NOVEL HIGH-ORDER ALLPASS FILTERS EMPLOYING VOLTAGE CONVEYORS

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The design procedure for a high-order allpass filter using a new active element (voltage conveyor) is described. Three types of voltage conveyors are given: generalized voltage conveyors, concrete voltage conveyors and universal voltage conveyors. The proposed network has a minimum number of active elements. It works in the voltage mode, can be loaded by an arbitrary one-port element and can be easily transformed into its adjoint counterpart.

Key words: voltage conveyors, allpass filters

1 INTRODUCTION

We try to introduce into the circuit theory an \( n \)-port \((n \geq 3)\) voltage conveyor as a counterpart to the current conveyor. The driving quantity is in this case the nodal voltage \( V^* \) applied only to one voltage port, the live terminal of which is denoted by symbol \( x \). An independent voltage \( V_2 = V^* \) is conveyed with the positive or the negative sign always to the output ports \( z_j \) \((j = 1, 2, \ldots, Z)\), but only sometimes to the current ports \( y_i \) \((i = 1, 2, \ldots, Y)\). Independent nodal currents \( I_{y_i} \) are always conveyed to the voltage port \( x \) with the plus or the minus sign, whereas the remaining independent nodal currents \( I_{z_j} \) are never conveyed to the voltage port \( x \). The sum of all independent currents is \( Y + Z = n - 1 \).

The number of current ports \( Y \) characterizes the conveyor kind, the number of current ports with conveyed voltages \( V^* \) specifies the conveyor class.

A useful tool for the design procedure is a generalized voltage conveyor (GVC), which represents a family of concrete voltage conveyors. For the design of the allpass filter a generalized four-port second-kind voltage conveyor, the schematic symbol of which is shown in Fig. 1, is suitable. This hypothetical device is defined by the following matrix equation:

\[
\begin{bmatrix}
I_x \\
V_{y1} \\
V_{y2} \\
V_{z1}
\end{bmatrix} =
\begin{bmatrix}
0 & a_1 & a_2 & 0 \\
b_1 & 0 & 0 & 0 \\
b_2 & 0 & 0 & 0 \\
c & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
V_x \\
I_{y1} \\
I_{y2} \\
I_{z1}
\end{bmatrix}
\tag{1}
\]

The letters \( a_1, a_2, b_1, b_2, c \) are the so-called conveyance coefficients. The choice of coefficients then determines a concrete voltage conveyor. A special case of the above GVC was published in [1] and denoted as CDBA (Current Differencing Buffered Amplifier). Its conveyance coefficients are: \( a_1 = +1, a_2 = -1, b_1 = b_2 = 0, c = +1 \).

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2 DESIGN PROCEDURE

Let us return to the high-order allpass filter (AF) design published recently in this journal [2]. It is possible to reduce the number of active elements if we use a new building block, i.e., the above VC. We have to realize a two-port network with the voltage transfer function as follows:

\[
H(s) = \frac{V_{out}}{V_{in}} = \frac{M(s)}{A(s)} = \frac{P(-s)}{P(s)} = \sum_{i=0}^{n} \frac{(-1)^i a_i s^i}{\sum_{i=0}^{n} \alpha_i s^i}, \tag{2}
\]
where \( P(s) \) is the polynomial of complex frequency variable \( s \) with coefficients \( \alpha_i \).

![Diagram](image1)

**Fig. 2.** A building cell containing a generalized voltage conveyor

We have proposed a building cell the circuit diagram of which is shown in Fig. 2. Let us suppose that the general voltage conveyor in the diagram is of class zero, \( i.e. \), \( b_1 = b_2 = 0 \). The subscript \( k \) in Fig. 2 relates to the serial cell-number, beginning from the input port. We can connect \( n \) such basic building cells in cascade.

![Diagram](image2)

**Fig. 3.** Symbols of concrete voltage conveyors of class zero and their adjoint counterparts

We have computed the transfer functions \( H(s) \) for different numbers of basic cells in cascade. The following relations are valid:

If \( n = 1 \), then

\[
M(s) = -a_{11}G_1 + sC_1 \quad (3)
\]

\[
A(s) = a_{21}G_1 + sC_1 \quad (4)
\]

The last digit in the conveyance-coefficient subscript corresponds to the serial number of the GVC element in the cascade.

When comparing (3) and (4) with the numerator and denominator in (2), we obtain: \( c_1 = -1, a_{11}c_1 = -1 \) and \( a_{21}c_1 = +1 \). Therefore \( a_{11} = +1 \) and \( a_{21} = -1 \). A similar comparison can be made for \( 2, 3, \ldots \).

![Diagram](image3)

**Fig. 4.** A third-order allpass filter using VCs working in the voltage mode

Let us consider the case when \( n = 3 \). Then

\[
M(s) = -a_{11}c_1a_{12}c_2a_{13}c_3G_1G_2G_3 + sC_1a_{12}c_2a_{13}c_3C_1G_2G_3 \nonumber
\]

\[
- s^2a_{13}c_3G_2G_3 + s^3C_3G_2G_3 \quad (5)
\]

\[
A(s) = a_{21}c_1a_{12}c_2a_{13}c_3G_1G_2G_3 - a_{21}c_2a_{13}c_3C_1G_2G_3 \nonumber
\]

\[
+ s^2a_{23}C_3G_2G_3 + s^3C_3G_2G_3. \quad (6)
\]

Comparing eqns (5) and (6) with the numerator and denominator of (2) we get:

\[
c_3 = -1, a_{13} = +1, a_{23} = -1, a_2 = +1, a_{12} = -1, a_{22} = +1, c_1 = -1, a_{11} = +1, a_{21} = -1.
\]

Finally, when considering an \( n \)th-order AF we can formulate a simple rule for defining conveyance-coefficients of concrete voltage conveyors connected in cascade:

\[
b_{1k} = b_{2k} = 0, a_{1k} = (1)^{k-1} \quad \text{and} \quad c_k = a_{2k} = (-1)^k, \quad (7)
\]

where the letter \( k = 1, 2, 3, \ldots, n \) denotes the position of each VC in the cascade.

The rule for creating the polynomial coefficients in (2) is also simple and may be generalized from eqns (5) and (6). We can write

\[
\alpha_0 = \prod_{i=1}^{n} G_i, \quad \alpha_k = \prod_{i=1}^{k} C_i \prod_{i=k+1}^{n} G_i, \quad \alpha_n = \prod_{i=1}^{n} C_i. \quad (8)
\]

For example, if \( n = 6 \), it holds:

\[
\alpha_0 = G_1G_2G_3G_4G_5G_6, \quad \alpha_1 = C_1G_2G_3G_4G_5G_6, \quad \alpha_2 = C_1C_2G_3G_4G_5G_6, \nonumber
\]

\[
\alpha_3 = C_1C_2C_3G_4G_5G_6, \quad \alpha_4 = C_1C_2C_3C_4G_5G_6, \quad \alpha_5 = C_1C_2C_3C_4C_5G_6, \nonumber
\]

\[
\alpha_6 = C_1C_2C_3C_4C_5C_6.
\]
Equation (7) shows that two different concrete second-kind voltage conveyors must be used to realize an nth-order allpass filter. A concrete conveyor lying in the odd places in the cascade is shown on the left side of Fig. 3a, the conveyor on the left side of Fig. 3b is connected in the even places in the cascade. A circuit diagram of a third-order allpass filter using the proposed concrete VCs is shown in Fig. 4.

All the designed allpass filters can be realized with the aid of a universal five-port voltage conveyor (UVC), the schematic symbol of which is drawn in Fig. 5. This element is described by the following equation set: $V_{z+} = V_{x+}$, $V_{z-} = -V_{x+}$, $I_{x+} = I_{y+} - I_{y-}$. The realization of the network in Fig. 4 using UVCs is shown in Fig. 6.

An arbitrary zero-class voltage conveyor (the independent voltage $V_{z+}$ is not conveyed to the current ports) has always its adjoint counterpart. We obtain it in the following way: The voltage-port $x+$ remains the same. The output ports (denoted as $z_i$) must be changed to current ports (denoted as $y_j$), the current ports (denoted as $y$) must be changed to the output ports (denoted as $z$). Simultaneously, all the conveyance-coefficient signs must be changed. Two examples of adjoint elements are shown in Fig. 3, where the symbol $\leftrightarrow$ denotes adjointness. Evidently, the second-kind and the first-kind four-port conveyors are mutually adjoint.

The circuit diagram of a two-port adjoint to that in Fig. 6 is shown in Fig. 7. This two-port network works therefore in the current mode.

3 CONCLUSION

The nth-order allpass filter just proposed has only n active elements (UVCs), 2n resistors and n capacitors, whereas the required number of active elements in [1] and [2] is $n + 1$ (CDBAs and UCCs, respectively), in [3] as many as $n + 2$ active elements (MO-OTAs). The transfer function of the proposed two-port network has a canonic form.

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References


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