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WEIGHTED ADAPTIVE LMS L–FILTER BASED ON CORRELATION COEFFICIENTS (LC–WOS)

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The weighted adaptive order-statistic LMS L-filter (L-WOS) based on the analysis of correlation coefficients (Lc-WOS) is presented in this paper. This filter is a combination of an L-filter and a weighted order-statistic (WOS) filter. The multiplicity of each pixel from an input observation is obtained by correlation relationship between sorted pixels grouped around this pixel and sorted pixels grouped around the central pixel in the input observation. This method can serve to identify the noise model in unknown images and at the same time an optimal weight mask and weight coefficients can be used to its elimination. The adaptation properties of Lc-WOS filter are studied by means of Gaussian white noise and the mixed (Gaussian plus impulsive) noise.

K e y w o r d s: adaptive LMS L-filter, weighted order-statistic, correlation coefficient

1 INTRODUCTION

The most widely known adaptive filters are the linear filters that have the form of a finite impulse response (FIR), but they are not suitable for non-linear applications. One of the best-known families of adaptive non-linear filters is based on the order statistics [1–4, 7]. This family contains the class of the adaptive L-filters that are defined as a linear combination of order statistics. Furthermore, they employ the least mean square (LMS) adaptation algorithm and they also minimize the mean square error (MSE) [2–6].

A new class of adaptive filters, called linear combination of weighted order statistics (L-WOS) filters, is introduced [1]. The acquirement of optimal weights is solved in this paper. This design is based on determining the correlation coefficients between sorted neighbour local areas in the input observation to the sorted pixels grouped around the central pixel.

The outline of this paper is as follows. In the next section, the theory of the design of the weighted order statistics LMS L-filter (Lc-WOS) is introduced. Section 3 involves our experiments and achieved results. The last section contains a brief filter results summary and future tasks that can improve the filtration results.

2 WEIGHTED LMS L–FILTER

The correlation-based adaptive weight order statistics (Lc-WOS) filter is shown in Fig. 1. Its design is based on two steps. First, by correlation analysis for a concrete noise the optimal weight mask is found and, second, it is applied to an adaptive LMS L-filter. In this paper the unconstrained local-invariant version of LMS L-filter (L-lu) was used [2–4].

The observed vector $\mathbf{x}(n)$ can be expressed as the sum of two components, the original image $\boldsymbol{d}(n)$ and the noise $\boldsymbol{\eta}(n)$. The $N=(2\xi+1)^2$ defines the filter dimension and the noisy two-dimensional observation vector ordered in the lexicographic order is given by

$$\mathbf{x}(n)=\big(x(k{-}\xi,l{-}\xi),\dots,x(k,l),\dots,x(k{+}\xi,l{+}\xi)\big)^{\top} \quad (1)$$

where n is the running index, which is used instead of the original pixel coordinates k, and l.

To determine the weights from correlation coefficients, the neighbour sorted pixels around the processed pixel must be assigned. Furthermore, these neighbour sorted vectors are defined by

$$\begin{aligned}
{}^{1}\mathbf{x}(n) &= \big(\mathrm{sort}\big\{x(k-\xi-\psi,l-\xi-\psi),\\
&\qquad \dots,x(k-\xi,l-\xi),\\
&\qquad \dots,x(k-\xi+\psi,l-\xi+\psi)\big\}\big)^{\top}\\
&\ \vdots \\
{}^{N}\mathbf{x}(n) &= \big(\mathrm{sort}\big\{x(k+\xi-\psi,l+\xi-\psi),\\
&\qquad \dots,x(k+\xi,l+\xi),\\
&\qquad \dots,x(k+\xi+\psi,l+\xi+\psi)\big\}\big)^{\top}
\end{aligned} \quad (2)$$

where the upper index of the input vectors denotes the pixel position in the input observation and $M=(2\psi+1)^2$ defines the dimension of the neighbour sorted vector. The ξ and η denote a pixel position in partial input observations. Furthermore, the inequality $\xi \ge \eta$ must be unbroken.

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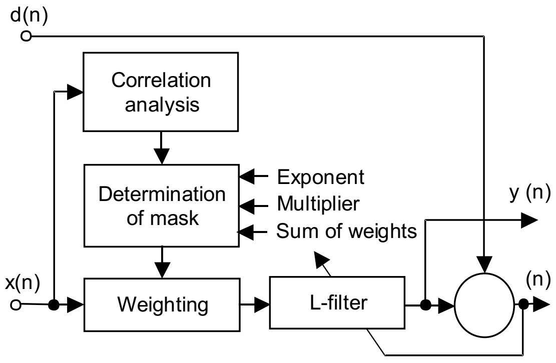

Fig. 1. Weighted adaptive LMS L-filter.

The correlation coefficient for j-th input pixel in n-th iteration is computed by

$$
{}^{j}r(n) = \frac{\sum_{j=1}^{N}\left\{\left({}^{j}\mathbf{x}(n) - {}^{j}\overline{\mathbf{x}}(n)\right)\right.}{\sqrt{\sum_{j=1}^{N}\left({}^{j}\mathbf{x}(n) - j\overline{\mathbf{x}}(n)\right)^{2}}}\cdot \\
\cdot\frac{\left.\left({}^{(N+1)/2}\mathbf{x}(n) - {}^{(N+1)/2}\overline{\mathbf{x}}(n)\right)\right\}}{\sqrt{\sum_{j=1}{}^{N}\left({}^{(N+1)/2}\mathbf{x}(n) - {}^{(N+1)/2}\overline{\mathbf{x}}(n)\right)^{2}}} \tag{3}
$$

where ${}^{j}\overline{\mathbf{x}}(n)$ and ${}^{(N+1)/2}\overline{\mathbf{x}}(n)$ are mean values (4) for j-th and $(N+1)/2$-th local sorted vectors, respectively.

$$
{}^{j}\overline{\mathbf{x}}(n) = \frac{1}{M}\sum_{i=1}^{M} {}^{j}_{i}x(n)\,. \tag{4}
$$

From all vectors of correlation coefficients can be obtained the weight integer by

$$
W_j = ROUND\left(m\cdot\left(\frac{1}{J}\sum_{n=1}^{J} {}^{j}r(n)\right)^{exp}\right) \tag{5}
$$

where m (*multiplier*) and exp (*exponent*) are integer set up parameters. The real number of iterations is defined by

$$
J = (K - 2\xi - 2\psi)\cdot(L - 2\xi - 2\psi) \tag{6}
$$

where K and L represent the image size.

Because the correlation coefficients were computed to the central local sorted vector, the multiplicity of the central pixel is strictly determined by m-parameter. Therefore, its multiplicity can be redefined by the sum of weight modification. Conversion of the achieved weights to the new weights is given by

$$
W'_j = ROUND\left(W_j\frac{\sum W}{\sum_{j=1}^{N} W_j}\right) \tag{7}
$$

where $\sum W$ (sum of weights) defines the desired sum of weights. By rounding of new the weights in (8) some deviation can arise between the desired and real sum of weights. Hence, this difference is added to the central weight by

$$
W''_{(N+1)/2} = W'_{(N+1)/2} + \sum W - \sum_{j=1}^{N} W'_j\,. \tag{8}
$$

Other weights are untouched.

The weight vector based on correlation coefficients has the form

$$
\boldsymbol{W}'' = \left(W'_1,\dots,W''_{(N+1)/2},\dots,W'_N\right). \tag{9}
$$

The input adaptation vector of the Lc- WOS filter is acquired by the combination of weight vector and input observed vector

$$
\boldsymbol{w}\mathbf{x}(n) = \left(W'_1\Diamond x_1(n), W'_2\Diamond x_2(n),\right. \\
\left.\dots, W'_{N-1}\Diamond x_{N-1}(n), W'_N\Diamond x_N(n)\right)^{\top} \tag{10}
$$

where $\Diamond$ denotes the pixel duplication, *ie*

$$
W_i\Diamond x_i(n) = \underbrace{x_i(n),\dots,x_i(n)}_{W_i\ \text{times}}\,. \tag{11}
$$

Let $\boldsymbol{w}\mathbf{x}_r(n)$ be the weighted ordered noisy input vector for n-th observation (*eg* in the ascending order) by next law

$$
wx_{r1}(n) \le wx_{r2}(n) \le \dots \le wx_{rN}(n) \tag{12}
$$

and the final weighted ordered input vector is given by

$$
\boldsymbol{w}\mathbf{x}_r(n) = \left(wx_{r1}(n), wx_{r2}(n),\dots,wx_{rN}(n)\right)^{\top}. \tag{13}
$$

For simplicity, the LMS unconstrained solution for the weight coefficients updating is used. Thus, the updating formula is written as follows

$$
\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + 2\mu\varepsilon(n)\boldsymbol{w}\mathbf{x}_r(n) \tag{14}
$$

where $\varepsilon(n)$ is the estimation error at n-th pixel, *ie* $\varepsilon(n) = d(n) - y(n)$. Equation (14) determines the adaptive unconstrained LMS L-filter [4]. The output of this L-filter is defined for each processed pixel as follows

$$
y(n) = \mathbf{w}^{\top}\boldsymbol{w}\mathbf{x}_r(n) \tag{15}
$$

where $\mathbf{w}^{\top}$ is the adapted coefficient vector and $\boldsymbol{w}\mathbf{x}_r(n)$ is the weighted ordered input vector for n-th observation.

3 EXPERIMENTS AND RESULTS

The experimental part is divided to two sets of experiments. The first set of experiments presents the design of optimal weight masks for typical test noisy images and the second set of experiments is devoted to the determination of the robust level of the designed filter method.

The power of weighted order statistics is demonstrated on two types of image complexity, namely, the Lena image and the second frame of the Trevor sequence, the so-called 2nd Trevor. The 2nd Trevor contains only a few image details along with many homogeneous areas. On the other hand, image of Lena has an opposite structure than the 2nd Trevor image. The original images are shown in Figs. 2a and 2d, respectively.

Table 1. The filter results of noisy Lena image corrupted by the noise G20

Method	Lena			
	MAE	MSE	NR	MAER
Identity	15.425	381.98	–	–
Median 5×5	8.782	182.35	−3.212	−4.893
L-lu 5×5	8.671	180.55	−3.254	−5.003
WM $exp = 26,\ m = 10,$ $\sum W = 45$	8.128	127.39	−4.768	−5.564
Lc-WOS $exp = 26,\ m = 10,$ $\sum W = 45$	7.862	116.64	−5.152	$d - 5.854$

Table 2. The filter results of noisy 2^{nd} Trevor image corrupted by the noise G20

Method	2^{nd} Trevor			
	MAE	MSE	NR	MAER
Identity	15.395	379.59	–	–
Median 5×5	6.535	82.03	−6.653	−7.442
L-lu 5×5	6.604	84.18	−6.541	−7.352
WM $exp = 27,\ m = 10,$ $\sum W = 46$	6.242	69.22	−7.391	−7.841
Lc-WOS $exp = 27,\ m = 10,$ $\sum W = 46$	6.143	65.12	−7.656	−7.979

To the original images two types of noise models were applied, namely, the Gaussian white noise model with variance $\sigma^2 = 400$ (marked as G20) and the mixed noise model which consists of the previous Gaussian model and impulsive noise with probability $p = 10\,\%$ (variable noise value, mixed noise model is marked as G20I10). The noisy images are shown in Fig. 2b or Fig. 2e for G20 noise model and in Fig. 2c or Fig. 2f for G20I10 noise model.

The mean absolute error (MAE), the mean square error (MSE), the noise reduction (NR) and the mean absolute error reduction (MAER) criterions were used to evaluation of achieved filtration results [3–4].

The experimental results achieved from first set of experiments are presented in the Table 1 for Lena and in the Table 2 for 2^{nd} Trevor images for G20 noise model. In the Table 3 for Lena and in the Table 4 for 2^{nd} Trevor images are presented the results for G20I10 noise model. The best results are printed bold. As it is seen, the adaptive Lc-WOS filter in comparison with unconstrained an adaptive local-invariant LMS L-filter (L-lu) offers an improvement around 0.8 (MAE) and 64 (MSE) for Lena (G20 noise). By filtration of the 2^{nd} Trevor (G20 noise) improvements around 0.5 (MAE) and 20 (MSE) were achieved. Filtered Lena and 2^{nd} Trevor images are presented in Fig. 3a and Fig. 3c, respectively.

By filtration of G20I10 noise model improvements around 0.8 (MAE) and 60 (MSE) were attained for Lena and around 0.7 (MAE) and 23 (MSE) for the 2^{nd} Trevor image. Filtered Lena and 2^{nd} Trevor images are presented in Fig. 3b and Fig. 3d, respectively.


a b c
d e f

Fig. 2. Lena image. (a) Original image, (b) Original image corrupted by mixed noise G20, (c) Original image corrupted by mixed noise G20I10. 2^{nd} Trevor image (d) Original image, (e) Original image corrupted by mixed noise G20, (f) Original image corrupted by mixed noise G20I10.

Fig. 3. Achieved filter results by Lc-WOS filter $\mu = 1.10^{-7}$. (a) Filtered noisy G20 Lena image by $exp = 26$, $m = 10$, $\sum W = 42$. (b) Filtered noisy G20I10 Lena image by $exp = 18$, $m = 8$, $\sum W = 45$. (c) Filtered noisy G20 2$^{\text{nd}}$ Trevor image by $exp = 27$, $m = 10$, $\sum W = 46$. (d) Filtered noisy G20I10 2$^{\text{nd}}$ Trevor image by $exp = 16$, $m = 13$, $\sum W = 81$.

Table 3. The filter results of noisy Lena image corrupted by the mixed noise G20I10

Method	Lena			
	MAE	MSE	NR	MAER
Identity	21.041	1120.2	–	–
Median 5×5	9.416	201.93	−7.441	−6.984
L-lu 5×5	9.273	196.03	−7.569	−7.116
WM $exp = 18$, $m = 8$ $\sum W = 42$	8.791	149.47	−8.747	−7.580
Lc-WOS $exp = 18$, $m = 8$ $\sum W = 42$	8.436	136.24	−9.149	−7.938

Table 4. The filter results of noisy 2$^{\text{nd}}$ Trevor image corrupted by the mixed noise G20I10

Method	2$^{\text{nd}}$ Trevor			
	MAE	MSE	NR	MAER
Identity	21.489	1240.2	–	–
Median 5×5	6.966	92.32	−11.282	−9.784
L-lu 5×5	7.480	103.09	−10.802	−9.165
WM $exp = 16$, $m = 13$ $\sum W = 81$	6.975	85.47	−11.616	−9.774
Lc-WOS $exp = 16$, $m = 13$ $\sum W = 81$	6.828	79.87	−11.911	−9.958

By the weight masks that are displayed in Fig. 4, best results were obtained of the adaptive Lc-WOS filter for used noise models. For G20 noise model, the difference between the two weight masks is small. The difference between the set up parameters is small, too. These parameters were found for Lena ($exp = 26$, $m = 10$, $\sum W = 45$) and for 2$^{\text{nd}}$ Trevor ($exp = 27$, $m = 10$, $\sum W = 46$).

As it is seen from Figs. 4c–4d, the weight masks for G20I10 noise model of both images were obtained at different parameters (for Lena $exp = 18$ $m = 8$ $\sum W = 42$ and for 2$^{\text{nd}}$ Trevor $exp = 16$ $m = 13$ $\sum W = 81$) and they are not similar. The obtained masks estimate the original image differently with reference to the higher image frequencies (image details, edges) or lower image frequencies (homogeneous regions). Therefore, to process unknown images Lena's weight mask and weight coefficients for more composite images may be applied, and the 2$^{\text{nd}}$ Trevor's weight mask and weight coefficients for processing simpler images.

The second sets of experiments determine the robust level of the proposed method. The optimal weight mask and weight coefficients from Lena image were used to the filtration of a noisy 2$^{\text{nd}}$ Trevor image and vice versa. The filter results are shown in Table 5 and sign "←" means the direction of application of the weight mask and weight coefficients (*eg* Lena ← Trevor, mask and coefficients from 2$^{\text{nd}}$ Trevor were applied to the Lena). By comparing these results with the results presented in a

1	1	2	1	1
1	2	3	2	1
1	2	9	2	1
1	2	3	2	1
1	1	2	1	1

b

1	1	2	1	1
1	1	3	2	1
2	3	6	2	1
1	2	3	2	1
1	1	2	1	1

c

1	1	1	1	1
1	2	3	2	1
1	2	8	2	1
1	2	3	2	1
1	1	1	1	1

d

2	2	3	2	2
2	3	5	3	2
3	5	13	5	3
2	3	5	3	2
2	3	3	2	2

Fig. 4. Found masks. (a) Lena $exp = 26$, $m = 10$, $\sum W = 45$ (b) 2^{nd} Trevor $exp = 27$, $m = 10$, $\sum W = 46$ (c) Lena $exp = 18$, $m = 8$, $\sum W = 42$ (d) 2^{nd} Trevor $exp = 16$, $m = 13$, $\sum W = 81$.

Table 5. The filter results achieved by using optimal weight mask and weight coefficients from the other image.

G20	MAE	MSE	NR	MAER
Lena ← Lena	7.862	116.64	–5.152	–5.854
Lena ← Trevor	8.027	133.61	–4.562	–5.673
Trevor ← Trevor	6.143	65.12	–7.656	–7.979
Trevor ← Lena	6.456	70.28	–7.325	–7.548

G20I10	MAE	MSE	NR	MAER
Lena ← Lena	8.436	136.24	–9.149	–7.938
Lena ← Trevor	8.582	146.34	–8.839	–7.789
Trevor ← Trevor	6.828	79.87	–11.911	–9.958
Trevor ← Lena	6.991	83.55	–11.715	–9.754

the first sets of experiment it was detected that by using parameters from the other image the achieved results are worse than by using original parameters, but still better than for other filters.

Figure 5 shows correlation coefficients achieved for all types of noise models. Furthermore, this method can serve to identify the noise model in unknown images.

In Table 1–4 the filter results of a weighted median (WM) filter are presented that were obtained after application of optimal weight masks achieved in the first sets of experiments. For all experiments square 5×5-filter windows were used. The adaptation parameter of adaptive filters was given $\mu = 10^{-7}$.

CONCLUSION

In this paper, the weighted adaptive LMS L-filter based on correlation analysis has been described. Determination of correlation coefficients, other set up parameters of test images and optimal weight masks were found. These optimal masks have been developed for Gaussian white and mixed noise models. Besides, this method can be used to identification of a noise model in the processing of unknown images. The designed filter achieved better result than Median, L-lu or WM filters. In general, the use of WOS filters is limited to applications requiring relatively small values of N (for example, $N < 30$) because at large values of N the design of the optimal mask requires more computational complexity. In future, the enhancement of adaptation properties will be obtained by changing the adaptation method.

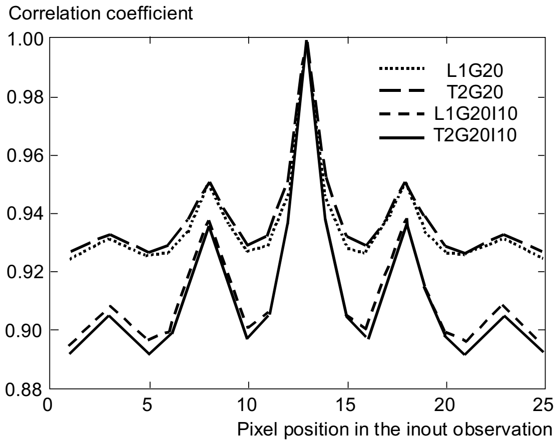

Fig. 5. Averaged correlation coefficients obtained for all noisy images (5×5-filter window).

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