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ANALYSIS OF THE SWITCHED RELUCTANCE MOTOR (SRM) PARAMETERS

Pavol Rafajdus [*] — **Ivan Zrak** [**] — **Valéria Hrabovcová** [*]

In the paper the model for analysis of the Switched Reluctance Motor (SRM) parameters is described. The inductances L, linkage flux Ψ, and torque T are calculated on the basis of geometrical dimensions and $B-H$ curves of the material. As the outputs the waveforms of the L, Ψ and T *vs* current and rotor position are gained.

Keywords: switched reluctance motor, analytical model, aligned and unaligned inductance, flux linkage, torque production, rotor position.

1 INTRODUCTION

To be able to investigate the SRM performances, including the design of the control circuits, it is necessary to know its parameters, such as inductance, flux linkage and torque. There are more possibilities how to do it, *eg* by measurements or calculation on the basis of electromagnetic field analysis. For this purpose it is possible to use the Finite Element Method (FEM) or to develop an analytical model. This paper is focused on the development of such an analytical model allowing calculation of SRM parameters on the basis of geometrical dimensions of its cross-section area and $B-H$ curve of the used iron material. This analytical model employs a numerical iteration to find the magnetic reluctance along the magnetic flux path and hence the inductance changing its value from aligned to unaligned rotor position for various values of excitation current. The inductance is then used to calculate the flux linkage and hence the torque given by the co-energy varying during the rotor movement.

2 ANALYTICAL MODEL

The model is developed for the 3-phase, 12/8 SRM, shown in Fig. 1. Its rating is as follows: 3.7 kW, 11.8 Nm, 3000 rpm, 540 V.

The motor has a 4-pole magnetic field, which means that the angle between aligned and unaligned positions is 22.5°. The rotor position $\theta = 0^\circ$, when the axis of the excited stator pole is identical with the axis of rotor pole, is defined as aligned [1]. The position $\theta = 22.5^\circ$, when the axis between two rotor poles is identical with the axis of the excited stator pole, is defined as unaligned position [1]. In the sequel, a detailed procedure for calculating the inductance and other parameters will be described.

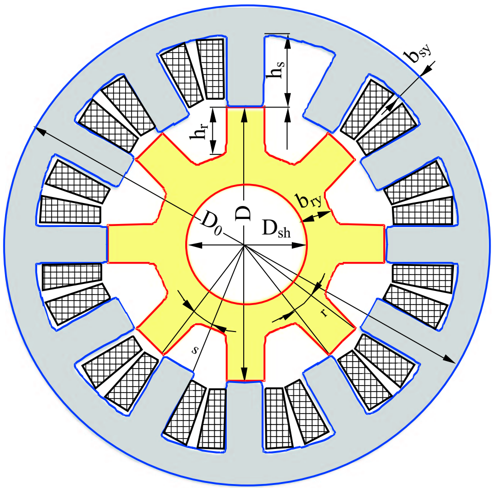

Fig. 1. 12/8 SRM cross-section.

2.1 Calculation procedure

The well-known expression for inductance calculation will be used:

$$L = \frac{(2N_c)^2}{R} \tag{1}$$

where N_c is the number of the turns of the excited coil, $2N_c$ means that inside the closed magnetic path there are two times N_c. R is the reluctance of the magnetic circuit in which the inductance is calculated. Since it is a complicated magnetic circuit, consisting of some parts, such as stator pole denoted with subscript sp, stator yoke (sy), air gap (δ), rotor pole (rp), rotor yoke (ry), the total reluctance will be the sum of single reluctances along the magnetic path. The reluctances will be calculated by means of geometrical dimensions and magnetic permeability:

$$R = \frac{l}{S\mu} = \frac{Hl}{BS} \tag{2}$$

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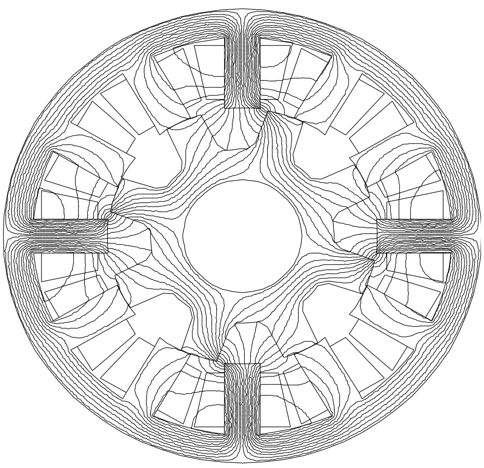

Fig. 2. Plotting of magnetic flux distribution in the 12/8 SRM cross-section.

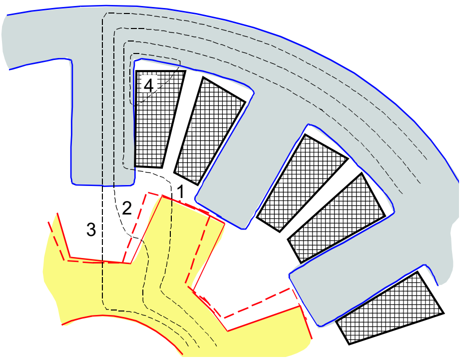

Fig. 3. Typical magnetic flux lines.

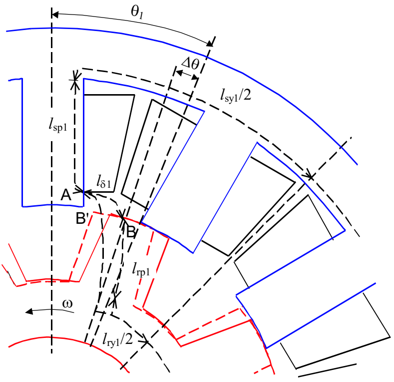

Fig. 4. Typical fux line No. 1 for two rotor positions shifted by angle $\Delta\theta$.

where l is the length of the magnetic path, S is the area which is penetrated by the magnetic flux, magnetic permeability μ is given by the values of B and H in the $B-H$ curve of the used material.

As the magnetic flux and, hence, the magnetic flux density B are given by the reluctance R, in a ferromagnetic material both R and B are unknown. This problem will be solved by means of magnetomotive force balance, given by Ampere's Circuit Law [2], which can be written for the SRM magnetic circuit in the form:

$$F = 2N_c i = \oint \vec{H} d\vec{l} = \sum_k H_k l_k \tag{3}$$

where k is gradually sp, sy, rp, ry, δ with their length l of the magnetic path and corresponding H (see also Fig. 6). If on the left side a product of the exciting current and of the number of turns equals to the product of magnetic field intensity H and the length of magnetic path l on the right side, then their difference is zero:

$$2N_c i - \sum_k H_k l_k = 0\,.$$

If this is not true, there will be a discrepancy, or better to say an error

$$\Delta F = 2N_c i - \sum_k H_k l_k\,. \tag{4}$$

The purpose is to look for such values of i and H, to minimize the ΔF, in ideal case to zero. It can be gradually made by means of numerical iteration with a prescribed value of error ΔF. If the prescribed accuracy is achieved, then for a given exciting current i the magnetic field intensity H on the investigated length l of magnetic circuit is known.

Then on the basis of the $B-H$ curve of the used ferromagnetic material the magnetic flux density B is determined and inserted into (2) to calculate reluctance R and then inductance according (1). In the next the calculation will be shown in detail.

2.2 Length of magnetic flux line and flux area calculation

To explain in greater details the calculation procedure an example will be given in this chapter. In Fig. 2 the plot of the magnetic flux distribution in the SRM cross-section area, gained by FEM, is shown.

For simplicity, all magnetic flux lines are divided in at least four typical groups, in which their form and length are very similar (Fig. 3). For each group one typical flux line is chosen. Its length will be calculated with the changing of the rotor position. Each typical flux line is divided onto intervals, corresponding to individual SRM parts (sp, sy, rp, ry, δ). The length of individual intervals is given as a sum of lines and arches creating the form of the flux line. The total length of one typical flux line is given by the sum of the lengths along all parts of the machine. The calculation starts at $\theta = 22.5°$, corresponding to the unaligned rotor position, and continues in the direction of rotor movement ω to the $\theta = 0°$ corresponding to the aligned rotor position. As an example some expressions derived for rotor position θ_1 of the typical flux line

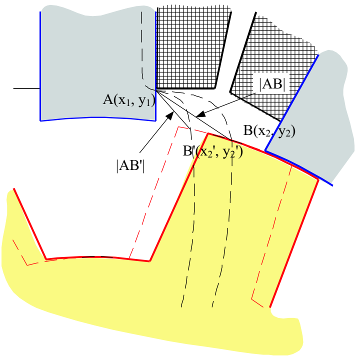

Fig. 5. Detail of flux line No. 1 in air-gap.

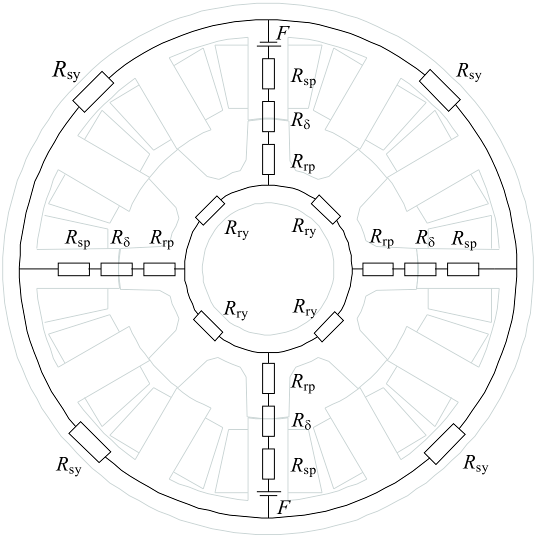

Fig. 6. Magnetic equivalent circuit for 4 — pole SRM, created for flux line No. 1.

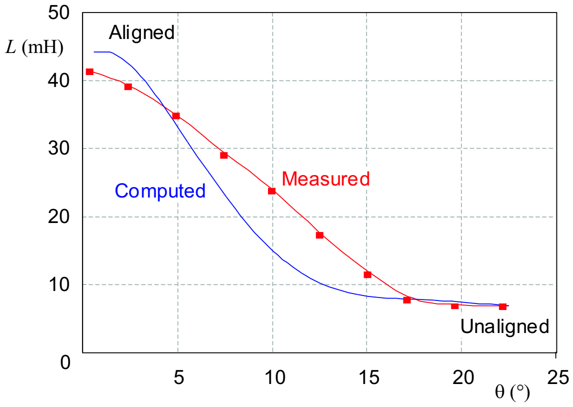

Fig. 7. Comparison of inductance calculated and measured values for current 3A.

No. 1 from Fig. 3 will be given. In Fig. 4, the flux line No. 1 is drawn for two rotor positions, shifted by angle $\Delta\theta$.

The length in the air-gap $l_{\delta 1}$ is given as an arch between points A and B, as it is seen in Fig. 5. The coordinates of points $A(x_1, y_1)$ and $B(x_2, y_2)$ are used for its calculation:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{5}$$

and then

$$l_{\delta 1} = arc\,AB = |AB|\frac{\pi}{2} \tag{6}$$

The lengths in the stator pole (see also Fig. 1 and Fig. 4)

$$l_{sp1} = \frac{7}{8}h_s \tag{7}$$

in the rotor pole

$$l_{rp1} = h_r \tag{8}$$

in the rotor yoke

$$l_{ry1} = \frac{2\pi}{n}\left(\frac{D}{2} - \delta - h_r\right) \tag{9}$$

when n is the number of coils connected in series, and the length in the stator yoke is:

$$l_{sy1} = \frac{2\pi}{n}\left(\frac{D}{2} + h_s + \frac{b_{sy}}{2}\right). \tag{10}$$

The second step is to determine the areas, which are penetrated by the magnetic flux belonging to individual typical flux lines. The area in the air-gap for flux line No. 1 is

$$S_{\delta 1} = \left[\frac{h_s}{4} + \frac{\beta_r}{4}\left(\frac{D}{2} - \delta\right)\right]\frac{l_{Fe}}{2} \tag{11}$$

in the stator pole

$$S_{sp1} = \frac{h_s}{4}l_{Fe} \tag{12}$$

in the rotor pole

$$S_{rp1} = \frac{\beta_r}{2}\left(\frac{D}{2} - \delta\right)l_{Fe} \tag{13}$$

in the rotor yoke

$$S_{ry1} = \left(\frac{D}{2} - \delta - h_r - \frac{D_{sh}}{2}\right)l_{Fe} \tag{14}$$

in stator yoke

$$S_{sy1} = b_{sy}l_{Fe} \tag{15}$$

where l_{Fe} is the axial length of the SRM.

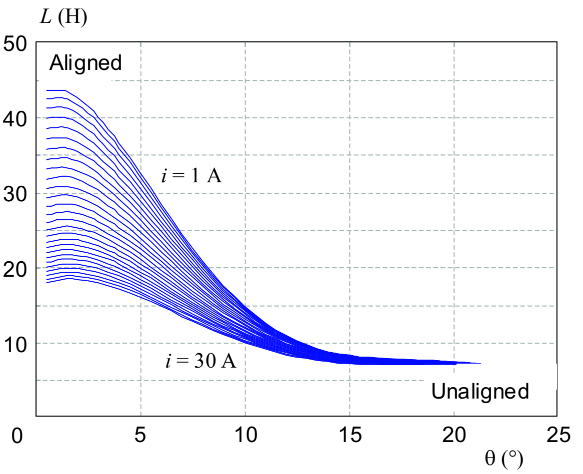

Fig. 8. Calculated inductance versus rotor position for various currents.

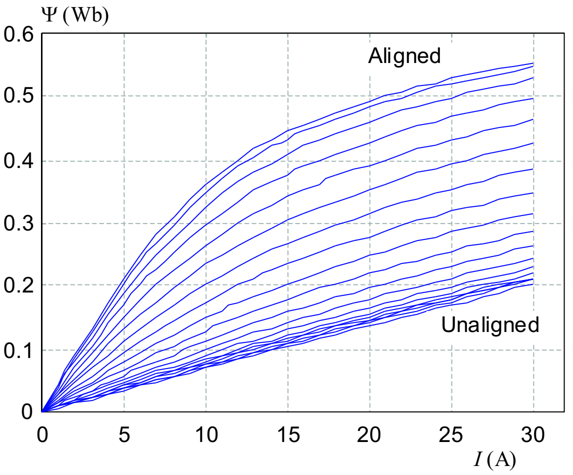

Fig. 9. Flux linkage versus rotor position for various currents.

2.3 Reluctance and inductance calculation

Now it is possible to calculate reluctances [3]. The reluctance in the stator pole for flux line No. 1 is

$$R_{sp} = \frac{l_{sp}}{S_{sp}\mu} = \frac{H_{sp} l_{sp}}{B_{sp} S_{sp}} \tag{16}$$

where H_{sp1} and B_{sp1} are determined by the procedure described in 2.1.

The reluctances in the other parts are calculated in a similar way. Finally, the inductance for flux line No. 1 is calculated as follows:

$$L_1 = \frac{(2N_c)^2}{(2R_{sp1} + 2R_{\delta 1} + 2R_{rp1} + R_{sy1} + R_{ry1})} \tag{17}$$

which is based on the magnetic equivalent circuit seen in Fig. 6, where F is given by (3).

The same procedure was applied to all typical flux lines and inductances L_2, L_3, L_4 have been calculated. Then the total inductance for the given rotor position θ_1 can be gained as follows:

$$L = \sum_{i=1}^{4} L_i \,. \tag{18}$$

Now the same calculation will be repeated for the next rotor position θ_2, *etc.* As the outputs, the values of inductance versus rotor position will be gained, from aligned to unaligned position, as it is seen in Fig. 7.

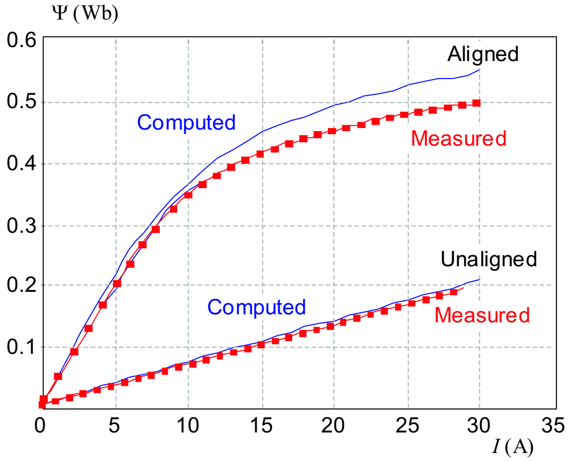

Fig. 10. Comparison of calculated and measured flux linkage values for unaligned and aligned rotor position versus current.

In Fig. 7 there is a comparison of the calculated and measured inductance versus rotor position for a current of 3 A. As it is seen, the coincidence is very good in unaligned position, quite good in aligned position, but certain discrepancy is in the positions where the rotor poles begin to overlap with the stator excited poles. Therefore some corrections must be made and special expressions derived for the region with pole overlapping. Under these conditions the inductance calculation for various currents between 1 and 30 A was made. As it is shown in Fig. 8, the waveforms confirm the prediction that the inductance does not change its value in the unaligned position where the air-gap is very big and no iron saturation takes place in the magnetic circuit. But in the aligned position, where the air-gap is very small, the iron saturation causes the changing of the inductance value with current: the higher the current, the lower the inductance. Therefore the inductance is a function of current and rotor position $L(i, \theta)$.

3 FLUX LINKAGE WAVEFORMS

As it is known, flux linkage and inductance are linked in the expression

$$\psi(\theta, i) = \frac{L(\theta, i) i}{N_c} \,. \tag{19}$$

Therefore if $L(i, \theta)$ is calculated by the procedure described in the previous chapter, on the basis of (19) it

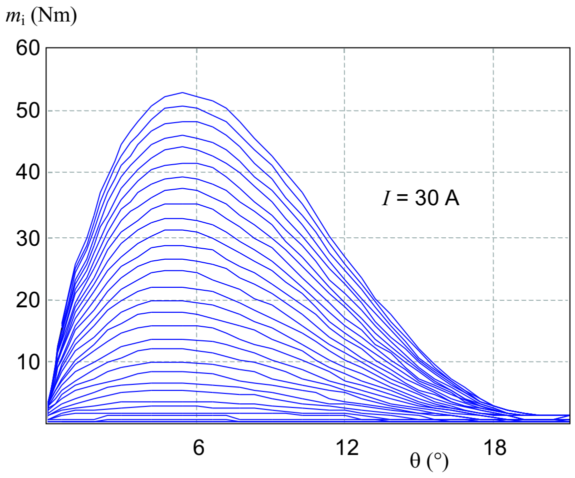

Fig. 11. Calculated torque versus rotor position for various currents.

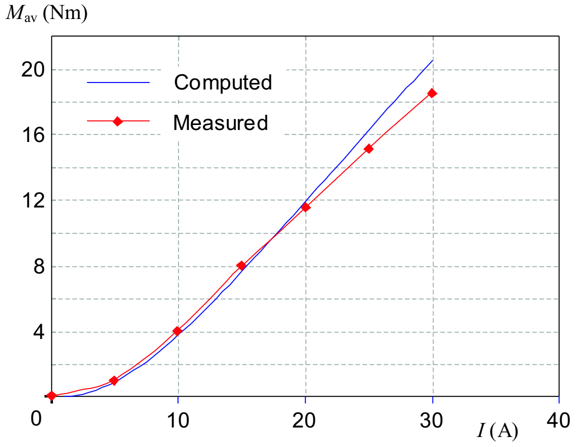

Fig. 12. Comparison of the calculated and measured values of average torque versus current.

is possible to calculate the flux linkage and its dependence on the rotor position and current [4]. The results are shown in Fig. 9. As it is seen, in the region close to the unaligned position the values of flux linkage are almost unchanged because of almost constant inductance and big air-gap.

To verify the calculated values of the flux linkage waveforms it is possible to compare them with the measured values. In Fig. 10 there are calculated and measured flux linkage values for unaligned and aligned position versus current. As it is seen, the coincidence is very good for unaligned position in the whole range of currents and also in aligned position for small currents where saturation does not take place. The discrepancy is increased with higher currents, when the saturation effect is dominant.

Most important for SRM performance prediction is the calculation of the torque. Therefore the gained values of the flux linkage will be used for torque calculation.

4 TORQUE PREDICTION

As it is known from the SRM theory [3], the instantaneous torque value can be calculated by means of expression

$$m_i(i,\theta) = \frac{\partial \int_0^i \Psi(i,\theta)\mathrm{d}i}{\partial\theta}\,. \tag{20}$$

The results are shown in Fig. 11.

Then the average torque values can be calculated in dependence on current. If the torque cogging is not important for performance prediction, it is enough to calculate the average values by means of the expression

$$M_{av} = \frac{1}{\theta_u}\int_0^{\theta_u} m_i \mathrm{d}\theta \tag{20}$$

here θ_u is the angle for unaligned position, $\theta_u = 22.5°$. The results are shown in Fig. 12.

To verify the rightness of the calculated values there are put also the measured values (see [4]). As it is seen, the coincidence of the calculated and measured values is very good in the whole range of current and the error does not exceed 10.2 % of the measured value. This means that the presented analytical method for inductance calculation using the description of the magnetic flux path and the areas under which magnetic flux penetrates the individual parts of the iron and air-gap is suitable to predict the performances of the SRM, mainly its torque production.

5 CONCLUSION

The paper presents an analytical method for calculation of some important SRM parameters such as inductance, flux linkage and torque. The calculation is based on the description of the magnetic flux line profile and its length and the areas under which the magnetic flux penetrates the iron and air-gap parts. The paper shows the procedure how to define typical magnetic flux lines and formulates the expression for their lengths and corresponding areas to be able to calculate the reluctance and, hence, inductance for a certain current and rotor position. The calculation continues with the values of flux linkage and torque production, which is the aim of the whole procedure.

The comparison with the measured values shows that the coincidence is very good. It means that the presented method could be employed for the SRM design procedure to predict its performances. The method has some advantages in comparison with FEM. First, the calculation time is much shorter and it is very easy to make changes of geometrical dimensions in the calculation procedure. Second, all outputs can be presented in graphical forms as well as data files which can be used in further calculations.

As a disadvantage can be regarded the fact that the description of the flux line profile is a bit complicated and cannot be done very precisely.

On the other hand, it is important that in spite of some inaccuracy in inductance calculation the calculated torque values are in good coincidence with the measured values. Therefore we can recommend to use this method in the SRM design procedure.

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