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ROBUST OUTPUT FEEDBACK CONTROLLER DESIGN: LMI APPROACH

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The paper addresses the problem of robust output feedback controller design with a guaranteed cost and quadratic, and parameter dependent Lyapunov function quadratic stability for linear continuous time affine systems. The proposed design methods lead to a non-iterative LMI based algorithm. Numerical examples are given to illustrate the design procedure.

K e y w o r d s: quadratic stability, parameter dependent quadratic stability, robust controller, output feedback

1 INTRODUCTION

Robustness has been recognized as a key issue in the analysis and design of control systems for the last two decades. During the last decades numerous papers dealing with the design of static robust output feedback control schemes to stabilize uncertain systems have been published ([1], [3], [4], [6], [8], [10], [12], [13], [15], [19], [21]). Various approaches have been used to study the two aspects of the robust stabilization problem, namely conditions under which the linear system described in state space can be stabilized via output feedback and the respective procedure to obtain a stabilizing or robustly stabilizing control law.

The necessary and sufficient conditions to stabilize the linear continuous time invariant system via static output feedback can be found in [11], [20]. In the above and other papers, the authors basically conclude that despite the availability of many approaches and numerical algorithms the static output feedback problem is still open.

Recently, it has been shown that an extremely wide array of robust controller design problems can be reduced to the problem of finding a feasible point under a Biaffine Matrix Inequality (BMI) constraint. The BMI has been introduced in [7]. In this paper, the BMI problem of robust controller design with output feedback is reduced to a LMI problem [2]. The theory of Linear Matrix Inequalities has been used to design robust output feedback controllers in [1], [3], [4], [8], [12], [19] and [20]. Most of the above works present iterative algorithms in which a set of LMI problems are repeated until certain convergence criteria are met. The V-K iteration algorithm proposed in [4] is based on an alternative solution of two convex LMI optimization problems obtained by fixing the Lyapunov matrix or the gain controller matrix. This algorithm is guaranteed to converge, but not necessarily, to the global optimum of the problem depending on the starting conditions.

The main criticism formulated by control engineers against modern robust analysis and design methods for linear systems concerns the lack of efficient easy to use and systematic numerical tools. This is especially true when analyzing robust stability as affected by highly structured uncertainty with BMI, for which no polynomial-time algorithm has been proposed so far [9].

This paper is concerned with the class of uncertain linear systems that can be described as

$$\dot{x}(t) = (A_0 + A_1\theta_1 + \dots + A_p\theta_k)x(t) \tag{1}$$

where $\theta = [\theta_1 \dots \theta_p] \in R^p$ is a vector of uncertain and possibly time varying real parameters.

The system represented by (1) is a polytope of linear affine systems which can be described by a list of its vertices

$$\dot{x}(t) = A_{ci}x(t), \quad i = 1, 2, \dots, N \tag{2}$$

where $N = 2^p$.

The system represented by (2) is quadratically stable if and only if there is a common Lyapunov matrix $P > 0$ such that

$$A_{ci}^{\top}P + PA_{ci} < 0, \quad i = 1, 2, \dots, N\,. \tag{3}$$

A weakness of quadratic stability is that it guards against arbitrary fast parameter variations. As a result, this test tends to be conservative for constant or slow-varying parameters θ, for polytopic systems. To reduce conservatism when (1) is affine in θ and the parameters of system are time invariant, in [5] the parameter-dependent Lyapunov function $P(\theta)$ has been used in the form

$$P(\theta) = P_0 + \theta_1 P_1 + \dots + \theta_p P_p\,. \tag{4}$$

Robust controller design with guaranteed cost and affine quadratic stability has been proposed in [21]. Other types

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of parameter-dependent Lyapunov functions have been proposed in [14] for the stability analysis of linear discrete time systems and for the analysis and design of continuous time systems with affine type uncertainties in [9], [17], [18]. In this paper, we introduce a new type of parameter-dependent Lyapunov functions which allows to reduce some important class of BMI problems to LMI. For guaranteed cost and system (2) this leads to a non iterative LMI based algorithm. The proposed design procedure guarantees with sufficient conditions robust parameter dependant Lyapunov function quadratic stability (PDQS) for closed loop systems.

The paper is organized as follows. In Section 2 the problem formulation and some preliminary results are brought. The main results are given in Section 3. In Section 4 the obtained theoretical results are applied.

We have used the standard notation. A real symmetric positive (negative) definite matrix is denoted by $P > 0$ ($P < 0$). Much of the notation and terminology follows the references of [5] and [11].

2 PRELIMINARIES AND PROBLEM FORMULATION

We shall consider the following affine linear time invariant continuous time uncertain systems

$$\begin{aligned} \dot{x}(t) &= A(\theta)x(t) + B(\theta)u(t)\,, \\ y(t) &= C(\theta)x(t), \quad x(0) = x_0 \end{aligned} \tag{5}$$

where $x(t) \in R^n$ is the plant state; $u(t) \in R^m$ is the control input; $y(t) \in R^l$ is the output vector of system; $A(\theta), B(\theta), C(\theta)$ are matrices of appropriate dimensions and

$$\begin{aligned} A(\theta) &= A_0 + A_1\theta_1 + \cdots + A_p\theta_p\,, \\ B(\theta) &= B_0 + A_1\theta_1 + \cdots + B_p\theta_p\,, \\ C(\theta) &= C_0 + C_1\theta_1 + \cdots + C_p\theta_p\,. \end{aligned}$$

Note that, in order to keep the polytope affine property, the matrix $B(\theta)$ or $C(\theta)$ must be precisely known. In general, a polytope description of uncertainties results in a less conservative controller design than other characterizations of uncertainty [2]. However, as the number of uncertain parameters increases, the number of vertices increases exponentially, and the design time increases exponentially too [1]. The system represented by (5) is a polytope of linear systems. The linear matrix inequality approach requires that system (5) be described by a list of its vertices, *ie*, in the form

$$\{(A_{v1}, B_{v1}, C_{v1}), \ldots, (A_{vN}, B_{vN}, C_{vN})\}\,. \tag{6}$$

The system represented by (6) is quadratically stable if and only if there is a Lyapunov matrix $P > 0$ such that [2]

$$A_{vi}^\top P + PA_{vi} < 0, \quad i = 1, 2, \ldots, N\,. \tag{7}$$

Consequently, system (6) is static output feedback quadratically stabilizable if and only if there is a Lyapunov matrix $P > 0$ and a feedback matrix F such that

$$\begin{aligned} &(A_{vi} + B_{vi}FC_{vi})^\top P \\ &\qquad + P(A_{vi} + B_{vi}FC_{vi}) < 0, \quad i = 1, 2, \ldots, N\,. \end{aligned} \tag{8}$$

If (8) holds for $P > 0$ and some F, then the vertices of the polytope (6) are said to be simultaneously quadratically stabilized by F. It is well known [2] that if P is a common Lyapunov matrix for the vertices of the polytope (6), it serves as a common Lyapunov function for the uncertain system (5) for all admissible uncertainties $\theta_i \in \langle \underline{\theta_i}, \overline{\theta_i} \rangle$, $i = 1, 2, \ldots, p$. In (6), each vertex is computed for different values of the p variables θ_i, alternatively taken at maximum and minimum values.

THEOREM 1. *Consider system* (6). *Then the following statements are equivalent.*

- *System* (6) *is static output feedback simultaneously quadratically stabilizable with a guaranteed cost*

$$\int_0^\infty (x^\top Qx + u^\top Ru)\, dt \le x_0^\top Px_0 = J^* \tag{9}$$

  and $P > 0$.
- *There exist matrices* $P > 0, R > 0, Q > 0$ *and matrix* F *such that the following inequality holds*

$$\begin{aligned} &(A_{vi} + B_{vi}FC_{vi})^\top P + P(A_{vi} + B_{vi}FC_{vi}) \\ &\qquad + Q + C_{vi}^\top F^\top RFC_{vi} \le 0 \end{aligned} \tag{10}$$

  for $i = 1, 2, ..., N$.
- *There exist matrices* $P > 0, R > 0, Q > 0$ *and matrix* F *that the following inequalities hold*

$$\begin{aligned} &A_{vi}^\top P + PA_{vi} - PB_{vi}R^{-1}B_{vi}^\top P + Q \\ &\quad + (B_{vi}^\top P + RFC_{vi})R^{-1}(B_{vi}^\top P + RFC_{vi})^\top \le 0 \end{aligned} \tag{11}$$

  for $i = 1, 2, ..., N$.

Proof. Consider the control algorithm with output feedback to have the form

$$u = Fy = FC_{vi}x$$

then for the closed loop system results

$$\dot{x} = (A_{vi} + B_{vi}FC_{vi})x, \quad i = 1, 2, \ldots, N\,.$$

For $V = x^\top Px$, the time derivative of V along the system (6) is

$$\frac{\mathrm{d}V}{\mathrm{d}t} = x^\top[(A_{vi} + B_{vi}FC_{vi})^\top P + P(A_{vi} + B_{vi}FC_{vi})]x\,.$$

If inequality (10) holds, then there exist matrices $P > 0$, $R > 0$, $Q > 0$ and F such that

$$\frac{\mathrm{d}V}{\mathrm{d}t} \le -x^\top(Q + C_{vi}^\top F^\top RFC_{vi})x < 0$$

for $i = 1, 2, ..., N$. Therefore the closed loop system is asymptotically stable. Furthermore, by integrating both sides of the inequality from 0 to T and using the initial condition x_0, we obtain

$$V(0) - V(T) \geq \int_0^{\top} x^\top (Q + C_{vi}^\top F^\top RFC_{vi}) x \,\mathrm{d}t \,.$$

As the closed loop system is asymptotically stable when $T \to \infty$, then

$$x(T)^\top P x(T) \to 0 \,.$$

Hence, we get

$$\int_0^\infty x^\top (Q + C_{vi}^\top F^\top RFC_{vi}) x \mathrm{d}t \leq x_0^\top P x_0 \qquad (12)$$

and the control algorithm $u = Fy$ is a guaranteed cost control law and

$$J^* = x_0 \top P x_0$$

is a guaranteed cost value for uncertain closed loop system. The equivalence of the second and the third statement is evident [20].

The following performance index is associated with system (5)

$$J = \int_0^\infty (x(t)^\top Q x(t) + u(t)^\top R u(t)) \mathrm{d}t \qquad (13)$$

where $Q = Q^\top \geq 0$, $R = R^\top > 0$ are matrices of compatible dimensions.

The problem studied in this paper can be formulated as follows: For a continuous time system described by (5) design a static output feedback controller with the gain matrix F and control algorithm

$$u(t) = Fy(t) = FC(\theta)x(t) \qquad (14)$$

so that the closed loop system

$$\dot{x} = (A(\theta) + B(\theta)FC(\theta))x(t) = A_c(\theta)x(t) \qquad (15)$$

is PDQS with guaranteed cost.

Definition 1. Consider system (5). If there exists a control law u^* and a positive scalar J^* such that closed loop system (15) is stable and the closed loop value cost function (13) satisfies $J \leq J^*$, then J^* is said to be the guaranteed cost and u^* is said to be the guaranteed cost control law for system (5).

3 MAIN RESULTS

In this paragraph we present a new procedure to design a static output feedback controller for affine continuous time linear systems (5) which ensure the guaranteed cost and PDQS of closed loop system.

Let us introduce different Lyapunov matrices $P_{vi} > 0$, $i = 1, 2, \ldots, N$ accompanies each of the vertices of the uncertainty polytope, thus eliminating the need for quadratic stability. In this note the solution of (10) for PDLF and polytopic systems is given. Using the following quadratic form

$$(P + BFC)\top(P + BFC) = PP + PBFC + (BFC)^\top P + C^\top F^\top B^\top BFC \qquad (16)$$

one can rewrite inequality (10) as follows

$$A^\top P + PA - PP + Q + (P + BFC)^\top (P + BFC) + C^\top F^\top (R - B^\top B) FC < 0 \qquad (17)$$

and LMI solution of (17) with an additional matrix Z and polytopic system is given as follows

$$\begin{bmatrix} L_i & L_{pi} & C_{bi} \\ L_{pi}^\top & -I & 0 \\ C_{bi}^\top & 0 & -(R - B_{vi}^\top B_{vi}) \end{bmatrix} < 0$$

where

$$\begin{aligned} L_i &= A_{vi}^\top P_{vi} + P_{vi} A_{vi} + Q + F_{vi}(Z, P_{vi}, D_{vi}) \,, \\ C_{bi} &= C_{vi}^\top F^\top (R - B_{vi}^\top B_{vi}) \,, \\ L_{pi} &= (P_{vi} + B_{vi} F C_{vi})^\top \,, \\ 0 &< P_{vi} < \varrho I \,, \quad i = 1, 2, \ldots, N \,, \end{aligned} \qquad (18)$$

with condition

$$R - B_{vi}^\top B_{vi} > 0 \quad i = 1, 2, \ldots, N \,. \qquad (19)$$

For LMI solution the term $-PP$ in (17) has to be replaced. In this note two possibilities are proposed.

1.

$$- P_{vi} P_{vi} \leq -P_{vi} D_{vi} - D_{vi} P_{vi} + D_{vi} D_{vi} + Z P_{vi}^{-1} Z - 2Z + P_{vi} = F_{vi}(Z, P_{vi}, D_{vi}) \qquad (20)$$

where $D_{vi} = D_{vi}^\top > 0$ is the initial value of matrix P_{vi} and $Z = Z^\top > 0$ is any positive definite matrix, a new LMI variable.

2.

$$-P_{vi} P_{vi} \leq (\varrho^2 - \varrho_1^2) I - Z = F_{vi}(Z) \qquad (22)$$

where

$$\varrho_1 I < P_{vi} < \varrho I \quad i = 1, 2, \ldots, N \,, \qquad (22)$$

$$\begin{bmatrix} Z & P \\ P & I \end{bmatrix} > 0 \,,$$

$$0 \leq Z \leq \varrho^2 \,. \qquad (23)$$

Another way how to find the solution of (10) directly may be as follows. With quadratic form $G^\top G$, where $G = A + BFC + P$, inequality (10) could be rewritten as follows

$$- PP - [A^\top A + A^\top BFC + (BFC)^\top A] + G^\top G + C^\top F^\top (R - B^\top B) FC < 0 \,. \qquad (24)$$

With condition (19), LMI solution of (24) is given as follows

$$\begin{bmatrix} L_{ki} & G_{vi}^\top & C_{vi}^\top F^\top \\ G_{vi} & -I & 0 \\ FC_{vi} & 0 & -(R - B_{vi}^\top B_{vi})^{-1} \end{bmatrix} < 0$$

where

$$\begin{aligned} L_{ki} = F_{vi}(Z, P_{vi}, D_{vi}) - [A_{vi}^\top A_{vi} + A_{vi}^\top B_{vi} F C_{vi} \\ + (B_{vi} F_{vi} C_{vi})^\top A_{vi}] + Q\,, \\ 0 < P_{vi} < \varrho I\,, \quad i = 1, 2, \dots, N\,. \quad (25) \end{aligned}$$

In the above equations instead of $F_{vi}(Z, P_{vi}, D_{vi})$ one can use $F_{vi}(Z)$ (21). If the solutions (18) or (24) are feasible with respect to Z, $P_{vi}, i = 1, 2, \dots, N$ and F, then the uncertain system (5) is parameter-dependant quadratically stable (PDQS) with a guaranteed cost control algorithm

$$u = Fy$$

and

$$J^* = x_0^\top P x_0$$

or

$$J^* = max_i(x_0^\top P_{vi} x_0)$$

is the guaranteed cost for the uncertain closed loop system.

4 EXAMPLES

In this example we consider the linear model of two cooperating DC motors. The problem is to design two PI controllers for a laboratory MIMO system which will guarantee PDQS of a closed loop uncertain system with guaranteed cost. The system model is given by (5) with a time invariant matrix affine type uncertain structure, where

$$A_0 = \begin{bmatrix} 0 & -.2148 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1.014 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.2605 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -.9107 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.1639 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -.8137 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.2279 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -.8251 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0 & -.025 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.1395 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.0938 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.2911 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .0188 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .0208 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.0333 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.1173 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & .0125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .0594 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .0116 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .0308 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.0188 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.0156 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & .0208 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.0333 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} .3148 & 0 \\ .0478 & 0 \\ 0 & -.1028 \\ 0 & -.0091 \\ -.0841 & 0 \\ -.0287 & 0 \\ 0 & .3676 \\ 0 & .2448 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} .0625 & 0 \\ -.0798 & 0 \\ 0 & -.0462 \\ 0 & -.0449 \\ .0016 & 0 \\ .0072 & 0 \\ 0 & .077 \\ 0 & -.005 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -.0094 & 0 \\ .0151 & 0 \\ 0 & .0019 \\ 0 & -.003 \\ -.0121 & 0 \\ -.03 & 0 \\ 0 & -.064 \\ 0 & .0189 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad C^\top = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The number of polytope systems is equal to 4 and the polytope vertices are computed for different values of the two variables θ_1, θ_2 alternatively taken at their maximum $\overline{\theta_i}$ and minimum $\underline{\theta_i}, i = 1, 2$. The decentralized control structure for the two PI controllers can be obtained by the choice of the static output feedback gain matrix F structure. It is given as follows

$$F = \begin{bmatrix} f11 & 0 & f13 & 0 \\ 0 & f22 & 0 & f24 \end{bmatrix}.$$

The results of calculation of a static output feedback gain matrix F for PDQS for different $Q = qI$, $R = rI$, $\theta_m = |\theta_1| = |\theta_2|$, ρ and ρ_1 are summarized as follows.

- Eqs. (18) and (20).
  For $\theta_m = 1$, $q = .00001$, $r = 1$ and $\rho = 50$ after 100 repeated calculations, that is $D_{vi}^{j+1} = P_{vi}^j$, $i = 1, 2, 3, 4$ and $j = 1, 2, \dots, 100$ gain matrix F, maximal closed-loop eigenvalue of all four polytopic systems $maxeig(CL)$ and the value of guaranteed cost $\lambda_M(P_{vi})$

  $$\max_i x_0^\top P_{vi} x_0 \le \max_i \|x_0\|^2 \lambda_M(P_{vi})$$

  are equal

  $$F = \begin{bmatrix} -1.6997 & 0 & -.1148 & 0 \\ 0 & -1.7519 & 0 & -.1863 \end{bmatrix},$$

$maxeig(CL) = -.0479$ and $\lambda_M(P_{vi}) = 48.2923$.

If the number of repeated calculations increases to 120, the results of calculations are changed as follows

$$F = \begin{bmatrix} -1.5191 & 0 & -.1427 & 0 \\ 0 & -1.8035 & 0 & -.2289 \end{bmatrix},$$

$maxeig(CL) = -.0658$ and $\lambda_M(P_{vi}) = 47.3849$. For the same case, V-K iterative method [4] gives

$$maxeig(CL) = -.0636\,,$$

$$F = \begin{bmatrix} -.8837 & 0 & -.1234 & 0 \\ 0 & -.7106 & 0 & -.0909 \end{bmatrix}.$$

All LMI solutions are feasible.

- Eqs. (25) and (20).
  For the same parameters as given above, after 120 repeated procedures the results of calculation are as follows.

  $$F = \begin{bmatrix} -2.2247 & 0 & -.3757 & 0 \\ 0 & -3.47776 & 0 & -.7672 \end{bmatrix},$$

  $maxeig(CL) = -.1424$ and $\lambda_M(P_{vi}) = 47.505$. For $\theta_m = 1.9$ one can obtain the following results

  $$F = \begin{bmatrix} -.8515 & 0 & -.3548 & 0 \\ 0 & -1.8185 & 0 & -.301 \end{bmatrix},$$

  $maxeig(CL) = -.1142$ and $\lambda_M(P_{vi}) = 49.7532$. For V-K iterative procedure for maximal closed-loop eigenvalue of all four polytopic systems we obtain $maxeig(CL) = -.00020036$. All LMI solutions are feasible.
- Eqs. (25) and (21).
  For the same parameters that are given in the first case and $\varrho = 50$, $\varrho_1 = 50/1.3$ we have got stable polytope systems with $maxeig(CL) = -.0062$ but the cost is not guaranteed. LMI solution is not feasible.

The second example has been borrowed from [1] to demonstrate the use of the algorithm given by (25) and (20). It is known that the presented system is static output feedback stabilizable. Let (A, B, C) in (1) be defined as

$$A = \begin{bmatrix} -0.036 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.010 & 0.0024 & -4.0208 \\ 0.1002 & q_1(t) & -0.707 & q_2(t) \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.4422 & 0.1761 \\ q_3(t) & -7.59222 \\ -5.520 & 4.490 \\ 0 & 0 \end{bmatrix}, \quad C = [0 \quad 1 \quad 0 \quad 0]$$

with parameters bounds $-0.6319 \le q_1(t) \le 1.3681$, $1.22 \le q_2(t) \le 1.420$, and $2.7446 \le q_3(t) \le 4.3446$. Find a stabilizing output feedback matrix F. The four vertices are calculated. The nominal model of (A, B) is given by the above matrices when we substitute for the entries $A(3,2) = 0.3681$, $A(3,4) = 1.32$ and $B(2,1) = 3.5446$. The affine model uncertainty (5) (A_1, A_2, B_1, B_2) are matrices with the following entries $A_1(3,2) = 1$, $A_2(3,4) = 0.1$ and $B_1(2,1) = 0.8$, $B_2 = 0$ with $\theta_i \in \langle -1, 1\rangle$, $i = 1,2$. Other entries of the above uncertain matrices are equal to zero. The nominal model is unstable with eigenvalues:

$$eig\{-2.0516, 0.2529 \pm 0.3247i\,,\ -0.2078\}\,.$$

Let the structure of F be defined as

$$F^\top = [F(1,1) \quad F(2,1)]\,.$$

- Eqs. (25), (20).
  For $Q = q * I$, $q = .00001$, $R = r * I$, $r = 123.3$, $\varrho = 100$ and $\theta_m = 1$ one obtain the following results.

  $$F^\top = [1.5801 \quad 3.6552]\,,$$
  $$maxeig(CL) = -.0707\,, \quad \lambda_M(P_{vi} = 93.5437\,.$$
- For $\theta_m = 2$, $r = 132.2$ results are as follows.

  $$F^\top = [1.6508 \quad 3.6051]\,,$$
  $$maxeig(CL) = -.0691\,, \quad \lambda_M(P_{vi} = 94.4187\,.$$
- For $\theta_m = 1$, $r = 123.3$, $\varrho = 100$ and different q the following results are obtained.

  a. $q = .1$

  $$F^\top = [1.931 \quad 3.6552]\,,$$
  $$maxeig(CL) = -.0686\,, \quad \lambda_M(P_{vi}) = 93.683\,.$$

  b. $q = 1$

  $$F^\top = [2.1851 \quad 3.6497]\,,$$
  $$maxeig(CL) = -.0663\,, \quad \lambda_M(P_{vi}) = 94.786\,.$$

  c. $q = 8$

  $$F^\top = [2.764 \quad 3.3253]\,,$$
  $$maxeig(CL) = -.0661\,, \quad \lambda_M(P_{vi}) = 99.011\,.$$

All LMI solutions are feasible. Note that for the above parameters the V-K iterative method does not give any reasonable solutions. Usually, the repeated procedure generates less conservative results than the first one. The convergence of the above special repeated procedure has not been proven yet, however, if the reasoning of [4] is taken account we can conclude that the proposed algorithm is guaranteed to converge but not necessarily to the global optimum of the problem, depending on the starting conditions.

5 CONCLUSIONS

In this paper, we have proposed a new procedure for robust output feedback controller design for linear systems with affine parameter uncertainty. The feasible solution of the proposed output feedback controller design procedure with sufficient conditions guarantees the parameter dependent Lyapunov function quadratic stability and guaranteed cost. The design procedure is based on necessary and sufficient conditions for output feedback stabilizability of linear systems and a non-iterative LMI based algorithm.

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