observed. Analytically, the resonance condition - setting the imaginary part of input impedance $\mathcal{Z}_1(f)$ to zero is practically equivalent to finding the extreme of $|\rho(f)|$ obtained by setting the derivative of both the real and imaginary part of $\rho(f)$ to zero, since the frequency at which $|\rho(f)|$ reaches its minimum coresponds to the resonant frequency for input impedance $\mathcal{Z}_1(f)$ defined in the reference plane AB, Fig. 1b. In resonance $\mathcal{Z}_1$ is high, so is the electric field intensity, the magnetic field intensity is low, and thus the resonant system can be modelled by a type of parallel L,R,C circuit.

3 EQUIVALENT CIRCUIT MODEL

A simple electromagnetic model, taking into account only the transverse wave components, is used to process the data. In this approximation the waveguide is treated as a transmission line. Then the well-known formulas for evaluation of the wave impedance of an empty waveguide (1), and waveguide filled with a lossy dielectric (2), can be used. Additionally, we have used the formula for real wavenumber of the dominant mode in the waveguide (3), and the formula for a complex wavenumber of the dominant mode in the waveguide filled with lossy dielectric (4).

$$\mathcal{Z}_g(f) = \frac{Z_0}{\sqrt{1-(\frac{f_c}{f})^2}}, \quad Z_e(f) = \frac{Z_0}{\sqrt{\varepsilon(\delta)-(\frac{f_c}{f})^2}} \tag{1,2}$$

$$k_g(f) = \sqrt{(\frac{2\pi f}{c})^2 - (\frac{2\pi}{\lambda_c})^2}, \quad k_e(f) = \frac{k_g(f)\mathcal{Z}_g(f)}{\mathcal{Z}_e(f)}. \tag{3,4}$$

Further, the load impedance (that of short-ended part of the empty waveguide length l_s) and appropriate reflection coefficient related to the right edge of the dielectric sample are

$$\mathcal{Z}_2(f,l_s) = \mathcal{Z}_g(f)\tanh(jk_g(f)l_s), \tag{5}$$

$$\rho_2(f,l_s) = \frac{\mathcal{Z}_2(f,l_s)-\mathcal{Z}_g(f)}{\mathcal{Z}_2(f,l_s)+\mathcal{Z}_g(f)}, \tag{6}$$

and finally, input impedance and reflection coefficient, as calculated in plane AB are

$$\mathcal{Z}_1(f,l_s) = \mathcal{Z}_g(f)\frac{1+\rho_2(f,l_s)\exp^{-j2k_e(f)l_e}}{1-\rho_2(f,l_s)\exp^{-j2k_e(f)l_e}}, \tag{7}$$

$$\rho(f,l_s) = \frac{\mathcal{Z}_1(f,l_s)-\mathcal{Z}_g(f)}{\mathcal{Z}_1(f,l_s)+\mathcal{Z}_g(f)}. \tag{8}$$

Here f is frequency, $\mathcal{Z}_g(f)$ is characteristic impedance of an empty waveguide, f_c and λ_c are cut-off frequency and cut-off wavelength of a rectangular waveguide, $\mathcal{Z}_e(f)$ is characteristic impedance of the waveguide filled by a sample, $\varepsilon(\delta) = \varepsilon_r(1-j\tan(\delta))$ is relative complex permittivity of the sample, $k_g(f)$ is wavenumber of an empty waveguide, $k_e(f)$ is complex wavenumber of the waveguide filled by a sample, c - is the velocity of the light in vacuum, and $Z_0 = 377\Omega$, is impedance of free space.

Only the dominant mode of the waveguide was considered, in spite of that the higher order modes may be excited, due to a simple geometry of the sample and frequency range used in our experiments we did not take them into account. The permittivity and the loss tangent are determined by fitting the experimental data to an equivalent circuit model in the whole measured frequency range.

4 ANALYSIS OF THE MEASUREMENT

As discussed above the estimation of the permittivity and the loss tangent is based on fitting the experimentally observed frequency dependence of $\rho(f,l_s)$ to the theoretical dependence. The two fitting parameters in this process are the permittivity and loss tangent. The criteria chosen for the fitting was the least-square-error (LSE). However, there is an intrinsic uncertainty in this procedure. This originates in the multiple solutions of the transcendental equation tgh(x)/x = y. The situation is similar to one discussed in [1-3] where the parameters were obtained from the standing wave ratio measurements. Practically, this means that although for different values of $\varepsilon_r^{(1)}$, $\varepsilon_r^{(2)}$, *etc*, there are always different numbers of the wavelengths through the thickness of the sample (here expressed by numbers $p^{(1)} = l_e/\lambda_e^{(1)}$, $p^{(2)} = l_e/\lambda_e^{(2)}$, *etc*), if their difference is a multiple of an integer there will be two different values of permittivity pertaining to the same resonant stage. In the presented measurement we circumvented this uncertainty by measuring the frequency dependence of $|\rho(f,l_s)|$ for different values of l_s.

The dependence of $\mathrm{FRA}(p(\varepsilon_r))$, the fractional part of number p (*ie*: $p-\mathrm{INT}(p)$, with $\mathrm{INT}(p)$ being the integral patrt of number p) and $p(\varepsilon_r) = l_e/\lambda_e$, as a function of the real part of relative complex permittivity (ε_r) for different values of $\mathrm{INT}(p)$ as a parameter, is shown in Fig. 2. The loss tangent, frequency, and the sample thickness correspond to a typical measurement values. For a certain sequence of ε_r we get a set of "$\mathrm{FRA}(p(\varepsilon_r))$", the roots, from which only one is correct. It can be shown that unless the loss tangent is too high, its influence on the position of these roots is negligible. The proposed evaluation method of measured dependence of $\rho(f, l_s)$ for a reasonable number of l_s allows to eliminate the uncertainty in finding the correct roots completely.

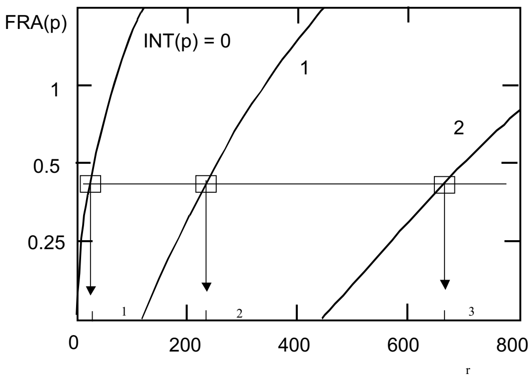

Fig. 2. Dependence of Fragment($p(\varepsilon_r)$)

5 RESULTS

We measured two types of materials: (i) a low loss material known as "Novodur," with ε_r around 4.8 to 4.9 and $\tan(\delta)$ approximately 64×10^{-3} to 80×10^{-3} in a good agreement with [5], (this was measured with different thickness l_e), (ii) a high-loss and high permittivity ceramic $Pb(Mn_{1/3}Sb_{1/3})$ sample of 1.6 mm in thickness, measured for different values of l_s. Figs. 3a, 3b show the comparison of measurement (symbols) with the fitting (solid line) using a correct root (the permittivity). The agreement of the measured and theoretical data is quite reasonable. Figs. 4a, 4b show the same experimental data as Figs. 3a, 3b, using the incorrect value of permittivity (incorrect roots). As it is seen in Fig. 4a there is a reasonable agreement even for incorrect value of permittivity, but when this incorrect value is used to fit the measured data with another l_s, the result of the fitting is quite different, see Fig. 4b. Clearly, the incorrect solution does not fit the experimental data for both values of l_s. In principle it would be possible to fit the experimental data to a satisfactory agreement in Fig. 4b (instead of Fig. 4a) but the resultant permittivity would be of a different value than that obtained by fitting in Fig. 4a. By other words, when using an incorrect root (value of permittivity) satisfactory agreement in one of the two above instances can be reached only.

More generally, one can get the correct value of permittivity by evaluating, for instance, the mean-square-error (MSE) between the theory and experiment. For all experiments (different l_s) the correct solution leads to the smallest MSE. Our measurements processed in this way are plotted in Fig. 5. The horizontal axis corresponds to the MSE values between the theory and experiment while on the vertical axis are the "fitted" permittivity solutions. From Fig. 5 it follows that the observed MSE for the roots corresponding to $\varepsilon_r \approx 220$ (upper left corner) are smaller than MSE of the roots of $\varepsilon_r \approx 20$ (lower right corner). This indicates that the value of the permittivity of measured material is $\varepsilon_r \approx 220$ and of course the best fit from the set in upper left corner should be chosen.

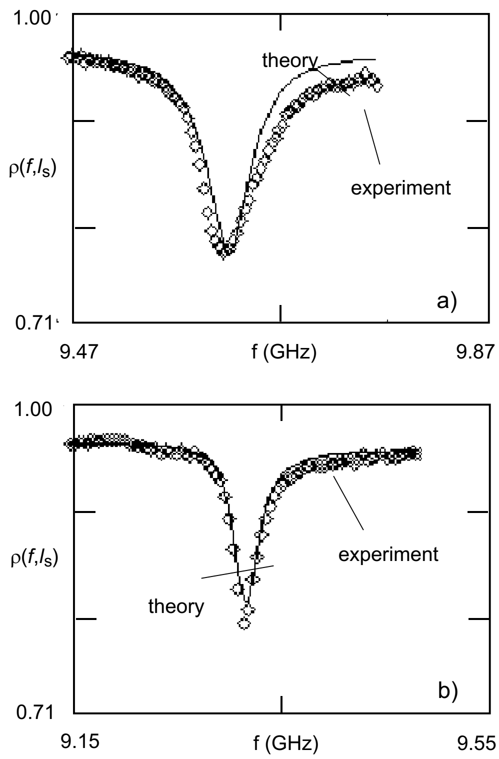

Fig. 3. Fitting the correct root (permittivity)

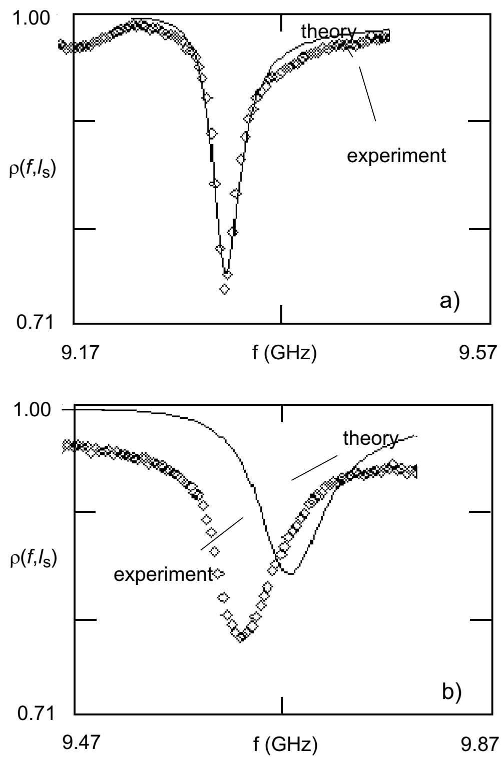

Fig. 4. Fitting the incorrect root (permittivity)

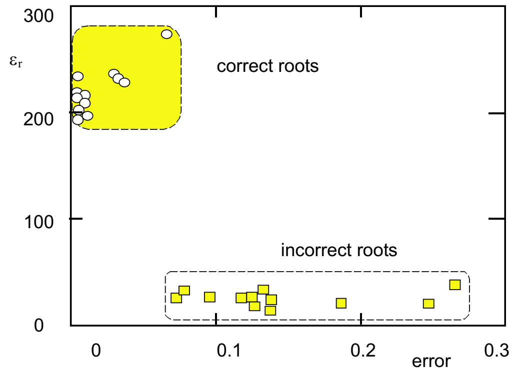

Fig. 5. Distinguishing the correct and incorrect roots

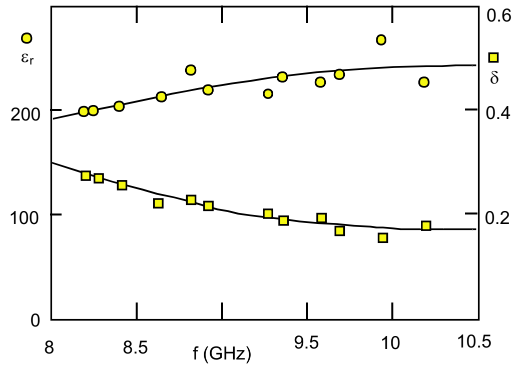

Fig. 6. Measured frequency dependence of the permittivity and loss tangent

The important question is, how many measurements have to be done to be able to distinguish the correct root from incorrect ones. The answer dependends on the precision of measurement. From our analysis it follows that the number of required measurements can be estimated from the MSE between the experiment and theory based on a clear distinction of MSE for different solutions. The number of measurements is sufficient when the set of solutions for a certain root has a smaller MSE (between the theory and experiment) as have the sets for any other roots.

6 CONCLUSIONS

The measurement of the complex permittivity was presented. The solutions and their MSE when fitting the experimental data have been found, calculated and plotted against each other for different values of l_s. This fitting procedure can be used for one (resonant) frequency or in a whole available frequency range. Both approaches lead to the correct results. In order to take into account possible frequency dependence of the permittivity we used the following approach. First, we were looking for a solution that fits to all measurements in a given frequency range. This result was than used as an initial condition for the root search while fitting one by one each of the observed resonances (after a change of l_s). In such a way the results obtained in fitting to individual resonance curves give the frequency dependence of the permittivity and loss tangent, shown in Fig. 6.

Our results are in agreement with the monochromatic measurements presented in [1], or in the case of "Novodur" with data published in [5]. The possible sources of error are those related to the SANA. The errors due to imperfect connections are mostly eliminated through the calibration procedure. The introduced method represents a modification of existing procedures allowing to measure permittivity and loss tangent of a large group of materials with a reasonable precision. Only one sample is measured and the ambiguity of the solution is eliminated by measuring the frequency dependence of the reflection coefficient for different values of parameter l_s.

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