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REVIEWS - LETTERS - REPORTS

IDENTIFICATION OF NONLINEAR DYNAMIC SYSTEMS WITH ASYMMETRIC INPUT NONLINEARITIES

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The paper deals with the identification of nonlinear dynamic systems having strongly asymmetric input nonlinearities. A special form of Hammerstein model with two-segment polynomials enables an iterative estimation of the model parameters on the basis of input/output records. The accuracy of resulting model is better, compared to the single polynomial models of higher degrees. The results of Monte Carlo analysis are also presented for the noisy systems.

K e y w o r d s: nonlinear systems, polynomials, Hammerstein model, identification

1 INTRODUCTION

Many nonlinear dynamic systems can be represented by the block-oriented models with linear dynamic and nonlinear static blocks [3]. Dynamic systems with input nonlinearities are usually modeled by the so-called Hammerstein model where the characteristics of nonlinear blocks are generally approximated by polynomials of proper degree (methods and applications see also in [1-2], [5-8], [10], [12]). However, the higher degrees of polynomials may be inconvenient, especially when the models should be used for control purposes.

Nevertheless, in some applications the polynomials of very high degree must be used to approximate the nonlinear characteristics with required accuracy. This is the case if the characteristics differ significantly for the positive and negative inputs, respectively. For such strongly asymmetric characteristics it may be reasonable to consider descriptions with two distinct maps, *ie*, two-segment descriptions. Then the nonlinearities can be approximated by proper two-segment polynomials of lower degrees than that of the equivalent single polynomials.

This paper deals with the parameter identification of discrete-time nonlinear dynamic systems having asymmetric input nonlinearities, which can be represented by the Hammerstein models with the nonlinear static block containing two-segment nonlinearities. The approach proposed by the author in [10] is applied to the systems with general two-segment nonlinear characteristics (possibly having discontinuous derivatives in the origin), which can be approximated by two polynomials.

The paper is organized as follows. First, the asymmetric nonlinear characteristics and the special form of Hammerstein models with two-segment polynomial nonlinearities are described. Then the iterative method proposed in [10] is used for the identification of nonlinear dynamic system with general asymmetric input nonlinearities. The resulting Hammerstein model with two-segment polynomial is compared with the models with single polynomials of different degrees. Finally, the results of Monte Carlo analysis for the identification of noisy system are presented.

2 ASYMMETRIC NONLINEARITY

Nonlinear characteristics can be generally approximated by polynomials of proper degree:

$$\gamma[u(t)] = \sum_{k=1}^{R} \gamma_k \, u^k(t) \tag{1}$$

However, in some cases, when the characteristics are strongly asymmetric, only the polynomials of higher degree can approximate the nonlinear characteristics adequately. The offset between the accuracy of approximation and lower degrees of approximating polynomials can be solved by using two-segment polynomial approximations [10].

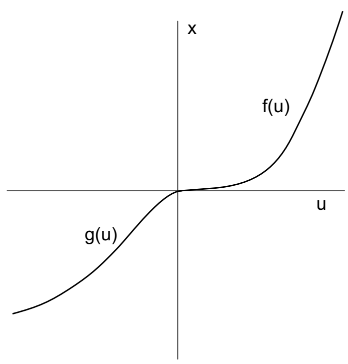

Fig. 1. The output of a nonlinear block.

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Assume the output of nonlinearity x(t) according to Fig. 1 depends on the sign and magnitude of input u(t) and can be written as

$$x(t) = \begin{cases} f[u(t)], \dots & \text{if} \quad u(t) \geq 0 \\ g[u(t)], \dots & \text{if} \quad u(t) < 0 \end{cases} \tag{2}$$

and the maps $f(.)$ and $g(.)$ significantly differ from each other. Introducing the following switching function

$$h(t) = h[u(t)] = \begin{cases} 0, \dots & \text{if} \quad u(t) > 0 \\ 1, \dots & \text{if} \quad u(t) < 0 \end{cases} \tag{3}$$

the relation between the inputs $\{u(t)\}$ and outputs $\{x(t)\}$ of assumed nonlinearity can be written as follows:

$$x(t) = f[u(t)] + \{g[u(t)] - f[u(t)]\}h(t) \tag{4}$$

similarly, as in the case of two-segment piecewise-linear asymmetric nonlinearities, where $h(t)$ switches between two segments of the nonlinear characteristics [4].

If the nonlinear maps f(.) and g(.) are approximable by proper polynomials

$$f[u(t)] = \sum_{k=1}^{r} f_k \, u^k(t) \tag{5}$$

$$g[u(t)] = \sum_{k=1}^{r} g_k \, u^k(t) \tag{6}$$

then (4) can be written as follows:

$$x(t) = \sum_{k=1}^{r} f_k \, u^k(t) + \sum_{k=1}^{r} p_k \, u^k(t) h(t) \tag{7}$$

where $p_k = g_k - f_k$.

Naturally, the nonlinearity (2) could be approximated by a single polynomial (1), however, to achieve adequate accuracy, the degree r of polynomials approximating $f(.)$ and $g(.)$ may be lower then the degree R of polynomial $g(.)$. This may be important in many control applications, where the degree of polynomials seldom exceeds the number three for computational reasons. Namely, the polynomials of the third degree have always at least one real root. From this point of view the same is valid for the polynomials of odd degrees. However, the solution (searching for the roots) of these polynomials is rather difficult. The proposed two-segment polynomial approximation can use lower degrees for reaching the required accuracy of the nonlinearity description.

3 HAMMERSTEIN MODEL

Dynamic systems with input nonlinearities can be represented by the Hammerstein model [3], which is given by the cascade connection of a static nonlinearity block followed by a linear dynamic system (Fig. 2). The description of its linear block representing the model output part with an additive noise $n(t)$ can be given as

$$B(q^{-1})y(t) = A(q^{-1})x(t) + \nu(t) \tag{8}$$

where $x(t)$ and $y(t)$ are the inputs and outputs, respectively, $A(q^{-1})$ and $B(q^{-1})$ are scalar polynomials in the unit delay operator q^{-1}.

$$A(q^{-1}) = a_0 + a_1 q^{-1} + \cdots + a_m q^{-m}, \tag{9}$$

$$B(q^{-1}) = 1 + b_1 q^{-1} + \cdots + b_n q^{-n}, \tag{10}$$

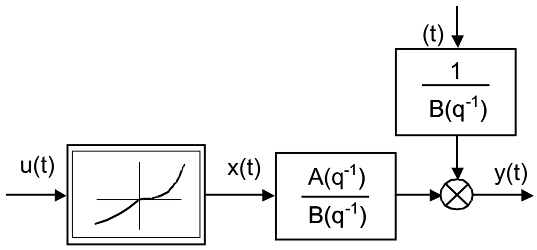

Fig. 2. Block scheme of the Hammerstein model

Assume the nonlinear block is described by the two-segment polynomial (7) and the nonlinear block input is identical with the linear block output. However, a direct substitution of the nonlinearity (7) into (8) would result in a complex expression, which is nonlinear in parameters.

To simplify this problem the so-called key term separation principle can be applied [8-11]. In the given model it is always possible to fix one of the parameters. If $a_0 = 1$, then (8) can be rewritten as follows:

$$\begin{aligned} y(t) = x(t) + [A(q^{-1}) - 1]x(t) + \\ + [1 - B(q^{-1})]y(t) + \nu(t). \end{aligned} \tag{11}$$

After choosing x(t) as the key term and then substituting (7) only for the separated key term $x(t)$, the model output will be

$$\begin{aligned} y(t) = \sum_{k=1}^{r} f_k u^k(t) + \sum_{k=1}^{r} p_k u_k(t) h(t) + \\ + [A(q^{-1}) - 1]x(t) + [1 - B(q^{-1})]y(t) + \nu(t). \end{aligned} \tag{12}$$

The equation (12) and that of (7), defining the internal variable x(t), represent a special form of the Hammerstein model with two-segment polynomial nonlinearity. The model has no redundancy of parameters and all of them enter the expressions linearly.

Note that the single polynomial description of the nonlinear block in the form (1) can be also incorporated into the Hammerstein model in this way leading to the following model equation:

$$y(t) = \sum_{k=1}^{R} \gamma_k u^k(t) + [A(q^{-1}) - 1]x(t) + \\ + [1 - B(q^{-1})]y(t) + \nu(t). \tag{13}$$

However, as already mentioned above, the degree r of polynomials in (12) may be lower than the degree R of polynomial in (13). Hence (12) may be more appropriate for control applications.

The nonlinear dynamic model given by (12) can be used for the identification of nonlinear dynamic systems with input nonlinearities in the way proposed in [10]. Defining the data vector

$$\Psi^\top(t) = [u(t), u^2(t), \ldots, u^r(t), u(t)h(t), \\ u^2(t)h(t), \ldots, u^r(t)h(t), x(t-1), \ldots, x(t-n), \\ -y(t-1), \ldots, -y(t-m)], \tag{14}$$

and the vector of parameters

$$\Theta^\top = [f1, f2, \ldots, f_r, p_1, p_2, \ldots, \\ p_r, b_1, \ldots, b_n, a_1, \ldots, a_m], \tag{15}$$

the Hammerstein model output equation can be written as

$$y(t) = \Psi^\top(t) + \nu(t), \tag{16}$$

and the parameter estimation can be solved like a linear estimation problem. As $\Psi(t)$ depends on the values of unmeasurable internal variable $x(t)$, the following iterative estimation method with internal variable estimation can be used. Assigning the estimated internal variable in the s-th step as

$${}^s x(t) = \sum_{k=1}^{r} {}^s f_k \, u^k(t) + \sum_{k=1}^{r} {}^s p_k \, u^k(t)h(t), \tag{17}$$

the error to be minimized is gained from (3.9) in the vector form

$$e(t) = y(t) - s\Psi^\top(t) \, {}^{s+1}\Theta, \tag{18}$$

where ${}^s\Psi(t)$ is the data vector with the corresponding estimates of internal variable according to (17) and ${}^{s+1}\Theta$ is the $(s+1)$-th estimate of the parameter vector. Then the steps in the iterative procedure are as follows:

Step 1
Minimizing a proper criterion based on (3.11) the estimates of both linear and nonlinear block parameters ${}^{s+1}\Theta$ are yielded using ${}^s\Psi(t)$ with the s-th estimates of internal variable.
Step 2
Using (3.10) the estimates of ${}^{s+1}x(t)$ are evaluated by means of the recent estimates of model parameters ${}^s f_k$ and ${}^s p_k$.
Step 3
Repeat steps 1 and 2 until the parameter estimates converge to constant values.

Hence, equally as in the case of single polynomial Hammerstein model [8], the parameters of linear and nonlinear blocks of the Hammerstein model can be iteratively estimated using input/output records and the values of estimated internal variable. These values result from the previous estimates of nonlinear block parameters.

4 IDENTIFICATION RESULTS

The presented method for the parameter identification of nonlinear dynamic systems with two-segment general (non-polynomial) nonlinearities using the Hammerstein model was implemented and tested by means of MATLAB. Several systems were simulated and the estimations of all the model parameters (those of linear and nonlinear blocks) were carried out on the basis of input/output records and the internal variables' estimates.

To illustrate the feasibility of the proposed approach, the following example shows the parameter estimation process using the Hammerstein model with two-segment polynomials given by the output equation (12). The efficiency of identification is compared to single polynomial Hammerstein models given by the output equation (13) for different values of R, because no comparable identification method using Hammerstein model with two-segment polynomials was published.

The input asymmetric nonlinearity was given by the following two-segment description:

$$x(t) = \begin{cases} u(t)/[0.1 + 0.9u(t)2]1/2, & \text{for } u(t) \le 0 \\ -u^2(t)[1 - exp(0.7u(t))], & \text{for } u(t) < 0. \end{cases}$$

and is shown in Fig. 3. The second order linear dynamic system was given by the difference equation

$$y(t+1) = \\ x(t) + 0.5\,x(t-1) + 1.5\,y(t) - 0.7\,y(t-1).$$

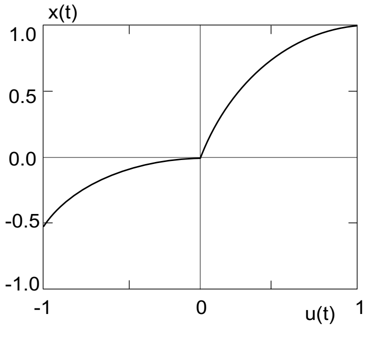

Fig. 3. The input asymmetric nonlinearity

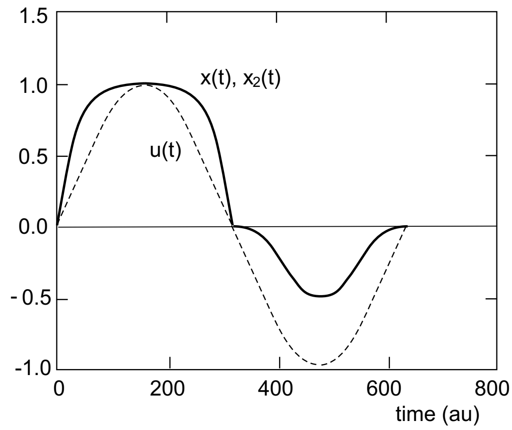

Fig. 4. Time dependence of internal variables $x(t)$ and $x_2(t)$

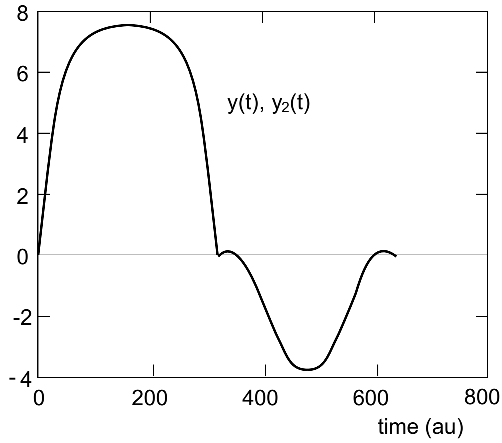

Fig. 5. Time dependence of the model outputs $y(t)$ and $y_2(t)$

4.1 Two-segment polynomial model

The identification using the Hammerstein model with two-segment polynomials in the form (12) with $r = 3$ was carried out on the basis of $N = 300$ samples of uniformly distributed random inputs and simulated outputs. The process of parameter estimation is included in Table 1. The initial values of parameters were chosen zero, hence in the first iteration the parameter a1 was not estimated. The iterative process finished after the mean squares error was less than 0.001. The resulting equations characterizing the blocks of Hammerstein model with two-segment polynomial are:

$$\begin{aligned} x_2(t) = &\\ &3.4585\,u(t) - 4.3151\,u^2(t) + 1.8741\,u^3(t)\\ &-3.4877\,u(t)h(t) + 4.1364\,u^2(t)h(t)\\ &-1.5188\,u^3(t)h(t),\\ y_2(t+1) = &\\ &x_2(t) + 0.4999\,x_2(t-1)\\ &-1.5001\,y_2(t) + 0.7002\,y_2(t-1), \end{aligned}$$

and the model equation in the form of (3.5) is

$$\begin{aligned} y_2(t+1) = &\\ &3.4585\,u(t) - 4.3151\,u^2(t) + 1.8741\,u^3(t)\\ &-3.4877\,u(t)h(t) + 4.1364\,u^2(t)h(t)-\\ &-1.5188\,u^3(t)h(t) + 0.4999\,x_2(t-1)\\ &-1.5001\,y_2(t) + 0.7002 y2(t-1), \end{aligned}$$

where $x_2(t)$ is the estimated model internal variable and the model output is assigned as $y_2(t)$ to distinguish it from that of simulated system. The accuracy of proposed model with two-segment polynomial is evident in Fig. 4 and Fig. 5, where the internal variables $x(t)$ and $x_2(t)$ and the outputs $y(t)$ and $y_2(t)$ of the system and the model for the same test signal $u(t)$ are shown.

4.2 Single polynomial models

For the same nonlinear system the identification was performed using the Hammerstein model with single polynomials. As for control purposes the degrees of polynomials are preferred to be odd to ensure at least one real

Table 1. Parameter estimation process

It	b_1	B_2	a_1	f_1	f_2	f_3	p_1	p_2	p_3
1	-1.6329	0.8140	-	-4.2793	-5.9417	2.7871	-5.1530	4.0594	-3.4055
2	-1.5014	0.7029	0.4880	3.1388	-3.6539	1.4802	-2.8542	4.0974	-0.7679
3	-1.4999	0.6995	0.5026	3.5895	-4.5862	2.0356	-3.7481	4.1526	-1.8257
4	-1.5002	0.7006	0.4986	3.4040	-4.2022	1.8067	-3.3794	4.1295	-1.3910
5	-1.5000	0.7001	0.5004	3.4801	-4.3598	1.9007	-3.5305	4.1392	-1.5693
6	-1.5001	0.7003	0.4997	3.4488	-4.2949	1.8620	-3.4683	4.1352	-1.4959
7	-1.5000	0.7002	0.5000	3.4616	-4.3216	1.8779	-3.4939	4.1368	-1.5261
8	-1.5001	0.7002	0.4999	3.4563	-4.3106	1.8714	-3.4833	4.1362	-1.5137
9	-1.5001	0.7002	0.4999	3.4585	-4.3151	1.8741	-3.4877	4.1364	-1.5188

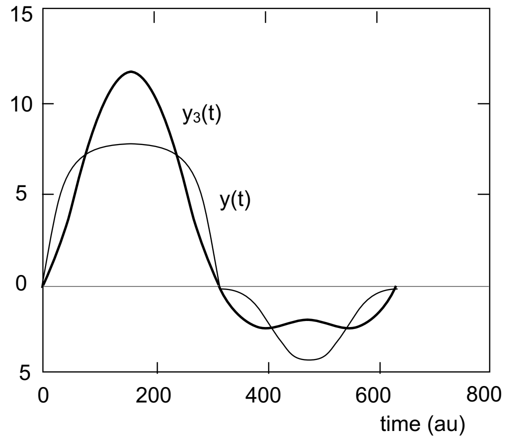

Fig. 6. Output of the sytem $y(t)$ and Hammerstein model $y_3(t)$

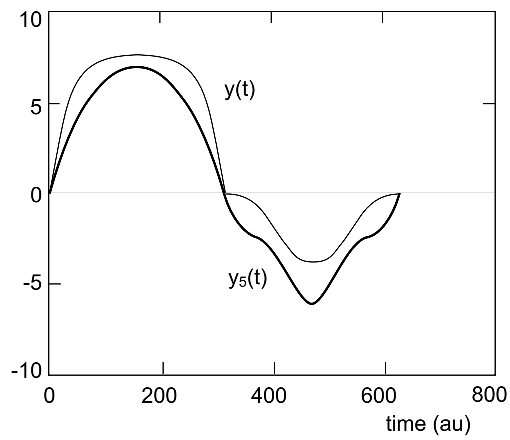

Fig. 7. Output of the $y(t)$ and Hammerstein model $y_5(t)$

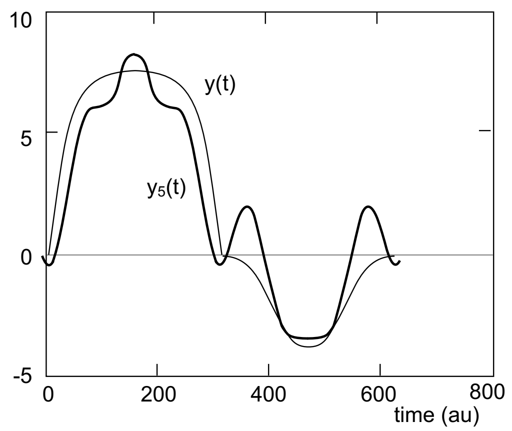

Fig. 8. Output of the sytem $y(t)$ and Hammerstein model $y_7(t)$

solution of respective control law, the Hammerstein models with the polynomials of degrees 3 and 5, requiring fewer parameters than the two-segment case, and also the polynomial of degree 7, requiring more parameters, were considered. The identification using the Hammerstein model (13) with the same input/output data gave the following results:

$$
\begin{aligned}
y_3(t+1) = {}& \\
& 1.1131\,u(t) + 0.5012\,u^2(t) - 0.4236\,u^3(t) + \\
& +0.4967\,x_3(t-1) - 1.5389\,y_3(t) + \\
& +0.6886\,y_3(t-1), \\
y_5(t+1) = {}& \\
& 1.2634\,u(t) + 1.7983\,u^2(t) - 1.2096 u^3(t) - \\
& -1.7599\,u^4(t) + 0.7070\,u^5(t) + 0.5163\,x_5(t-1) \\
& 1.5065\,y_5(t) + 0.6843\,y_5(t-1), \\
y_7(t+1) = {}& \\
& 1.3794\,u(t) + 3.1399\,u^2(t) - 2.2893\,u^3 t) - \\
& -6.5411\,u^4(t) + 2.9110\,u^5(t) + 3.7343\,u^6(t) \\
& -1.2541\,u^7(t) + 0.5121\,x_7(t-1) - 1.5001\,y_7(t) \\
& +0.6888\,y_7(t-1),
\end{aligned}
$$

where $x_i(t)$ and $y_i(t)$, $i = 3, 5, 7$, are the internal variables and outputs of Hammerstein models of the form (13) with $R = 3, 5, 7$. Notable differences in the accuracy of approximation using two-segment polynomial

Table 2. Monte Carlo analysis

		SNR = 100		SNR = 50	
	Estimate	*Mean*	*STD*	*Mean*	*STD*
$b1$	−1.5001	−1.4985	0.0009	−1.4940	0.0020
$b2$	0.7002	0.6987	0.0008	0.6945	0.0018
$a1$	0.4999	0.5015	0.0047	0.5059	0.0094
$f1$	3.4563	3.4621	0.0301	3.4584	0.0599
$f2$	−4.3106	−4.3242	0.1047	−4.3117	0.2086
$f3$	1.8714	1.8815	0.0803	1.8720	0.1601
$p1$	−3.4833	−3.4935	0.0306	−3.4906	0.0610
$p2$	4.1362	4.1288	0.1496	4.1030	0.2981
$p3$	−1.5137	−1.5420	0.0770	−1.5465	0.1537

compared to the single polynomial cases can be observed by the comparison of the outputs of different types of Hammerstein models to the output of simulated system for the same input signal $u(t)$ as above. This is shown in Fig. 6 to Fig. 8, where $y(t)$ is the output of a real system, while $y_3(t)$, $y_5(t)$, and $y_7(t)$ are the outputs of Hammerstein models with single polynomials. (Note that the scales in Fig. 4 to Fig. 8 are not identical and are imposed by MATLAB.) The differences are caused by the inaccurate approximations of strongly asymmetric characteristic in the input block, which consequently influence the estimates of linear dynamic block parameters.

4.3 Monte Carlo analysis

The estimation process for the above example of Hammerstein model with two-segment polynomial nonlinearity was also tested with additive output noise and Monte Carlo simulation studies of more runs were performed. It is worthwhile to note that using Monte Carlo is not fully correct in this case because of the iterative method of parameter estimation. Namely, the iterative processes may stop after different numbers of steps and it is impossible to ensure the same conditions for the Monte Carlo analysis. For this reason the number of iterations was limited to 10.

Normally distributed random noises with zero mean and different signal-to-noise ratios (SNR - the square root of the ratio of output and noise variances) were added to the simulated outputs. The results of 20 runs for N $= 1000$ samples of input/output data are in Table 2 for SNR $=$ 100 and SNR $= 50$, respectively. The Hammerstein model parameters are given by the mean values (Mean) and the standard deviations (STD). The results are satisfactory in both cases and the mean values are close to the values of corresponding parameters' estimates (Estimate) gained in the noiseless case.

5 CONCLUSIONS

An approach to the identification of nonlinear dynamic systems with asymmetric input nonlinearities using the Hammerstein model with two-segment nonlinearities has been presented. It has been shown, that the two-segment polynomial approximation of input nonlinearity may be in some cases of strongly asymmetric characteristics (possibly having discontinuous derivatives) better than that of single polynomials with higher degrees.

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