STUDY ON THE CONTACT MODEL OF ULTRASONIC MOTOR CONSIDERING SHEARING DEFORMATION

Li Huafeng* — Zhao Chunsheng* — Gu Chenglin**

The interface contact model between the stator and rotor of a travelling wave type ultrasonic motor is presented, assuming that the rigid stator contacts the flexible rotor over an area, and the frictional material is regarded as a distributed linear spring and without shearing deformation from which the mechanical characteristic of the motor and the interface transmission efficiency are deduced. On this basis, the interface contact model taking account the shearing deformation of the frictional material, which better agrees with the practical situation, is proposed. Then the influences of the motor’s parameters on its performance are investigated. The effectiveness of the proposed model is validated by other researchers.

Keywords: ultrasonic motor, contact model

1 INTRODUCTION

As a novel motor, ultrasonic motors motors (USM) exhibit advantages over conventional electromagnetic motors. For example, USM can produce a relatively high torque at a low speed with high efficiency, and the torque produced per unit weight is high. Since the frictional force between the stator and the rotor is used to drive the motor, the property of the frictional material and the characteristic of the contact area will govern the motor’s working character and operating life. How to increase the frictional force and the transmission efficiency of the contact interface between the stator and the rotor is a key in the study of the ultrasonic motor. Establishing the frictional driving model between the stator and the rotor to predicate the motor’s output characteristic will be helpful to the USM’s design.

Up to now, the analyses of the USM’s friction and many conclusions were achieved only based on experiments and tests [1-3]. In 1992, Maeno et al employed FEM to calculate the motor’s mechanical characteristic, in which the static friction coefficient and kinetic one were used to introduce the shear deformation of the contact layer [4,5]. After then, they adopted LDA to measure the dynamic tangential and normal displacement of the volume parts in the contact area [6]. Nesbit presented an energy analysis model by introducing a symbolic function expressing the relationship of the relative velocity of the stator/rotor volume parts in the contact area. However, Takashi et al. found in their experiments that only using the dynamic coefficient lower than that measured in experiment in the equations taking no account of the shearing deformation could match with the experiments [3]. Nevertheless, they did not explain this phenomenon analytically. In this paper, we analyze the contact mechanism while considering shearing deformation, deduce the motor’s mechanical characteristics, and discuss the influences of the motor’s parameters upon its performance, which will be helpful to the design of USM.

Fig. 1. The contact model between the stator and rotor

2 MECHANICAL CHARACTERISTIC OF AN ULTRASONIC MOTOR

The travelling wave ultrasonic motor consists of a stator generating a travelling wave, and a rotor with adhered frictional material. The travelling wave on the surface of the stator drives the rotor through a tangential force. In the contact area between the stator and rotor, the frictional material deforms due to the preload. When the rigid stator contacts the compliant rotor, the waveform of the stator does not change, as shown in Fig. 1. To analyze the drive mechanism through the frictional force, the frictional material is regarded as a distributed linear spring [7], and the mechanical characteristic of the

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motor is deduced on this basis. Actually, the stator contacts the rotor in a state of stick/slide alternately, and shearing deformation exists. In the following parts, we will deduce the motor’s mechanical characteristics under different contact conditions.

2.1 Taking no account of shearing deformation

When taking no account of shearing deformation of the frictional material, a simplex slide friction exists between the stator and rotor. Introducing one piece of ceramic, that is half wavelength \((L)\), to analyze the tangential force produced by the stator.

Fixing the coordinate to the stator, the longitudinal displacement of the stator under forced vibrations can be expressed by

\[
u_y = A_m \cos \left( \frac{\pi x}{L} \right) = A_m \cos(\omega t)
\]  

where \(k = \frac{\pi}{L}\) is the wave number.

When the preload applied to the half wavelength of the rotor is \(F_c\), according to Fig. 1 the pressure gained by the rotor within the contact area per length will be

\[
p(x) = k_f A_m (\cos kx - \cos kx_0)
\]  

where \(k_f = E_f/h\) is the elastic coefficient of the frictional material, \(E_f\) is Young’s modulus of the frictional material, \(h\) is the thickness of the frictional material, and \(2x_0\) is the contact width of the stator and rotor within half wavelength circumference.

Therefore, the preload can be written as

\[
F_c = b \int_{-x_0}^{x_0} p(x)dx = \frac{2bE_fA_m}{h} \left( \frac{1}{k} \sin kx_0 - x_0 \cos kx_0 \right)
\]  

where \(b\) is the width of the contact area, So the contact length \(x_0\) is a function of preload \(F_c\).

According to the Coulomb friction model and Fig. 1, the tangential force per length applied by the volume parts on the surface layer of the stator to the rotor is

\[
f_1 = \mu_d p(x) \cos^2 \theta
\]  

where \(\mu_d\) is the dynamic frictional coefficient between the stator and frictional material, \(\theta = \frac{\partial u_y}{\partial x}\) is the deflection angle.

Since \(\theta\) is very small, and \(\cos \theta \approx 1\), (4) can be rewritten as

\[
f_1 = \mu_d p(x)
\]  

The rotor slides on the surface of the stator. When a volume part in the surface layer of the stator has a tangential speed greater than that of the rotor, it pushes the rotor. Otherwise, it drags the rotor. Thus the tangential force applied by the stator to the rotor is

\[
F = b \int_{-x_0}^{x_0} \text{sign}[\nu_y(x) - \nu_r] f_1 dx
\]  

where \(\nu_y(x)\) is the tangential speed of the small volume part in the surface layer of the stator, \(\nu_r\) is the rotor’s speed and \(\text{sign}(x)\) is the sign function.

The tangential speed \(\nu_y(x)\) is:

\[
\nu_y = \frac{\partial u_y}{\partial t} = -h \frac{d^2 u_y}{\partial t \partial x} = v_{sm} \cos(\omega t - kx)
\]  

where \(v_{sm} = kA_m h \omega\) is the amplitude of \(\nu_y(x)\), \(\omega\) is the angular velocity of vibration, \(h\) is the distance between the upper surface and the intermediate plane of the vibrator ring, which is a function of stator’s dimensions.

At one moment, the tangential speeds of the volume parts are

\[
\nu_y = v_{sm} \cos kx
\]  

Substituting (5), (8) into (6), we will get

\[
F = 2b \int_{-x_0}^{x_0} \text{sign}[\nu_y(x) - \nu_r] f_1 dx
\]

\[
= 2b \int_{-x_r}^{x_r} f_1 dx - 2b \int_{x_r}^{x_0} f_1 dx
\]

\[
= \frac{2b \mu_d E_f A_m}{h} \left( \frac{2}{k} \sin kx_r - 2x_r \cos kx_0 \right)
\]

\[
- \frac{1}{k} \sin kx_0 + x_0 \cos kx_0
\]

where \(x_r\) is the horizontal coordinate of a volume part in the surface layer of the stator whose speed is equal to the rotor’s speed.

The motor’s output torque is

\[
T = Fr
\]  

where \(r\) is the radius of the contact region.

The input power from the vibrator to the rotor is

\[
P_{in} = 2b \int_{-x_0}^{x_0} \text{sign}[\nu_y(x) - \nu_r] f_1 3v_y(x) dx
\]

\[
= \frac{2b E_f \mu_d A_m v_{sm}}{h} \left[ \frac{x_r - x_0}{2} \right]
\]

\[
+ \frac{1}{4k} \left( 2 \sin 2kx_r - \sin 2kx_0 \right)
\]

\[
- \frac{1}{k} \cos kx_0 (2 \sin kx_r - \sin kx_0)
\]

and the output power of the motor is

\[
P_{out} = F \nu_r
\]

so we can get the conversion efficiency of the contact surface

\[
\eta = \frac{P_{out}}{P_{in}}
\]
2.2 Considering Shearing Deformation

Since the frictional material is softer than the stator, there must exist shearing deformation within the contact area. When the speed of the volume part in the surface layer of the stator is less than that of the rotor, just sliding friction exists between them. When they are equal, the frictional material produces shearing deformation in this region, and is pushed by the volume parts. At this time, sticking engenders between the stator and the rotor. Though the force applied by the stator to the local rotor is static friction, it does not push the whole rotor to move for the frictional material is relatively soft. This procedure lasts to a certain shearing angle, and gliding friction appears again in the contact area.

This procedure can be expressed visually by Fig. 2. Figure 2(a) is a heap of cotton with uniform density. When force \( F \) is applied to it, it does not push the whole heap cotton since it is very soft. As a result, the density and the rigidity of cotton increases around the forced region locally, shown in Fig. 2(b). The deeper the force, the wider the area with higher density. When the force \( F \) goes deep into an angle \( \theta \) (called shearing angle), the area with higher density is so wide that the whole heap cotton can be pushed to move, shown as Fig. 2(c). It can be seen from Fig. 2(b) that the shearing angle \( \theta \) is related to the cotton’s density. The lower the density, the bigger \( \theta \). Within the contact area, the density of frictional material contacted with the stator will become bigger for the reason of squeezing by the stator. As the density near the wave crest is the biggest, \( \theta \) is related to coordinate \( y \), too. Since the amplitude of the vibrator is rather small compared with the thickness of the frictional material (\( \mu \text{m versus mm} \)), \( \theta \) is a function of density simply.

Therefore, let the shearing angle \( \theta \) be

\[
\theta = \theta_0 - k_q \rho_f
\]

where \( k_q \) is a proportionality constant related to the density, \( \rho_f \) is the density of the frictional material, and \( \theta_0 \) is the initial shearing angle.

The sticking region is

\[
\Delta x = h \tan \theta
\]

The contact model between the stator and rotor taking into account the shearing deformation is shown in Fig. 3. The stator sticks the rotor in the region of \([-x_r, -x_{r1}]\), and slides it again after \(-x_{r1}\).

\[
x_{r1} = x_r - \Delta x
\]

It can be seen from Fig. 3 clearly that the existence of the stick area leads to a decrease of the area within witch the stator pushes the rotor, so as to the decrease the torque.

The force \( F \) in Fig. 3 is:

\[
F = b \int_{-x_0}^{x_0} \text{sign}(v_s(x) - v_r) f_1 dx =
\]

\[
b[- \int_{-x_0}^{-x_r} f_1 dx + \int_{-x_r}^{x_{r1}} f_1 dx - \int_{x_{r1}}^{x_0} f_1 dx]
\]

\[
= \frac{bE_f A_m h_d}{h} \left[ \frac{1}{k} \left( 3k \sin kx_r + \sin kx_{r1} - 2 \sin kx_0 \right) - \cos kx_0 (3x_r - 2x_0 + x_{r1}) \right]
\]

The motor’s torque, output power, input power and the conversion efficiency are given in (10) to (13).

3 SIMULATIONS

To discover the qualitative relationship between the motor’s parameters and its performance, some simulations are carried out based on the above equations. The motor’s parameters are given in Table 1, and the simulation results are shown from Fig. 4 to Fig. 8.
Table 1. Constructional parameters of travelling wave type ultrasonic motor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer diameter</td>
<td>80 mm</td>
</tr>
<tr>
<td>Inner diameter</td>
<td>60 mm</td>
</tr>
<tr>
<td>Thickness of the vibrator</td>
<td>6 mm</td>
</tr>
<tr>
<td>Thickness of PZT</td>
<td>1 mm</td>
</tr>
<tr>
<td>Width of the vibrator</td>
<td>10 mm</td>
</tr>
<tr>
<td>Width of PZT</td>
<td>10 mm</td>
</tr>
<tr>
<td>Young’s modulus of the vibrator</td>
<td>1.05×10¹¹ N/m²</td>
</tr>
<tr>
<td>Young’s modulus of PZT</td>
<td>7.5×10¹⁰ N/m²</td>
</tr>
<tr>
<td>Shear modulus of the vibrator</td>
<td>3.8×10¹⁰ N/m²</td>
</tr>
<tr>
<td>Shear modulus of PZT</td>
<td>2.8×10¹⁰ N/m²</td>
</tr>
<tr>
<td>Density of the vibrator</td>
<td>8.5×10³ kg/m³</td>
</tr>
<tr>
<td>Density of PZT</td>
<td>7.5×10³ kg/m³</td>
</tr>
<tr>
<td>Deepness of the groove</td>
<td>2.3 mm</td>
</tr>
<tr>
<td>Width of tooth</td>
<td>1 mm</td>
</tr>
<tr>
<td>Amplitude of driving voltage</td>
<td>100 V</td>
</tr>
<tr>
<td>Piezoelectric strain constant d_{31}</td>
<td>1.23×10⁻¹⁰ m/V</td>
</tr>
<tr>
<td>Number of teeth</td>
<td>72</td>
</tr>
<tr>
<td>Vibration mode</td>
<td>9</td>
</tr>
</tbody>
</table>

Fig. 5. The mechanical characteristics with different thickness of frictional material

Fig. 6. The mechanical characteristics with different preloads

Fig. 7. Comparison of mechanical characteristics with different circumstance

Fig. 8. The relationships of maximum efficiency and output power with the density of frictional material

4 CONCLUSION

Therefore, we can draw some conclusions from the above analysis:

1. An increase of the preload results in an increase of the contact angle, maximum torque and maximum output power, but a decrease of the runaway speed and efficiency.

2. The motor’s maximum torque is constant under certain preload.

3. Young’s modulus of the frictional material influences the motor’s performance greatly. Bigger Young’s modulus yields a higher output power and higher efficiency.

4. The thinner the frictional material, the better of the mechanical characteristics, and the higher the runaway speed.

5. The bigger the vibrator’s Young’s modulus and rigidity, the lower the runaway speed, maximum torque and efficiency.

6. When the shearing deformation is considered, the motor’s torque, speed, output power and efficiency decrease. With an increase of the density of the frictional...
material, the motor’s maximum power and efficiency increase significantly. This conclusion is consistent with the findings by Takashi, which validates effectiveness of this model indirectly. Therefore, it is better to use the vibrator with a lower Young’s modulus and a thinner frictional material with a higher Young’s modulus. As to the preload, it is necessary to balance the weights of torque, runaway speed and efficiency.

REFERENCES


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