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ADAPTIVE PID CONTROLLER WITH ON–LINE IDENTIFICATION

Jiří Macháček * — **Vladimír Bobál** **

A digital adaptive PID controller algorithm which contains recursive identification is presented in this paper. It is performed for three types of discrete model: second order, third order and first order with time delay. The controller parameter design is derived from the ultimate (critical) gain and ultimate period of the system. The main part of the paper consists of simple algebraic expressions used to calculate the ultimate values. The basic condition for this is that the poles of the closed loop with ultimate gain lie on the stability boundary. This procedure can be implemented in all controller design methods based on the knowledge of the ultimate values, such as the well-known Ziegler-Nichols (Z-N) method.

K e y w o r d s: Adaptive control, PID controllers, Ziegler-Nichols method, recursive least squares method, time delay system

1 INTRODUCTION

PID controllers are still very popular in practical control applications for their good properties: they are simple and sufficiently robust, their behavior is easily understood and several methods for setting their parameters have been published. Aström and Hägglund [1] give a good survey of this topic. Tuning PID controllers using the knowledge of the ultimate (critical) gain and ultimate period, suggested by Ziegler and Nichols (Z-N) [2], are used in the proposed adaptive PID controller algorithm. This adaptive controller algorithm consists of three basic parts, which are repeated in each sampling period:

- Recursive identification
- Ultimate gain and period calculation
- Discrete PID controller parameter design.

On-line identification gives the parameters of the discrete model from which the ultimate values are calculated. The method is derived for three types of discrete model: Second order, third order and first order with time delay. Calculation of the ultimate values assumes that the closed control loop poles are placed on the stability boundary. The ultimate gain K_u of the proportional controller and the ultimate period of oscillations T_u are determined on this basis. The PID controller can then be designed using a combination of one of the variations of the Z-N method and one of the several methods of digitizing a PID controller. Simple algebraic relations were derived for 2$^{\rm nd}$ and 3$^{\rm rd}$ order models in [3–7] and for a 1$^{\rm st}$ order model with time delay in [8–9]. The main ideas, suggested by the authors in previous works, are surveyed and compared in this paper.

The paper is organized as follows. Section 2 gives the generalized approach to the computing of the ultimate values. Concrete relationships for the chosen types of simple models are derived in Section 3. A short description of both recursive identification and the control algorithm is given in Sections 4 and 5. The properties of this algorithm are shown on simulation examples in Section 6.

2 COMPUTING THE ULTIMATE VALUES FROM THE TRANSFER FUNCTION — COMMON PROCESS

Let the process be described by discrete time linear model

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) \tag{1}$$

where $A(q^{-1})$ and $B(q^{-1})$ are polynomials with the following structure

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \\ B(q^{-1}) &= b_1 q^{-1} + \cdots + b_m q^{-m} \end{aligned} \tag{2}$$

and d is the time delay expressed as a multiple of sampling period T. The proportional controller with gain K_c is connected in feedback to find the stability boundary. The closed loop transfer function for set point changes can be written as

$$G(q^{-1}) = \frac{q^{-d}K_cB(q^{-1})}{A(q^{-1})+q^{-d}K_cB(q^{-1})}\,. \tag{3}$$

The poles of this transfer function can be calculated from the characteristic equation which, after transformation on the positive powers q, has the form

$$q^{nm+d}\left[A(q^{-1})+K_c\,q^{-d}B(q^{-1})\right] = 0 \tag{4}$$

* Department of Process Control and Computer Techniques, Faculty of Chemical Technology, University of Pardubice, nám. Čs. legií 565, 532 10 Pardubice, Czech Republic, E-mail: jiri.machacek@upce.cz

** Department of Control Theory, Institute of Information Technologies, Faculty of Technology, Tomas Bata University in Zlín, nám. T.G. Masaryka 275, 762 72 Zlín, Czech Republic, E-mail: bobal@ft.utb.cz

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where $nm = \max(n, m)$. At least one root of equation (4) must lie on the unit circle with the rest inside the unit circle of the q-plane when the loop is on the stability boundary ($K_c = K_u$). There are two possible sites for the roots on the unit circle: either a pair of complex conjugated roots, making quadratic three-term

$$q^2 + cq + 1 \tag{5}$$

where

$$c = -2 \cos \omega_u T \tag{6}$$

or a root -1 making two-term

$$q + 1 . \tag{7}$$

Equation (4) must be divisible by either term (5) or (7) and it leads to the solution of the following polynomial equations:

$$q^{nm+d} \left[A(q^{-1}) + K_u\, q^{-d} B(q^{-1})\right] = (q^2 + cq + 1)\, q^{nm+d-2} D(q^{-1}) , \tag{8}$$

$$q^{nm+d}[A(q^{-1}) + K_u\, q^{-d} B(q^{-1})] = (q + 1)\, q^{nm+d-1} E(q^{-1}) \tag{9}$$

where

$$D(q^{-1}) = 1 + d_1 q^{-1} + \cdots + d_{n+d-2} q^{-(nm+d-2)} , \tag{10}$$

$$E(q^{-1}) = 1 + e_1 q^{-1} + \cdots + e_{nm+d-1} q^{-(nm+d-1)} . \tag{11}$$

The solution which has both the minimum positive ultimate gain and stable roots must be chosen. The ultimate frequency for the instances, where the roots are complex, can be calculated from (6). Equation (9) can be solved in a simple way. Root -1 forms the oscillating component $(-1)^k$, which corresponds to a continuous function $\cos(\pi t/T)$ with ultimate frequency

$$\omega_u = \frac{\pi}{T} \tag{12}$$

and ultimate period

$$T_u = 2T . \tag{13}$$

The ultimate gain is then given by

$$K_u = -\frac{A(-1)}{(-1)^d B(-1)} . \tag{14}$$

The following chapters deal with computing the ultimate values for the chosen three types of models.

3 COMPUTING THE ULTIMATE VALUES FOR THE CHOSEN TYPES OF SIMPLE MODELS

Models for which a relatively simple algebraic solution of the ultimate values can be derived were chosen. The following three models well approximate most industrial processes:

- Second order model with discrete transfer function

$$G_1(q^{-1}) = \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1} + a_2 q^{-2}} , \tag{15}$$

- third order model with discrete transfer function

$$G_2(q^{-1}) = \frac{b_1 q^{-1} + b_2 q^{-2} + b_3 q^{-3}}{1 + a_1 q^{-1} + a_2 q^{-2} + a_3 q^{-3}} , \tag{16}$$

- first order plus time delay model with discrete transfer function

$$G_3(q^{-1}) = q^{-d} \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1}} . \tag{17}$$

The time delay in model (17) is expressed as an integer part d (in sampling times T) together with its fractional part τ_z in the range of $0 \le \tau_z < T$, which is included in parameter b_2.

3.1 Second order model

The ultimate gain can be found as a solution of the closed loop characteristic equation (4)

$$q^2 + (a_1 + K_u b_1) q + a_2 + K_u b_2 = 0 . \tag{18}$$

This equation has two solutions:
Case 1: A pair of complex conjugated roots $q_{1,2} = \alpha \pm i\beta$ for which condition $\alpha^2 + \beta^2 = 1$ is valid. The polynomial on the left side of (18) must be in form (5), where $c = a_1 + K_u b_1$ and $a_2 + K_u b_2 = 1$. The ultimate gain is then given by

$$K_u = \frac{1 - a_2}{b_2} . \tag{19}$$

Case 2: Two real roots: $q_1 = -1$, $|q_2| \le 1$ where the polynomial on the left side of (18) must be divisible by root factor $(q + 1)$. The ultimate gain is from (14) given by

$$K_u = \frac{a_1 - a_2 - 1}{b_2 - b_1} . \tag{20}$$

The above relationships derived to calculate the ultimate gain K_u and those for the ultimate period T_u, calculated according to Eqs. (6) and (13) are given in Table 1.

3.2 Third order model

The relationships were derived in the same way as for the second order model from characteristic equation

$$q^3 + (a_1 + K_u b_1) q^2 + (a_2 + K_u b_2) q + a_3 + K_u b_3 = 0 \tag{21}$$

and are given in Table 2.

Table 1. Ultimate values for model G_1

Conditions	$\beta^2-4\le 0\,;\ B=\frac{a_1b_2+b_1-a_2b_1}{b_2}$	$\beta^2-4>0$
K_u	$\frac{1-a_2}{b_2}$	$\frac{a_1-a_2-1}{b_2-b_1}$
T_u	$\frac{2\pi T}{\cos^{-1}\left(-B/2\right)}$	$2T$

Table 2. Ultimate values for third order model

Conditions	$B^2-4AC>0$ where $A=b_3^2-b_1b_3 \quad B=b_2-b_1b_3+b_3(2a_3-a_1)$ $C=a_2+a_3(a_3-a_1)-1$ $K_u=\min(K_{u1},K_{u_3})>0$ $d_1=a_3+K_ub_3\,,\ \lvert d_1\rvert<1$	$K_{u,3}>0$ $K_{u3}<K_{u1}$ $K_{u3}<K_{u2}$
K_u	$K_{u\,1,2}=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$	$K_{u3}=\frac{1-a_1+a_2-a_3}{b_1-b_2+b_3}$
T_u	$\frac{2\pi T}{\cos^{-1}\left(-c/2\right)}$ where $c=a_1-a_3+K_u(b_1-b_3)$	$2T$

Table 3. Ultimate values for model G_3 and delay $d\langle 0;1\rangle$

Con-ditions	$d=0$		$d=1$
	$a_1+\frac{b_1}{b_2}>2$	$a_1+\frac{b_1}{b_2}\le 2$	
K_u	$\frac{1-a_1}{b_1-b_2}$	$\frac{1}{b_2}$	$\frac{a_1b_2-b_1+\sqrt{(b_1-a_1b_2)^2+4b_2^2}}{2b_2^2}$
T_u	$2T$	$\frac{2\pi T}{\cos^{-1}\left(-\frac{b_1}{2b_2}-\frac{a_1}{2}\right)}$	$\frac{2\pi T}{\cos^{-1}\left(\frac{b_2K_u}{2}-\frac{a_1}{2}\right)}$

Table 4. Ultimate values for model G_3 and delay $d>1$

Conditions	$\omega_u\tau\le 1$ where $\omega_u=\frac{2\pi}{T_u}$	$\omega_u\tau>1$
T_u	$\frac{4\overline{\tau}_d+\pi\tau}{2}$	$\frac{8\pi\tau\overline{\tau}_d}{\pi\tau+\sqrt{\pi^2\tau^2+4\pi\tau\overline{\tau}_d}}$
K_u	$\frac{\sqrt{1+\omega_u^2\tau^2}}{K_p}$	

3.3 First order plus time delay model

The placement of the roots on the unit circle depends on the time delay values. Calculation of the ultimate values is then divided over three cases according to the magnitude of the time delay d.

3.3.1 Time delay in the range 0 to T, $(d=0)$

The ultimate gain can be found as a solution of the closed loop characteristic equation (4)

$$q^2+(a_1+K_ub_1)q+K_ub_2=0\,. \tag{22}$$

For systems without time delay ($b_2=0$) the root is equal to -1 and the ultimate gain is

$$K_u=\frac{1-a_1}{b_1}\,. \tag{23}$$

When the time delay increases from zero, root $q_1=-1$ does not change and ultimate gain is equal to

$$K_u=\frac{1-a_1}{b_1-b_2}\,. \tag{24}$$

When the condition

$$a_1+\frac{b_1}{b_2}>2 \tag{25}$$

is fulfilled, a pair of complex conjugated roots arises and the ultimate gain is computed according to

$$K_u=\frac{1}{b_2}\,. \tag{26}$$

According to (6), (22) and (26) the ultimate frequency ω_u for the complex roots equals

$$\omega_u=\frac{1}{T}\cos^{-1}\left(-\frac{b_1}{2b_2}-\frac{a_1}{2}\right) \tag{27}$$

and, where root $q_1=-1$, equation (12) is applied. The values of the ultimate gain and period are given in the left part of Table 3.

3.3.2 Time delay in the range $T<\tau_d\le 2T$, $(d=1)$

Here the characteristic equation of the closed loop has the form

$$q^3+a_1q^2+K_ub_1q+K_ub_2=0\,. \tag{28}$$

The roots lying on the unit circle may only be complex conjugate. The expressions for ultimate gain and ultimate period are given in the right part of Table 3.

3.3.3 Time delay greater than $2T$, $(d>1)$

The analytical solution of the ultimate values yields very complicated expressions and optimization methods are not suitable for real time computing. The following procedure was therefore suggested. It is based on the approximation of the zero-order-hold by time delay which has a magnitude of half the sampling period T. Discrete model (17) is replaced by continuous one

$$G_3(s)=\frac{K_p}{\tau\,s+1}\,e^{-\overline{\tau}_ds} \tag{29}$$

where K_p is the static process gain given as

$$K_p = \frac{b_1 + b_2}{1 + a_1} \tag{30}$$

τ is the time constant

$$\tau = -\frac{T}{\ln(-a_1)} \tag{31}$$

and $\overline{\tau}_d$ is the time delay

$$\overline{\tau}_d = dT + \tau_z + \frac{T}{2} \tag{32}$$

which contains the integer part of the time delay dT, fractional part τ_z

$$\tau_z = T\Big(1 - \frac{\ln C}{\ln(-a)}\Big) \quad \text{where} \quad C = \frac{b_2 - ab_1}{b_1 + b_2} \tag{33}$$

and an approximation of the zero-order-hold by half of the sampling period T.

The ultimate values of this modified continuous system are then determined from the amplitude and phase conditions for the stability boundary

$$|G(i\omega_u)|\; K_u = 1 \tag{34}$$

$$\arg G(i\,\omega_u) = -\pi\,. \tag{35}$$

Equations (34) and (35) for system (29) take the form

$$\frac{K_p K_u}{\sqrt{1 + \omega_u^2 \tau^2}} = 1 \tag{36}$$

$$-\operatorname{tg}^{-1}(\omega_u \tau) - \omega_u \overline{\tau}_d = -\pi\,. \tag{37}$$

Equation (37) can be solved more easily if the following approximation is used for the tg^{-1} function (see *eg* [10]):

$$\operatorname{tg}^{-1} \omega_u \tau = \begin{cases} \frac{\pi}{4}\omega_u \tau & \text{for } \omega_u \tau \le 1 \\ \frac{\pi}{2} - \frac{\pi}{4\omega_u \tau} & \text{for } \omega_u \tau > 1\,. \end{cases} \tag{38}$$

The resulting equations to calculate the ultimate values are given in Table 4.

3.4 Example of an ultimate values computation

The first two examples show the error involved in the approximation mentioned in section 3.3. In both examples it is assumed that the system is exactly described by the model. If the system has a different structure to the model, further errors occur due to this approximation. The magnitude of error in the ultimate values calculation is shown in example 3.

Table 5. Ultimate values of the system from example 1

Model	K_u	T_u[sec]
Discrete model (40)	1.3249	13.0876
Continuous model (39)	1.4293	11.7310
Continuous model with $\overline{\tau}_d = 4.1$ sec, exact solution	1.2988	13.1036
Continuous model with $\overline{\tau}_d = 4.1$ sec, approximation function tg^{-1}	1.2665	13.4996

Table 6. Parameters of approximate model G_3

j	d	b_1	b_2	a_1
3	1	0.0713	0.1057	−0.8290
4	3	0.0876	0.0737	−0.8411
5	4	0.0653	0.0659	−0.8707

Table 7. Ultimate values of approximate functions

j	Discrete transfer functions to (41)		Exact values for models G_3		Suggested method for models G_3	
	K_u	T_u [sec]	K_u	T_u [sec]	K_u	T_u [sec]
3	4.8550	4.6442	4.7240	3.7203	4.7240	3.7203
4	3.2647	7.0274	2.9747	6.5131	2.8385	6.7157
5	2.5752	9.2933	2.8473	8.2316	2.7403	8.4816

Example 1. The first order system with time delay has gain $K_p = 2$, time constant $\tau = 5$ sec and time delay $\tau_d = 3.6$ sec

$$G(s) = \frac{2}{5s + 1}\, e^{-3.6s} \tag{39}$$

has discrete model (17), which for sampling interval $T = 1$ sec has these parameters: $a_1 = -0.81873$, $b_1 = 0.15376$, $b_2 = 0.20877$, $d = 3$:

$$G(q^{-1}) = q^{-3}\frac{0.15376 q^{-1} + 0.20877 q^{-2}}{1 - 0.81873 q^{-1}} \tag{40}$$

The calculated ultimate values are given in Table 5.

The correct values for the discrete model, computed using the MATLAB optimization function, are given in the first row and those for the continuous model are given in the second row. The approximation according to equations (32) to (37) gives those values in the third row and the values for further simplification using (38) are given in the fourth row. The accuracy of the estimated ultimate values is better than 5 %, including the function tg^{-1} approximation (3^{rd} and 4^{th} rows). In comparison with the relay autotune method suggested by Åström and Hägglund [11], this method gives better results and, in addition, can be used for recursive controller tuning.

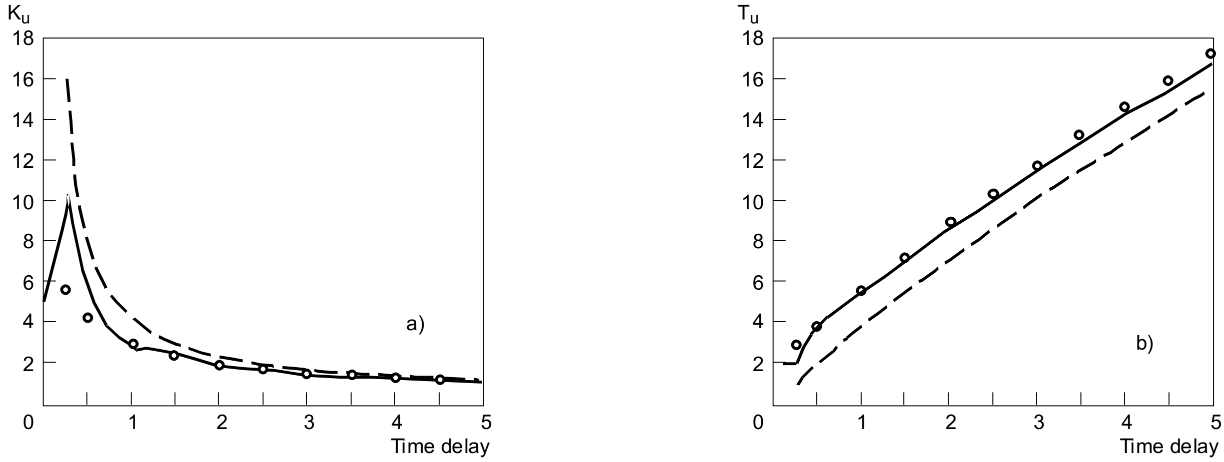

Fig. 1. Ultimate gain K_u and ultimate period T_u for various time delay τ_d (solid lines: exact values, dashed lines: exact continuous values, asterisk: approximation by equations in Table 4)

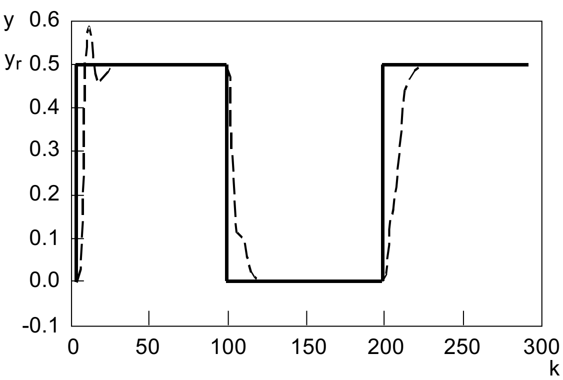

Fig. 2. Control process for model G_2 (setpoint y_r and controlled variable y)

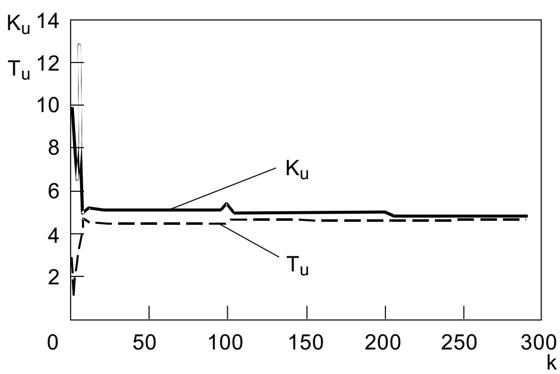

Fig. 3. Calculated ultimate values in the course of control (K_u ultimate gain, T_u ultimate period)

Example 2. Consider a system with transfer function (39), but with time delay in the range 0 to 5 sec. The actual and estimated ultimate gain and period (calculated according to Table 4) are presented in Fig. 1 where the exact solution is compared with the approximation. The accuracy of the estimated values in range $\tau_d = 2$ to 5 sec is better than $3\,\%$. For a smaller τ_d, the exact ultimate values must be calculated using Table 3.

Example 3. Systems with transfer function

$$G(s) = \frac{1}{(s+1)^j}\,, \qquad j = 3, 4, 5 \tag{41}$$

are approximated by model (17) with sampling period $T = 0.5$ sec. For example, if $j = 3$, then the discrete transfer function (41) has the form

$$G_2(q^{-1}) = \frac{0.0144q^{-1} + 0.0397q^{-2} + 0.0068q^{-3}}{1 - 1.8196q^{-1} + 1.1036q^{-2} - 0.2231q^{-3}} \tag{42}$$

and its first order approximation is

$$G_3(q^{-1}) = q^{-1}\frac{0.0713\,q^{-1} + 0.1057\,q^{-2}}{1 - 0.8290\,q^{-1}}\,. \tag{43}$$

All approximation model parameters for time delay $j = 3, 4, 5$ are given in Table 6. These values are taken as the results of the identification algorithm. The ultimate values for discrete form transfer function (41) and for the approximate models with parameters from Table 6 (exact and according to suggested method) are shown in Table 7.

Most of the errors arise from approximation discrete models to (41) by the first order plus time delay system, as can be seen from a comparison between the 1st and 2nd columns in Table 7. The maximum error occurs for $j = 3$, where T_u has error of $20\,\%$. The error of the proposed method is smaller (compare the exact values in 2nd column and those calculated according to the suggested method in 3rd column); for $d = 1$ the error is zero. Successful application of this method depends above all on

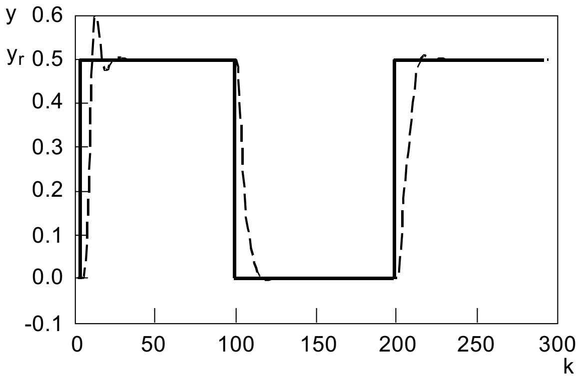

Fig. 4. Control process for model G_3 (setpoint y_r and controlled variable y)

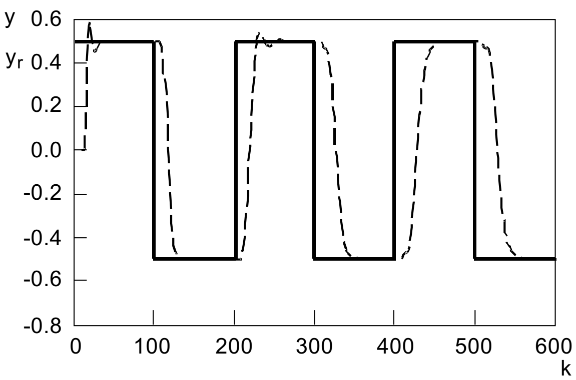

Fig. 6. Control process for model G_2 (setpoint y_r and controlled variable y) with system change at the 150^{th} step

the quality of the approximation. The selected example with multiple time constants is the worst-case of all the tested transfer functions.

4 RECURSIVE IDENTIFICATION

The well-known recursive least squares method together with a forgetting strategy is used to estimate the process model parameters, as part of the general control algorithm. Parameter vector Θ for the $(k+1)$-th time interval is estimated using the following recursive equations:

$$\hat{\Theta}(k+1) = \hat{\Theta}(k) + \boldsymbol{m}(k)[y(k+1) - \Phi^\top(k+1)\hat{\Theta}(k)] \quad (44)$$

$$\boldsymbol{m}(k) = \boldsymbol{C}(k)\Phi(k+1)[\varphi + \Phi^\top(k+1)\boldsymbol{C}(k)\Phi(k+1)]^{-1} \quad (45)$$

$$\boldsymbol{C}(k+1) = \frac{1}{\varphi}[\boldsymbol{C}(k) - \boldsymbol{m}(k)\Phi^\top(k+1)\boldsymbol{C}(k)]\,. \quad (46)$$

For model G_1 the parameter and data vectors have the form

$$\Phi^\top(k+1) = [-y(k) \quad -y(k-1) \;\; u(k) \;\; u(k-1)]$$
$$\Theta^\top(k+1) = [a_1 \;\; a_2 \;\; b_1 \;\; b_2]$$

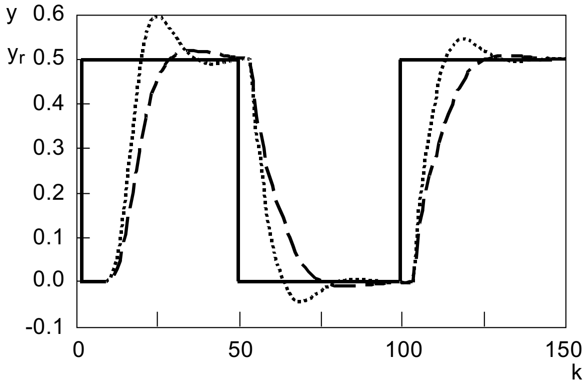

Fig. 5. Control process for model G_3 (setpoint y_r and controlled variable y: solid line — normal algorithm, dashed line — modified ultimate values)

for model G_2 this becomes

$$\Phi^\top(k+1) = [-y(k) - y(k-1) - y(k-2) \;\; u(k) \;\; u(k-1) \;\; u(k-2)]$$
$$\Theta^\top(k+1) = [a_1 \;\; a_2 \;\; a_3 \;\; b_1 \;\; b_2 \;\; b_3]$$

and for model G_3, if time delay d is known,

$$\Phi^\top(k+1) = [-y(k) \;\; u(k-d) \;\; u(k-d-1)]$$
$$\Theta^\top(k+1) = [a_1 \;\; b_1 \;\; b_2]$$

φ is the forgetting factor $\varphi \le 1$. The algorithm is started with diagonal matrix $\mathbf{C}$, which has large values on its diagonal (10^4) and an arbitrary initial parameter vector which does not lead to division by zero in the control algorithm. For model G_3 time delay d must be known.

5 ALGORITHM OF ADAPTIVE PID CONTROLLER

A suitable model based on a priory knowledge about the controlled system must be first chosen (G_1 to G_3). The algorithm then consists of the following steps in each sampling period:

Step 1. Parameter estimation (see Chapter 4).

Step 2. Calculation of the ultimate values (see Chapter 3).

Step 3. Calculating the PID controller parameters. The used controller has continuous form

$$u(t) = K\Big\{[\beta y_r(t) - y(t)] + \frac{1}{T_i}\int_0^t e(\tau)\mathrm{d}\tau + T_d\frac{\mathrm{d}e(t)}{\mathrm{d}t}\Big\} \quad (47)$$

where β is the setpoint weighting constant, K is the proportional gain, T_i is the integral time, T_d is the derivative time, y_r is the setpoint and e is the control error ($e = y_r - y$). The Z-N tuning rules do not perform well

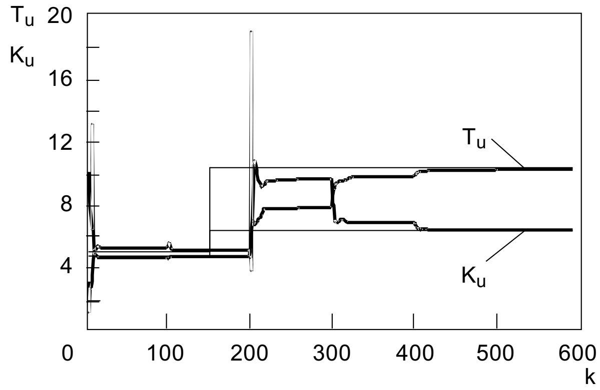

Fig. 7. Ultimate gain K_u and ultimate period T_u for model G_2 with system change at the 150th step (dashed lines — right values)

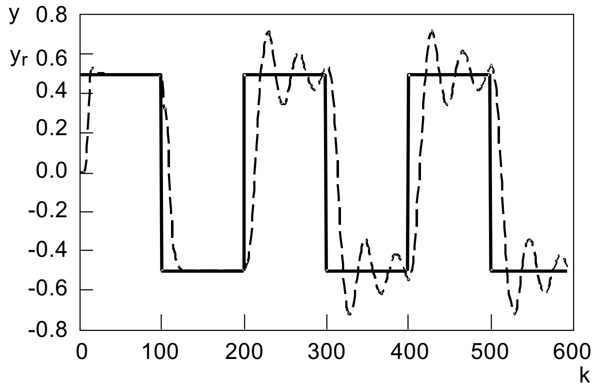

Fig. 8. Control process for model G_2 (setpoint y_r and controlled variable y) with system change at the 150^{th} step, nonadaptive algorithm

for all processes. Improvement in control performance can be achieved by modifying this method. The third parameter, gain ratio κ

$$\kappa = \frac{1}{K_p K_u} \tag{48}$$

together with ultimate values are used to characterize the process dynamics. For given maximum sensitivity $M_s = 1.4$ the controller parameters are calculated using the following terms ([1]):

$$K = 0.33\, K_u e^{-0.31\kappa-\kappa^2} \tag{49}$$

$$T_i = 0.76\, T_u e^{-1.6\kappa-0.36\kappa^2} \tag{50}$$

$$T_d = 0.17\, T_u e^{-0.46\kappa-2.1\kappa^2} \tag{51}$$

$$\beta = 0.58\, e^{-1.3\kappa+3.5\kappa^2} \tag{52}$$

Step 4. Calculating the controller output. Various forms of the discrete controller can be derived from continuous form (47). In the following examples the incremental form of the discrete controller is used with approximation of the integral term using the trapezium method and the derivative term using the first difference. The algorithm also contains a limitation control variable.

6 CONTROL SIMULATION

EXAMPLE 4. System (42) was controlled with sampling period $T = 0.5$ sec. The third order model G_2 was chosen for identification and calculation of the ultimate values. The response to setpoint changes is shown in Fig. 2. Recursive identification yields an error in the model parameters and, consequently, in the ultimate values (see Fig. 3). After 250 identification steps the ultimate gain is $K_u = 5.214$ and the ultimate period $T_u = 4.55$ sec (compared with Table 7 the error is 7.4 % in K_u and -2.03 % in T_u — circles in Fig. 3). The relay autotuning method gives the following values: $K_u = 4.1072$ and $T_u = 5$ sec, which represent error -15.4 % and 7.7 %. With exception of parameter $b_3 = 0.1$ identification was started from a zero parameter vector.

A similar simulation was made for approximation model G_3. The ultimate values change in greater boundaries than for model G_2, but the control process does not differ significantly (see Fig. 4).

EXAMPLE 5. The first order system with time delay (39), (40) was used to investigate robustness against changes in the ultimate values. In each step the ultimate gain increased by 130 % and the ultimate period decreased by 70 %. Responses to setpoint changes are shown in Fig. 5. The response with modified ultimate values has higher overshoots, but is still very good.

The adaptive behavior of the control algorithm is shown in the following example.

EXAMPLE 6. The system (42) increased its time constant from 1 sec to 2.5 sec at the 150^{th} step. The response to setpoint changes and estimated ultimate values are given in Fig. 6 and 7. If the controller has fixed parameters calculated exactly for time constant 1 sec, then the response oscillates (see Fig. 8).

7 CONCLUSION

The proposed self-tuning PID controller can be used to control of a wide spectrum of industrial processes. The method is relatively simple and sufficiently robust. The controller has been tested by simulation on various types of processes with good results, especially when the chosen model approximates the controlled system without great errors. This controller has also been successfully used to control real systems such as a distillation column [12], 200 MW power plant [13], and laboratory through-flow air heating equipment [8].

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Jiří Macháček (Doc, Ing, CSc) was born in Svítkov, Czech Republic, in 1939. He graduated in 1966 from the Faculty of Electrical Engineering, Czech Technical University, Prague. He received his CSc degree in Technical Cybernetics at VŠCHT Pardubice in 1975. From 1958 to 1969 he worked as a research worker at Tesla Pardubice. From 1969 to 1990 he worked as a Senior Lecturer and since 1990 as Associate Professor at the University of Pardubice, Faculty of Chemical Technology, Department of Process Control and Computer Techniques. His current work is in following areas: control of multivariable systems, control of chemical technology processes, PID controllers and system identification. He published over 80 contributions in scientific journals and conference proceedings.

Vladimír Bobál (Prof, Ing, CSc) was born in Slavičín, Czech Republic, in 1942. He graduated in 1966 in Faculty of Mechanical Engineering from Brno University of Technology. He received the CSc degree in Technical Cybernetics at the Institute of Technical Cybernetics, Slovak Academy of Sciences, Bratislava, Slovak Republic. He worked from 1969 as a Senior Lecturer, Associate Professor and Professor in the Department of Automatic Control at Faculty of Technology, Tomas Bata University in Zlín, Czech Republic. His research interests are adaptive control systems, system identification and CAD for self-tuning controllers. He published over 130 contributions in scientific journals and conference proceedings.