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ROBUST CONTROLLER DESIGN TO CONTROL A WARM AIR–DRYING CHAMBER

Ján Danko * — Magdaléna Ondrovičová * — Vojtech Veselý **

The aim of the contribution is the presentation of a robust controller design for a warm air-drying chamber. Static and dynamic characteristics of the system are identified, and the robust controller is designed for linear interval system and affine linear SISO systems using extremal transfer functions and D partition. The proposed algorithms are computationally simple and utilize the well-known approaches of the control theory only.

K e y w o r d s: Warm air-drying chamber, static and dynamic characteristics, robust controller, affine system, extremal transfer functions, D-partition

1 INTRODUCTION

The warm air-drying chamber is one of the real processes built at the Department of Information Engineering and Process Control. The technical equipment allows to control the drying process by the classical and advanced control methods. The drying chamber at the present time is used to dry cuprexid plates with a photoactive emulsion. The warm air-drying chamber connection with the computer is in Fig. 1 [1]. Ventilator 2 to chamber 1 transports the air. Important parameters of the air flow are the temperature and moisture. Electric heaters 3 heat the air. The power consumption of each electric heater is 800 W. Behind the electric heaters are situated the products of drying. The values of temperature and moisture of the drying air are measured by temperature and humidity sensors 5, 6. Signals of the sensors are taken into control computer 9 through converters 7 and 8. The control computer controls by the triac controller 10 the power consumption of the electric heaters. Flaps 11 control the air flow fed through the drying chamber. The input-output card PCL 812 PG ensures communication between the warm air-drying chamber and the control computer. The computer software Real Time Toolbox for Matlab allows to control the process in real time.

2 REAL PLANT IDENTIFICATION

The warm-air drying chamber was identified by step responses. They were measured in five working states by the step change of the power from 0-20 % to 80-100 % of the range. The step responses were identified by the Hudzovič method and the transfer functions are in the form:

$$\text{0-20\%} \qquad G(s) = \frac{10.6}{12893s^3 + 8058s^2 + 226.7s + 1} \tag{1}$$

$$\text{20-40\%} \qquad G(s) = \frac{10.5}{2078s^3 + 10346s^2 + 230.9s + 1} \tag{2}$$

$$\text{40-60\%} \qquad G(s) = \frac{11.4}{40184s^3 + 10506s^2 + 255.6s + 1} \tag{3}$$

$$\text{60-80\%} \qquad G(s) = \frac{12.8}{13681s^3 + 11606s^2 + 279.4s + 1} \tag{4}$$

$$\text{80-100\%} \qquad G(s) = \frac{8}{32324s^3 + 10370s^2 + 276.9s + 1} \tag{5}$$

The identified and measured step response is in Fig. 2. The comparison of the step responses confirms the convenience of the identification method.

Based on the above, we can obtain the transfer function parameters of two models of uncertainty — affine model and linear interval model. The transfer function of the linear interval model is given as follows

$$G(s) = \frac{B(s)}{A(s)} = (8 - 12.8)\big/\big((12893 - 32324)s^3 + (8058 - 11606)s^2 + (226.7 - 279.4)s + 1\big) \tag{6a}$$

and for the warm air-drying chamber the following affine transfer function is obtained

$$G(s) = \frac{\overline{b}_0 + \overline{b}_1 q_1 + \overline{b}_2 q_2}{\overline{a}_0 + \overline{a}_1 q_1 + \overline{a}_2 q_2} \tag{6b}$$

where $\overline{b}_0 = 11.45$, $\overline{b}_1 = 2.2$, $\overline{b}_2 = 1.25$,
$\overline{a}_0 = \big(3.0131s^3 + 1.0426s^2 + 0.00243s + 0.0001\big)\times 10^4$,
$\overline{a}_1 = \big(3.93s^3 + 0.068s^2 + 0.0106s\big)\times 10^3$,
$\overline{a}_2 = \big(-6.123s^3 - 0.012s^2 - 0.023s\big)\times 10^3$.

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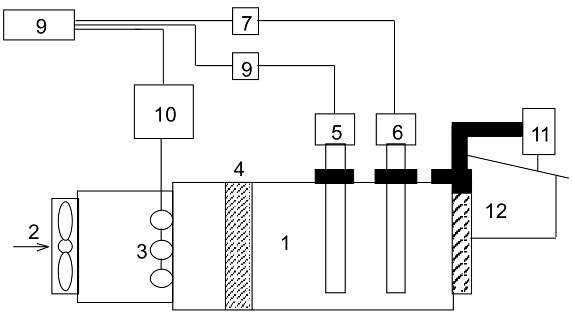

Fig. 1. Control scheme of the drying chamber.

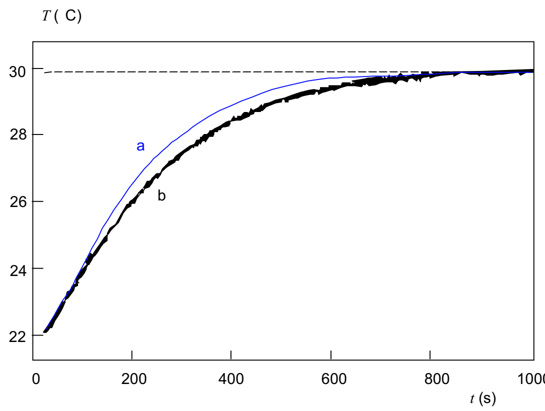

Fig. 2. Step response of the drying chamber: a – step response obtained by simulation, b – step response obtained by measurement.

3 ROBUST CONTROLLER DESIGN

3.1 First approach

Let the transfer function of the robust controller be

$$R(s) = \frac{C(s)}{D(s)} \tag{7}$$

where $C(s)$ and $D(s)$ are polynomials with constant coefficients. The characteristic equation of the closed loop system is in the form

$$p(s) = B(s)C(s) + A(s)D(s) .$$

Polynomials $B(s)$ and $A(s)$ are linear on the interval and from this point of view the closed loop control is described by an infinite number of characteristic equations. The set of characteristic equations will be referred to as the family of characteristic equations. In general, if we describe the characteristic equation of the closed-loop in the form

$$p(s) = F_1(s)P_1(s) + F_2(s)P_2(s) , \tag{8}$$

for a general case we can obtain

$$p_1(s) = \sum_{i=1}^{p} F_i(s)P_i(s) , \tag{9}$$

where

$$p_i(s) = p_{0i} + p_{1i}s + \dots + p_{di}s^{di} , \tag{10}$$

$p_{ij} \in \langle p_{ji}, \overline{p}_{ji} \rangle$, $j = 0, 1, \dots, d_i$, $i = 1, \dots, p$.
$F_i(s)$, $i = 1, 2, \dots, p$, is a polynomial with constant coefficients.

The i^{th} uncertainty box of the i^{th} polynomial is given as follows

$$Q_i = \left\{ p_i : \underline{p_{ij}} \le p_{ij} \le \overline{p_{ij}}, \quad j = 0, 1, \dots, d_i \right\} , \tag{11}$$

where p_i is the parameter vector of polynomial $P_i(s)$. The uncertainty box for the family of characteristic equations is

$$Q = Q_1 \times Q_2 \times \dots \times Q_p . \tag{12}$$

We can say that $F(s) = [F_1(s), \dots F_2(s)]$ stabilizes $P(s) = [P_1(s), \dots P_2(s)]$ if the family of characteristic equations

$$p(s) = \{ \langle F(s), P(s) \rangle : p \in Q \} , \tag{13}$$

where $p = [p_1, \dots p_p]$ is stable.

In our case $p = 2$. The stability of the family of characteristic equations (13) is solved by the Generalized Kharitonov theorem [2]:

THEOREM 1. *Let a polynomial vector* $F(s) = [F_1(s) \dots F_2(s)]$ *with constant coefficients be given.* $F(s)$ *stabilizes the family of polynomials* $P(s) = [P_1(s) \dots P_2(s)]$, *when the extremal characteristic equation*

$$\Delta_E(s) = \bigcup_{l=1}^{2} \Delta_E^l(s) \tag{14}$$

is stable.

In equation (14) let us denote

$$\begin{aligned} \Delta_E^1(s) &= F_1(s)S_1(s) + F_2(s)K_2(s) \\ \Delta_E^2(s) &= F_1(s)K_1(s) + F_2(s)S_2(s) \end{aligned} \tag{15}$$

where $K_i(s)$, $S_i(s)$, $i = 1, 2$ are Kharitonov polynomials and segments of the polynomial $P_i(s)$.

On the basis of (15), if $F_1(s)$, $F_2(s)$ are the controller polynomials with constant coefficients then the extremal transfer function for linear interval system is given as follows

$$G_E(s) =$$

$$\left\{ \frac{K_1^i(s)}{\lambda K_2^j(s) + (1-\lambda)K_2^k} ; \frac{\lambda K_1^i(s) + (1-\lambda)K_1^j(s)}{K_2^k(s)} \right\}, \tag{16}$$

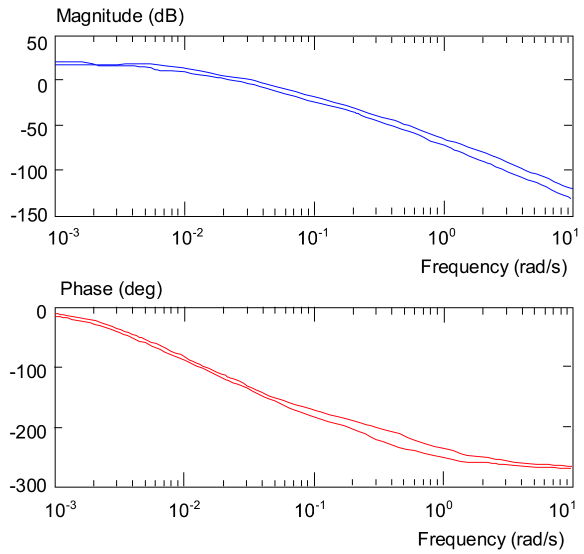

Fig. 3. The Bode frequency characteristics of the interval system

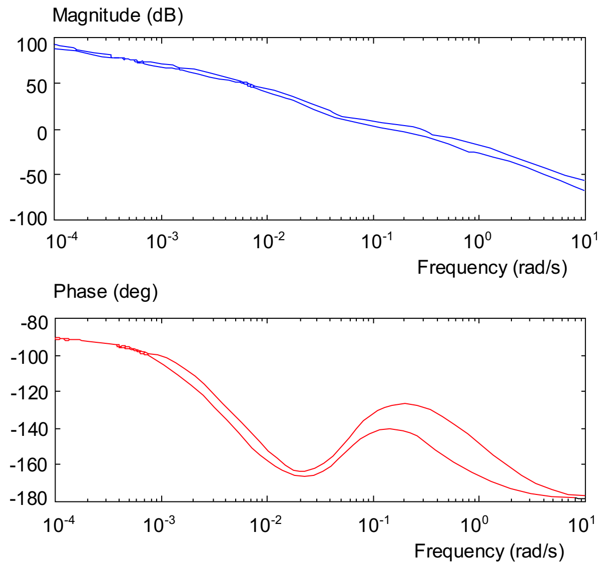

Fig. 4. The Bode diagram of the interval system plant with PID controller

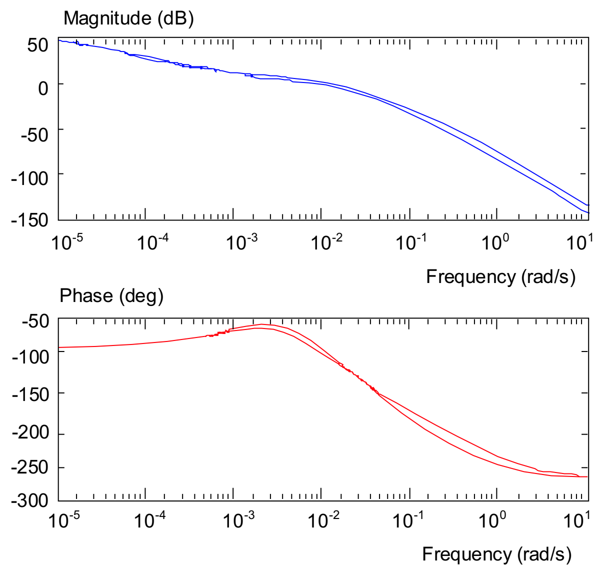

Fig. 5. The Bode diagrams of the interval system plant with PI controller.

where $K_i(s) = \left[K_i^1(s)\, K_i^2(s)\, K_i^3(s)\, K_i^4(s)\right]$, $i = 1, 2$ is the vector of Kharitonov nominator and denominator polynomials of the system and $\lambda \in \langle 0, 1\rangle$ is an unknown parameter. On the basis of the extremal transfer function (16) which solves the closedloop stability question with a general type of controller (7) PID and PI controllers were designed that stabilize the linear interval system (6). The D-curves method has been used for the design of the robust controller. This method is generally well-known and from this point of view we will publish only the obtained results.

For the transfer function of the controller in the form

$$R(s) = k\Big(1 + \frac{1}{T_i s} + T_d s\Big)$$

the following parameters were obtained by the D-curves method:

$k = 15$, $T_i = 37.5$ s, $T_d = 11.66$ s.

The maximum value of the real part roots of the extremal characteristic equations (13) is $\operatorname{Re}\lambda_M = -0.04709$. If the value of the derivative part of the controller is decreasing, the change of the $\operatorname{Re}\lambda_M$ is

$$T_d = 6.66\, s \qquad \operatorname{Re}\lambda_M = -0.0183$$

$$T_d = 3.33\, s \qquad \operatorname{Re}\lambda_M = \text{unstable process}$$

The dynamics of the process is relatively fast and when the closed-loop output signal is too noised, the designed gain of the derivative part may make the process control impossible. The Bode frequency characteristics of the interval system plant without controller are in Fig. 3. The Bode frequency characteristics of the interval plant with designed PID controller are in Fig. 4.

The PI controller was designed on the basis of the Bode frequency characteristics with demanded phase margin $p_M = 70^\circ$. The designed parameters of the PI controller are

$$R(s) = 0.205\Big(1 + \frac{1}{833.33s}\Big)$$

and the Bode diagram of the interval system plant with designed PI controller is in Fig. 5.

The closed-loop dynamics with the designed controller is expressively sluggish but the control loop quality with appearance to safety in phase for the first (20°) and second (70°) case is better. Practical experience with the control of the warm air-drying chamber with designed controller parameters are in Fig. 6 and Fig. 7. The control responses certificate the ability of using this method to control of the real process.

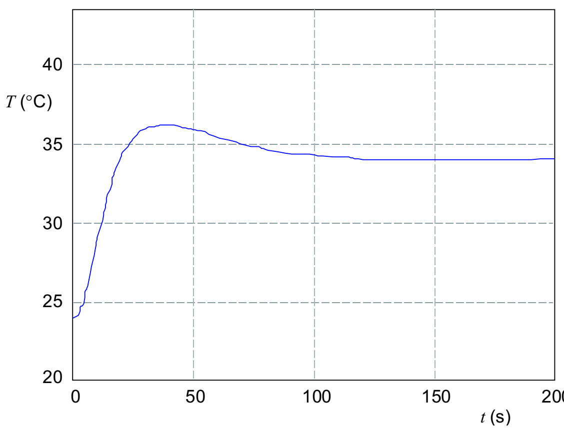

Fig. 6. Control with PID controller

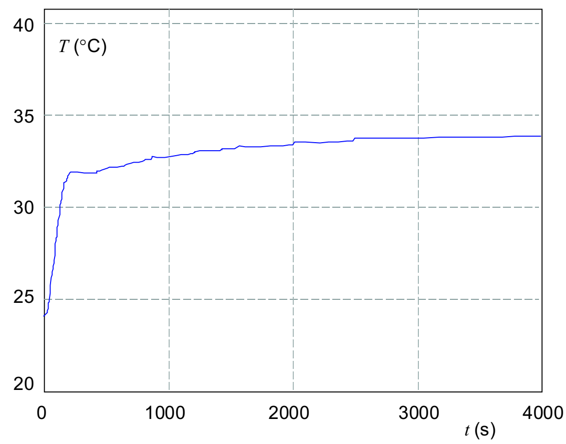

Fig. 7. Control with PI controller

3.2 Second approach

We have the affine model (6b) in the form

$$G(s) = \frac{b_0(s) + \sum_{i=1}^{p} q_i b_i(s)}{a_0(s) + \sum_{i=1}^{p} q_i b_i(s)} = \frac{B(s)}{A(s)}, \tag{17}$$

where the uncertainty box Q is defined as

$$Q = \{q_i \colon q_i \langle \underline{q_i}\ \overline{q_i}\rangle,\ i = 1, 2, \ldots, p\}\ . \tag{18}$$

The control problem can be formulated as follows:

Design the structure and parameters of the robust controller

$$R(s) = \frac{C(s)}{D(s)} \tag{19}$$

such that the closed-loop system

$$G_c(s) = \frac{R(s)G(s)}{1 + R(s)G(s)} \tag{20}$$

is stable and exhibits desirable performance.

The characteristic polynomial of system (20) with respect to (17) is as follows

$$p(s), q) = p_0(s) + \sum_{i=1}^{p} p_i(s) q_i\ . \tag{21}$$

The closed-loop system (20) for all $q \in Q$ is stable if and only if the set of the segments given below are stable. Denote the vertices of Q and corresponding characteristic polynomial as follows

$$p_v(s,q) = \begin{Bmatrix} p(s,q) \colon q_i = \underline{q_i} \\ \text{or } q_i = \overline{q_i}\ ,\ i = 1, 2, \ldots, p \end{Bmatrix} = \{v_1(s), \ldots, v_N(s)\} \tag{22}$$

where $N = 2^p$.

According to (17) and (19) the vertex characteristic polynomial $v_i(s)$ is follows

$$v_i^{(0)} = C(s) v_{Bi}(s) + D(s) v_{Ai}(s), \quad i = 1, 2, \ldots, N \tag{23}$$

where $v_{Bi}(s)$, $v_{Ai}(s)$ are vertex polynomials of $B(s,q)$ and $A(s,q)$.

The set of segments is given as

$$E_i(s) = \lambda v_i + (1 - \lambda) v_j \tag{24}$$

and after substituting (23) to (24) we can obtain

$$\begin{aligned} E_i(s) = \lambda\big(C(s)v_{Bi}(s) + D(s)v_{Ai}(s)\big) &+ (1-\lambda)C(s)v_{Bj}(s) \\ + D(s)v_{Aj}(s) = C(s)\left[\lambda v_{Bi}(s) + (1-\lambda)v_{Bj}(s)\right] & \\ + D(s)\left[\lambda v_{Ai}(s) + (1-\lambda)v_{Aj}(s)\right] & \end{aligned}$$

where $\lambda \in \langle 0, 1\rangle$.

For all entries of $E(s)$ the characteristic polynomial of closed-loop system (21) is as follows

$$p(s) = \frac{C(s)}{D(s)} \frac{\lambda v_{Bi}(s) + (1-\lambda) v_{Bj}(s)}{\lambda v_{Ai}(s) + (1-\lambda) v_{Aj}(s)}. \tag{25}$$

From (25) the extremal transfer function of the plant (17) is

$$G_E(s) = \frac{\lambda v_{Bi}(s) + (1 - -\lambda) v_{Bj}(s)}{\lambda v_{Ai}(s) + (1-\lambda) v_{Aj}(s)} = \frac{B_E(s) \quad i \neq j}{A_E(s) \quad i,j = p2^{p-1}}. \tag{26}$$

Let the transfer function of PID controller be given as

$$R(s) = \frac{a_0 + a_1 s + a_2 s^2}{s} = K\big(1 + \frac{1}{T_i s} + T_D s\big) = \frac{C(s)}{D(s)} \tag{27}$$

and the problem is to determine a_I, $i = 0, 1, 2$. The characteristic polynomial of the closed-loop system with extremal transfer function is

$$p(s) = B_E(s)C(s) + D(s)A_E(s) = \big(a_0 + a_1 s + a_2 s^2\big) B_E(s) + s A_E(s)\ . \tag{28}$$

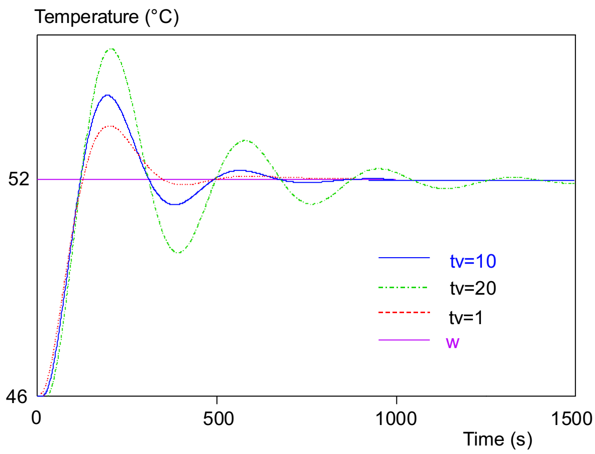

Fig. 8. Control responses of PSD controller

Using the D-partition approach when $s = j\omega$ for PID controller parameters a_l, $l = 0, 1, 2$ are obtained

$$a_0 = -\frac{sA_E(s)}{B_E(s)}\Big|_{s=j\omega},$$
$$a_1 = -\frac{sA_E(s)}{a_0 B_E(s)}\Big|_{s=j\omega},$$
$$a_2 = -\frac{sA_E(s)}{(a_0 + a_1 s)B_E(s)}\Big|_{s=j\omega}, \quad i = 1, 2, \dots, N.$$

By this method, from step response in Fig. 2 the following form of the PID controller is obtained

$$R(s) = \frac{0.05s^2 + 0.3s + 0.002}{s} = 0.3\big(1 + \frac{1}{150s} + 0.1666s\big). \qquad (29)$$

The influence of the data acquisition sample time upon the process control quality was investigated in the power range 40–60 % (46–52 °C) of the whole working range of the drying chamber. The transfer function of the process in this working range is described by equation (3). The time constant is $T_s = 240$ s. To the control of the drying chamber temperature the PSD controller is used.

Figure 8 shows control responses that are obtained using the PSD controller. The sample time t_v was changed from 1 s to 20 s. The stability limit is the sample time 37 s. The value of the sample time depends on the value of the proportion temperature/power in the working range. The control response obtained by controller (29) and the control response obtained by PSD controller with sample time $t_v = 1$ are equal.

4 CONCLUSIONS

Static characteristics and step responses of the warm air-drying chamber exhibit that it is an uncertain system. We can choose the variable which we can control from two possibilities — the temperature in the drying chamber or relative air-moisture of the chamber. The control of both variables provides good results. The warm air-drying chamber is an interesting model in the education process and in the research too.

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