MATHEMATICAL MODEL OF TEMPERATURE DEPENDENCES OF THE DIELECTRIC CHARACTERISTICS OF INSULATION SYSTEMS

Pavol Šandrik — Ján Zlatovský *

The contribution treats a model of the temperature dependences of dielectric quantities needed for evaluating the diagnostic measurements on the insulating systems of rotating electric machines. The model is based on experimentally determined dependences. The choice of the model is based on the coefficient of determination tested for statistical significance. The outcome is a mathematical model, which can be used to process the results of diagnostic measurements.

Keywords: dielectric properties of insulation system, temperature conditions, conversion to a reference value, insulation resistance, dielectric losses, permittivity, partial discharges, electrical strength, mathematical model.

1 INTRODUCTION

The effort of operators to maintain electrical equipment in a good operating state and early to identify a worsened condition leads to diagnostics. Except for visual inspection of the measured insulation and of the attempts for its structural analysis, applied diagnostics of high voltage insulation is based mainly on monitoring its electrical characteristics. All the same, none of the methods currently used has the ability to evaluate the examined insulation system satisfactorily. The estimation of the remaining time of operation is presently made by comparing the results of consecutive measurements. The attempts to use expert systems [4] fail because of a lack of knowledge of the process of ageing. Specialists dealing with these issues have selected a group of parameters, which are then compared. However, comparisons of consecutively measured characteristics are meaningful only if obtained under identical conditions or at least if they were recalculated. The first precondition for such a conversion is that the identical conditions can be qualitatively described, thus the given state can be characterized by a corresponding measured quantity. The measured dielectric properties of insulation in operating conditions are strongly affected by temperature and moisture. The temperature of insulation can be found by measurements. As yet, the dampness of insulation cannot be defined. A certain indicator may be the magnitude of the polarization index, which depends on the magnitude of the conductivity component of the charging current. If it is known that no significant contamination of the insulating system occurred since the last measurement and if we neglect the change in the conductivity current due to degradation, roughly identical values of the polarization index may mean approximately the same dampness.

Naturally, based on this reasoning one can not make far-reaching conclusions, still the magnitude of the polarization index plays its non-negligible role in the process of comparing consecutive measurements. The parameter followed to keep identical conditions for comparing the measured values is the temperature of insulation. When measuring the temperature, assumption is made automatically that the temperature of the insulation system is the same in its whole volume and equal to the measured value. Maintaining the same conditions for the compared quantities can be achieved in two ways — by measuring always at the same temperature or by measuring the parameters of insulation under given temperature conditions and eliminating the effect of temperature by conversion to a reference value. Owing to a limited time in diagnostic measurements, these have to be made under actual temperature conditions and then the measured values must be recalculated to reference conditions. This procedure, however, requires knowing the dependence of a given quantity on temperature. Nowadays, for none of the dielectric quantities an exact formula has been derived expressing its temperature dependence. Theoretical derivation of such a formula is complicated by multifactorial effects, which, at present, cannot be precisely described.

2 CURRENT STATUS OF THE TOPIC

In the diagnostics of high voltage insulation of rotating electric machines, thus of the so-called hard insulation, the problem of recalculating the diagnosed parameters is resolved using an empirical expression [1] which has a general form

$$V_{\theta_r} = V_{\theta_m} \cdot \exp[K_{\theta} \cdot (\theta_r - \theta_m)]$$

(1)
The coefficient of conversion, $K_v$, for a given quantity is given by relation

$$K_v = \frac{1}{\theta_r - \theta_m} \ln \frac{V_v}{V_m} \quad (2)$$

where are: $V_{\theta_v}$ - value of the diagnostic quantity recalculated to the reference temperature, $V_{\theta_m}$ - value of this quantity measured at temperature $\theta_m$, $V_v$ - value of the quantity measured in the initial, measurement at temperature $\theta_v$, $V_m$ - value of the quantity measured in a particular measurement at temperature $\theta_m$, $K_v$ - coefficient of conversion for a given dielectric quantity, $\theta_r$ - reference temperature, $\theta_m$ is the temperature of the measured insulation in a particular measurement, $\theta_v$ - the temperature of the measured insulation in the initial measurement.

Expressions (1) and (2) can be used to recalculate the measured values of insulation resistance, loss factor $\tan \delta$ and of the capacitance. Until now, however, diagnostic practice has raised several objections against these expressions: - actual experience with the use of these formulae has not led to fully convincing results, - the use of these expressions is based on the initial values which, as a rule, are not available, - one can easily see that Eqn. (2) was derived from Eqn. (1); since the dielectric quantities vary due to degradation, a question arises whether coefficient $K_v$ determined in this way describes the nature of this change properly.

The best evaluation criterion and at the same time the best possibility how to answer the objections against using Eqn. (1) is an experiment. This proved procedure was applied to temperature characteristics of dielectric parameters of the insulating systems of actual objects in power engineering.

### 3 EXPERIMENTAL DETERMINATION OF THE MATHEMATICAL MODEL

The electrical properties of insulation, thus the insulation resistance, dielectric losses, permittivity and electrical strength depend on temperature. One of the possible ways that may lead to resolution of this problem is experimental determination of these dependences and their mathematical processing.

It is assumed that the dependences searched for are determined by the material of which the insulation is made and, hence, the will be typical for a given type of insulation. Relying on this assumption it might appear that it is enough to find the temperature dependence by examining the given type of insulation under laboratory conditions. However, since many measurements conducted until now in laboratory conditions and their outcomes for real operating conditions of insulation have led to false results, we have decided to perform direct measurements on insulating system in normal operation.

The objects of measurement were stator windings of hydrogenerators the insulating system of which was made of insulators of: - thermoplastic type, temperature class B, - thermosetting type, temperature class F. To obtain the needed temperature dependences, the following procedure was chosen. The object of measurement was heated to the maximum admissible temperature in normal operation, then it was put out of operation. During cooling, measurements of the dielectric parameters were carried out. For these measurements to lead to practical results, it was necessary to choose the conditions of experiment in such a way as not to change the other parameters, just the temperature of insulation, which plays a decisive role. Having in mind considerable sizes of particular objects, the question arises of accurately defining the conditions of experiment.

To minimize the effect of moisture upon the results of measurements, such climatic conditions were chosen that guarantee a low value of atmospheric humidity. These conditions are best approximated by cloudless sonny weather. To achieve rapid cooling and as low temperature as possible, the measurement were performed under cool and stable weather. In order to create good conditions for cooling, the cooling unit was switched off and the vent shafts were closed. This prevented also contingent bedewing of the surface of insulation. The change of temperature was achieved by unassisted cooling. The temperature was measured first by built-in devices for temperature monitoring. The probes and instruments were calibrated prior to experiment. This way of temperature measurement was chosen because in diagnostic measurements the temperature is measured in the same way, then it is checked by a suitable contact thermometer or thermovision pistol. As found later, the chosen method of defining the temperature of the object under study was unsatisfactory [2] and the temperature was evaluated by measuring the winding resistance of a selected phase.

After shutting down and securing the object, it was necessary to disconnect the outlet and node of the generator rapidly and rapidly to make it accessible for measurements. The following parameters were measured: - one minute polarization index at a voltage of 2 kV, - insulation resistance at a voltage of 2 kV, - insulation resistance at a voltage of 10 kV, - loss factor $\tan \delta$ and capacitance at voltages from 0.2 Un up to $U_n$ with a step of $0.2 U_n$, - initial voltage of partial discharges, - magnitudes of apparent charge and current of partial discharges in the range from the initial voltage of partial discharges to $U_n$ with a step of 1 kV.

First, the values of the one-minute polarization index and insulation resistances were measured on all three phases. Then, in the same way, the values of the loss factor and capacitance and at the end the initial voltage of partial discharges and voltage dependence of the apparent charge and partial discharge current. In measuring each quantity, the temperature of insulation was recorded. After completing one cycle, the measurement was repeated. At higher temperatures of insulation, when its decrease was faster, the cycles of measurement followed immediately one after another, later it was necessary to wait for
the temperature to drop to the desired value. Because of conducting the measurements on weekend days and sometimes also because of weather, the measurements had to be made within roughly 48 hours. During this period it was possible to repeat the cycle described above five or six times, hence 5 to 6 points of temperature dependences were obtained. Reaching the lower temperatures was very time consuming.

Since the measured dependences are reliably valid for the particular measured object, the formulae derived thereof can only be used for this particular case. However, if one believes that the temperature dependences of the measured parameters are the same for a given type of insulation, thus they are a kind of materials characteristics, then the formulae can be applied also to other objects with the same type of insulation. In order to verify or ruling out of this assumption, the measurements were subsequently performed on several objects. The measured objects were chosen in such a way as to be able to detect the contingent effect of the power of the given object or the effect of the duration of operation.

4 METHODOLOGY OF PROCESSING THE MEASURED TEMPERATURE DEPENDENCES

The mathematical model by which it will be possible to recalculate the measured values to the reference temperature was derived from the measured temperature dependences. The way of obtaining the measured curves does guarantee that a particular value is not affected by a random error. This error may be transferred also to the derived values and this should be borne in mind when processing the measured data.

Based on the assumption that the measured values of any dielectric quantity characterizing the insulation in dependence on temperature create a two-dimensional statistical set, the statistical dependences were retrieved by regression analysis. A special case of statistical dependence is the functional dependence, and this is of our interest. To allow further processing, the statistical set was assumed to have two-dimensional normal distribution. This assumption is substantiated because we deal with repeated measurement of the same quantity under identical conditions. So as to apply an unbiased approach to data processing, no particular function was assumed by definition that should describe the measured curve. In processing each of the measured dependences, a function was chosen — by means of linear regression — that best describes the behaviour of the given parameter in dependence on temperature. The reason why linear regression analysis was chosen for data processing is the fact that if the selected mathematical model is to be used to recalculate the measured value to the reference temperature, then one has to start from only the measured point and a known conversion coefficient. This can be achieved only by a linear model.

It is clear from the measured curves that the equation of a line does not match the measured curve with sufficient accuracy and therefore the linear model with one autonomous variable was chosen in the general form:

\[ y = a_0 + a_1 \cdot g(x). \] (3)

Function \( g(x) \) may be also non-linear, e.g. \( g(x) = x^j \), where \( j = 2, 3, \ldots \), it can also be hyperbolic or a logarithmic function. Coefficients \( a_0, a_1 \) are then retrieved by the least square method.

The choice of the most suitable function was made by linear regression analysis. We have studied functions:

\[ \begin{align*}
  y &= a_0 + a_1 \cdot x, \quad y = a_0 + a_1 \cdot 1/x \quad (4,5) \\
  y &= a_0 \cdot a_1^x \quad y = a_0 \cdot x^{a_1} \quad (6,7) \\
  y &= a_0 \cdot \exp(a_1/x). \quad (8)
\end{align*} \]

Linear regression can only be used for linear functions. Some of the functions given above, however, do not meet this requirement and therefore they had to be transformed to a suitable form. Then one has to remember that the least square technique minimizes the sum of squares for the transformed function rather than for the initial one. In spite of that the transformation is advantageous because it makes possible to change the initially non-linear function into a linear one. As a consequence, we get rid of difficulties with solving a set of non-linear equations.

The procedure of linear regression consists in retrieving the coefficients of the regression function for its known analytical form and measured data points by means of the least square technique. During the calculation, all the calculated values, thus the coefficient of correlation, coefficients of the regression function \( a_0 \) and \( a_1 \) and the regression function itself are tested for statistical significance at a level of significance \( p = 0.01 \). The result is the chosen function with known coefficients \( a_0 \) and \( a_1 \), which best describes the measured temperature dependence.

The written programme makes the choice of the most suitable mathematical model. After entering the identification signs of the measured temperature dependence and of the measured data, it first evaluates the first proposed regression function and returns the calculated values of the coefficient of correlation, of coefficients \( a_0 \) and \( a_1 \), and also the results of the tests for statistical significance. The procedure is repeated for all selected regression functions. After completing the whole cycle of regression analysis, the program returns the chosen regression function, thus the best mathematical model. It allows subsequent processing of further measured temperature dependences and creating a statistical set of regression coefficient \( a_1 \), thus of the values of the conversion coefficient \( K_v \). Creation of such a statistical set is possible only if the result of regression analysis is the same type of function. If the resulting regression is different, the programme allows to change it. After approving the regression function chosen by the programme or after changing it, the regression coefficient \( a_1 \) is stored in a statistical set.
Since the statistical set is obtained from experimental data, the programme performs Grubbs’s and Dixon’s tests of extreme deviations with a level of significance \( p = 0.01 \). This level of significance was chosen so as to eliminate an element of the statistical set with a strictest criterion. The programme recommends to eliminate an element loaded by a large error. After accepting the recommendation, a new statistical set is displayed and the test of extreme deviations is offered again.

The statistical set is characterized by the values of the arithmetical mean, variation width, dispersion and standard deviation. For a better orientation in the set and possibility to trace various effects, each element of the statistical set is accompanied by a coefficient of correlation, hours of operation of the facility, and an identification shortcut.

**DISCUSSION OF THE DEFINED MODEL**

After processing more that 80 temperature dependences obtained on both real objects in operation and disassembled generator coils under laboratory conditions, the best mathematical model of the temperature dependence of dielectric quantities of the insulation system turned out to be the function

\[
V_{\theta r} = V_{\theta m} \cdot K_{V}^{(\theta_r - \theta_m)}.
\]

(9)

It is necessary to say that in processing the measured temperature characteristics the programme has chosen this model in practically all curves. Hence, we deduce that expression (9) can be used to recalculate the measured data to an arbitrary temperature in the range of common operation temperatures.

The analytical form of this model is identical with that used for temperature dependences of oil-paper insulating systems [3], thus in diagnostics of transformers.

It follows from the measurements evaluated until now that the basis of the exponential function in the selected model, thus coefficient \( K_{V} \), is independent of the size of the insulation system, number of hours of operation of the facility, and even of the type of material of which the insulation system is made. This statement is supported by the fact that all numerical values of \( K_{V} \) determined by linear regression analysis have been classified into the same statistical set. This is also confirmed by direct comparisons of single obtained values, their dispersion is really small.

**CONCLUSION**

The empirically determined mathematical model of the temperature dependence of dielectric quantities (9) can make easier the process of evaluating diagnostic measurements. However, its application is bond to the accuracy of defining the actual state, as far as temperature is concerned, in a given measurement. One can assume that the form of the mathematical model remains intact and it is necessary to make it more accurate by additional temperature measurements and their evaluation, thus by increasing the number of elements in the statistical set of coefficients \( K_{V} \).

Our experiences with using the defined model suggest slightly unexpected results. An analysis of the whole issue of reproducibility of diagnostic measurements reveals that a similarly important role as that of temperature belongs also to the moisture of the insulation system being studied. This phenomenon, however, is beyond the scope of this contribution and is subject of study in each subsequent temperature measurement.

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**References**


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**Pavol Šandrik** (Doc, Ing, CSc) was born in Bánovce, Slovakia, on 6 November 1938. He graduated at the Faculty of Electrical Engineering of the Slovak University of Technology, from the branch of Power Engineering, in 1963. His employment experience includes research and pedagogical work. Now he is with the Slovak University of Technology, Faculty of Electrical Engineering and Information Technology. His special field of interest includes diagnostics of high voltage insulation.

**Ján Zlatovský** (Doc, Ing, CSc) was born in Ivanovce, Slovakia, on 21 May 1937. He graduated at the Faculty of Electrical Engineering of the Slovak University of Technology, from the branch of Power Engineering, in 1960. His employment experience includes research and pedagogical work. Now he is with the Slovak University of Technology, Faculty of Electrical Engineering and Information Technology. His special field of interest includes diagnostic of high voltage insulation.