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LINEAR TRANSFORMER WITH TWO COILS CONNECTED IN SERIES

Ladislav Hruškovič *

The paper presents linear transformers which have instead of one full pitch coil two half pitch coils connected in series. Using this type of winding on the stator it is possible to reduce the weight of the linear transformer. Using this type of winding on the rotor it is possible to improve the linearity of the output volt-angle characteristic of the brushless linear transformer.

K e y w o r d s: rotating transformer, linear transformer, brushless rotating transformer

1 INTRODUCTION

Linear transformers are used to obtain a linear dependence of the output voltage on the rotor turning angle. The stator winding of the linear transformer consists of one full pitch coil put in two slots. The coil is connected to an alternating network. The rotor winding also consists of one full pitch coil put in two slots. The rotor winding is an output winding. It is connected to rotor terminals through slip rings and brushes [1], [2].

In the paper there are presented some new types of linear transformers for the cases when the linear output volt-angle characteristic with required functional accuracy is enough in the range of rotor turning angles from $-40°$ to $+40°$ (*eg* for measuring of a lean of a crane). In these types are used instead of one coil two coils connected in series. This allows to reduce the weight of the linear transformer or to improve the linearity of the output volt-angle characteristic of brushless linear transformers.

A developed sketch of this linear transformer with two full pitch coils connected in series is in Fig. 1, where is also shown the space distribution of the flux density of the magnetic field in the air gap of both coils at marked directions of currents. τ_p is the pole distance.

The same result can be obtained using two half pitch coils according to Fig. 2. The coils in Fig. 2 have, however, shorter end-turns than are the end-turns of coils in the Fig. 1.

The parameters of presented linear transformers are determined on the basis of the linear transformer EL40H11. Parameters of this linear transformer were obtained from [1], by calculations and measurements. These parameters are: terminal voltage $U_S = 115$ V, resistance of the stator winding $R_S = 55\,\Omega$, self reactance of the stator winding $X_S = 1148.27\,\Omega$, resistance of the rotor winding $R_R = 25\,\Omega$, self reactance of the rotor winding $X_R = 628.54\,\Omega$, mutual reactance of the stator and rotor windings $X_{RS} = 811.30\,\Omega$, exciting power input $P = 2$ W, iron losses $\Delta P_{\mathrm{Fe}} = 1.45$ W, core weight $G_{\mathrm{Fe}} = 0.41$ kg, copper weight $G_{\mathrm{Cu}} = 0.044$ kg, weight of the linear transformer $G = 0.87$ kg. The error of function accuracy in the range of rotor turning angles from $-85°$ to $+85°$ is up to $0.5\,\%$, that means it satisfies the class accuracy Selection.

With these values of parameters the characteristics of a reference linear transformer were calculated with which the presented new types of linear transformers are compared. There are compared the slope of the output volt-angle characteristic, the weight of the core, the weight of copper and the weight of the linear transformer.

The slope of the output volt-angle characteristic of the reference transformer, loaded by a resistance of $50\,\mathrm{k}\Omega$ and calculated with respect to the above mentioned values of parameters is $0.9008\,\mathrm{V}/1°$.

The error of function accuracy of the presented types of linear transformers is calculated according to expression

$$L = \frac{U_L - U_Z}{U_{Z\,42.5}} 100\ [\%] \qquad (1)$$

where U_L is the output voltage at the given rotor turning angle when the output volt- angle characteristic is a theoretical straight line which connects the initial point with the value of the output voltage at a rotor turning angle of $30°$, U_Z is the output voltage of the linear transformer (voltage on the loading impedance), $U_{Z\,42.5}$ is the output voltage at a rotor turning angle $42.5°$. Values $30°$ and $42.5°$ were chosen because the linearity of the output volt-angle characteristic is appreciated only in the range of rotor turning angles from $-45°$ to $+45°$.

There were calculated temperature rises of the stator and rotor cores and stator and rotor windings of the reference transformer and the values of terminal voltages, and in one case the value of the rotor loading impedance of the presented linear transformers were chosen so that their temperature rise of the stator and rotor cores and windings do not exceed the values of reference transformer.

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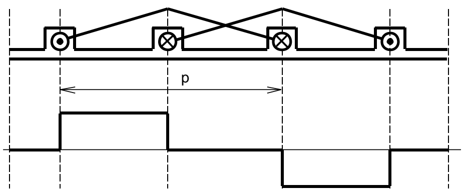


Fig. 1.

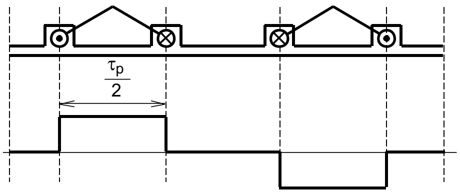


Fig. 2.

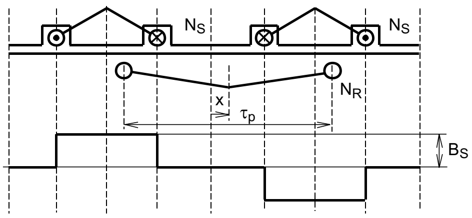
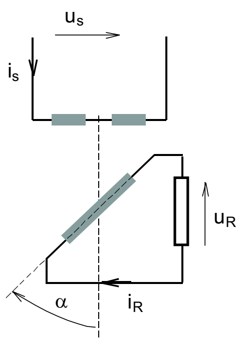


Fig. 3.



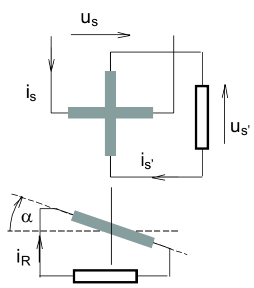


Fig. 4. **Fig. 5.**

The influence of slot openings on reactances was taken into account by means of Carter's air-gap coefficient. The influence of iron on reactances was taken into account by means of the saturation coefficient.

2 LINEAR TRANSFORMERS WITH TWO COILS ON THE STATOR CONNECTED IN SERIES

2.1 Iron losses

A developed sketch of this type of linear transformer is in Fig. 3, where x is a displacement of the rotor winding, N_S is the number of stator coil turns, equal to the number of the stator coil turns of the reference linear transformer, N_R is the number of rotor coil turns, equal to the number of the rotor coil turns of the reference linear transformer. The stator coils are connected in series so that directions of current are as shown in Fig. 3. In Fig. 3 is also shown the space flux density distribution of the stator magnetic field.

The magnetic flux in this case at the same terminal voltage as on the reference linear transformer is a half because there are two coils connected in series and the number of turns is double. But the cross-section of the tooth is also a half. Therefore the flux density in the tooth does not change. Because the weight of tooth is a half, the teeth iron losses are half as small. The yoke cross-section does not change. Therefore the yoke flux density is half as small and the yoke iron losses are four times smaller. This allows either to shorten the core at the given terminal voltage or to increase the voltage at the given core.

The change of the core length will be expressed by coefficient k which is defined as

$$k = \frac{l}{l_r} \tag{2}$$

where l is the core length of the presented linear transformer, l_r is the length of the reference linear transformer.

Calculation of iron losses showed that the iron losses of a linear transformer with two stator coils connected in series designed on the basis of the linear transformer EL40H11 can be expressed as

$$\Delta P_{\mathrm{Fe}} = 0.4102\Big(\frac{U_S}{115}\Big)^2 \frac{l}{k}\,. \tag{3}$$

2.2 Theory

Electrical scheme is in Fig. 4.

The displacement of the rotor will be expressed by quantity x', defined as

$$x' = \frac{2x}{\tau_p}\,. \tag{4}$$

Then the rotor turning angle is

$$\alpha = x' \cdot 90^\circ\,. \tag{5}$$

Voltage equations in the range $x' = -0.5 \div +0.5$ are

$$\begin{aligned} \mathbf{U} &= (R_S + jX_S)\mathbf{I}_S + jX_{RS}x'\mathbf{I}_R \\ 0 &= (R_{tR} + jX_{tR})\mathbf{I}_R + jX_{RS}x'\mathbf{I}_S \end{aligned} \tag{6}$$

where

$$\begin{aligned} R_{tR} &= R_R + R_{ZR} \\ X_{tR} &= X_R + X_{ZR} \end{aligned} \tag{7}$$

and R_{ZR} (X_{ZR}) is the resistance (reactance) of the loading impedance, R_{tR} (X_{tR}) is the total resistance (reactance) of the rotor circuit.

Table 1.

$\alpha\,(^\circ)$	0	15	30	45
I_S (A)	0.096	0.096	0.096	0.096
I_R (A)	0	0.00027	0.00054	0.00081
P (W)	1.296	1.296	1.296	1.296
U_{ZR} (V)	0	13.54	27.09	40.64
L_R (%)	0	−0, 0014	0	0.0072

Table 2.

$\alpha\,(^\circ)$	0	15	30	45
I_S (A)	0.091	0.091	0.091	0.091
I_R (A)	0	0.00049	0.00099	0.00148
P (W)	1.88	1.88	1.88	1.88
U_{ZR} (V)	0	24.80	49.61	74.41
L_R (%)	0	−0.0012	0	0.007

Solving equations (6) we get

$$\mathbf{I}_S = \mathbf{U}_S \frac{R_{tR} + jX_{tR}}{\mathbf{D}} \tag{8}$$

$$\mathbf{I}_R = -\mathbf{U}_S \frac{jX_{RS}x'}{\mathbf{D}} \tag{9}$$

$$\mathbf{U}_{ZR} = \mathbf{U}_S \frac{jX_{RS}x'(R_{ZR} + jX_{ZR})}{\mathbf{D}} \tag{10}$$

where

$$\mathbf{D} = (R_S + jX_S)(R_{tR} + jX_{tR}) + X_{RS}^2 x'^2 \,. \tag{11}$$

2.3 Characteristics

Resistances and reactances R_S, X_S, R_R, X_R, X_{RS} were calculated considering their dependence on the core length. Also was taken into account the shortening of the end-turns of stator coils according to Fig. 3 and their connection in series. Characteristics were calculated for the loading resistance $R_{ZR} = 50\,\mathrm{k}\Omega$.

2.3.1 Characteristics of linear transformer with shortened core

The same temperature rise of the core and windings as the reference linear transformer at terminal voltage $U_S =$ 115 V is obtained at $k = 0.52$. Resistances and reactances in this case have values: $R_S = 54.52\,\Omega$, $X_S = 1190.38\,\Omega$, $R_R = 20.28\,\Omega$, $X_R = 330.90\,\Omega$, $X_{RS} = 842.79\,\Omega$. Calculated characteristics are in Table 1.

As follows from Table 1 the slope of the output volt-angle characteristic is $0.9032\,\mathrm{V}/1^\circ$, or it is almost the same as for the reference linear transformer. The error of function accuracy is deeply under 0.5 %.

If the ratio of the weight of a relevant part of presented linear transformer and the weight of this part of reference linear transformer is marked g, then $g_{\mathrm{Fe}} = 0.5$, $g_{\mathrm{Cu}} = 0.94$, $g = 0.73$. The weight of core is smaller than 0.52-multiple because this type of linear transformer has a double number of stator slots in comparison with the reference linear transformer.

2.3.2 Characteristics of linear transformer with the same core length and higher voltage

The same temperature rise of the core and windings as for the reference linear transformer is obtained at terminal voltage $U_S = 207$ V. Resistances and reactances in this case ($k = 1$) have values: $R_S = 65.30\,\Omega$, $X_S = 2251.58\,\Omega$, $R_R = 25\,\Omega$, $X_R = 627.85\,\Omega$, $X_{RS} = 1620.76\,\Omega$. Calculated characteristics are in Table 2.

As follows from Table 2, the slope of the output volt-angle characteristic is $1.6537\,\mathrm{V}/1^\circ$, or it is 1.83-multiple of the slope of the reference linear transformer. The error of function accuracy is deeply under 0.5 %. The relative weights are in this case $g_{\mathrm{Fe}} = 0.96$, $g_{\mathrm{Cu}} = 1.13$, $g = 0.98$.

3 BRUSHLESS LINEAR TRANSFORMER

3.1 Description

This type of linear transformer has on the stator two full pitch coils with their axis displaced 90 electrical degrees in space. One coil (marked S) is connected to the network and the second coil (marked S') is the output winding. The rotor winding, without being connected with the outer circuit through rings and brushes, is either permanently connected to the impedance or short circuited. It is a bearer of transformation between the two stator windings. Such a linear transformer is a subject of patent [3].

3.2 Brushless linear transformer with one coil on the rotor

Electrical scheme is in Fig. 5.

Voltage equations in the range $x' = 0 \div 1$ are

$$\begin{aligned} \mathbf{U}_S &= (R_S + jX_S)\mathbf{I}_S + jX_{RS}(1 - x')\mathbf{I}_R \\ 0 &= (R_{tS'} + jX_{tS'})\mathbf{I}_{S'} - jX_{RS}x'\mathbf{I}_R \\ 0 &= (R_{tR} + jX_{tR})\mathbf{I}_R + jX_{RS}(1 - x')\mathbf{I}_S - jX_{RS}x'\mathbf{I}_{S'} \end{aligned} \tag{12}$$

where

$$\begin{aligned} R_{tS'} &= R_{S'} + R_{ZS'} \\ X_{tS'} &= X_{S'} + X_{ZS'} \end{aligned} \tag{13}$$

and $R_{S'}$ ($X_{S'}$) is the resistance (reactance) of the stator winding S', $R_{ZS'}$ ($X_{ZS'}$) is the resistance (reactance) of the loading impedance, $R_{tS'}$ ($X_{tS'}$) is the total resistance (total reactance) in the winding S' circuit.

Table 3.

$\alpha\,(°)$	0	10	20	30	40
$U_{ZS'}$ (V)	0	7.62	15.64	23.65	30.88
$L_{S'}$ (%)	0	0.81	0.39	0	2.02

Table 4.

$\alpha\,(°)$	0	15	30	45
I_S (A)	0.116	0.116	0.116	0.116
$I_{S'}$ (A)	0	0.00039	0.00079	0.00119
I_R (A)	0.075	0.075	0.075	0.075
P (W)	1.568	1.568	1.567	1.565
$U_{ZS'}$ (V)	0	19.86	39.70	59.50
$L_{S'}$ (%)	0	−0.021	0	0.114

Solution of equations (12) gives for the output voltage the expression

$$\mathbf{U}_{ZS'} = -\mathbf{U}_S \frac{X_{RS}^2(1-x')x'}{\mathbf{D}}(R_{ZS'} + jX_{ZS'}) \qquad (14)$$

where $\mathbf{D} = (R_S + jX_S)(R_{tS'} + jX_{tS'})(R_{tR} + jX_{tR}) + X_{RS}^2(1-x')^2(R_{tS'} + jX_{tS'}) + X_{RS}^2 x'^2(R_S + jX_S)\,.$ (15)

Resistances and reactances are again calculated from the values of the reference linear transformer considering a different length of the end-turns. The best linearity of the output volt-angle characteristic was obtained only with a condenser permanently connected in the rotor circuit. Calculated characteristics for values $U_S = 97\,\mathrm{V}$, $R_S = 57.92\,\Omega$, $X_S = 1135.01\,\Omega$, $R_{S'} = 64.05\,\Omega$, $X_{S'} = 1140.84\,\Omega$, $R_R = 25\,\Omega$, $X_R = 619.56\,\Omega$, $X_{RS} = 799.30\,\Omega$, $R_{ZS'} = 50\,\mathrm{k}\Omega$, $X_{ZS'} = 0$, $R_{ZR} = 0$, $X_{ZR} = -880\,\Omega$, are in Table 3.

As follows from Table 3, in this case it was not possible to obtain the class of accuracy Selection. There was obtained only the 1-st class of accuracy with the error of function accuracy to 1 % and only in the range of rotor turning angles from 0 to 37°. At $\alpha = 37°$ the error of function accuracy was 0.96 %, for higher rotor turning angles exceeded 1 %. In addition, at $\alpha = 0$ the slots of the stator winding S and the slots of the rotor winding have opposite positions. This can cause an inadmissible nonlinearity of the output volt-angle characteristic at the initial point.

These imperfections can be removed by using two rotor coils connected in series.

3.3 Brushless linear transformer with two coils on the rotor connected in series

3.3.1 Construction and theory

The construction is shown in the developed sketch in Fig. 6.

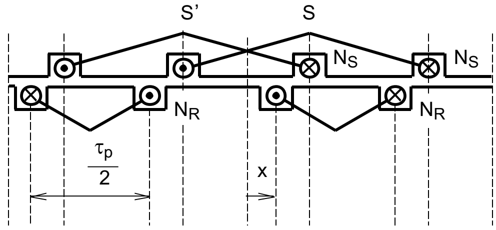


Fig. 6.

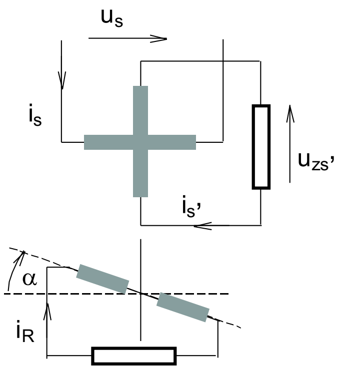


Fig. 7.

The rotor coils are connected in series in this way to get current directions as shown in Fig. 6. N_S is the number of stator coil turns equal to the number of stator coil turns of the reference linear transformer, N_R is the number of rotor coil turns equal to the number of the rotor coil turns of the comparing linear transformer. Stator coils are full pitch coils, rotor coils are half pitch coils.

Electrical scheme is in the Fig. 7.

Voltage equations in the range $x' = -0.5 \div +0.5$ are

$$\begin{aligned} \mathbf{U}_S &= (R_S + jX_S)\mathbf{I}_S + jX_{RS}\mathbf{I}_R \\ 0 &= (R_{tS'} + jX_{tS'})\mathbf{I}_{S'} - jX_{RS}2x'\mathbf{I}_R \qquad (16) \\ 0 &= (R_{tR} + jX_{tR})\mathbf{I}_R + jX_{RS}\mathbf{I}_S - jX_{RS}2x'\mathbf{I}_{S'} \end{aligned}$$

where $R_{tS'}$, $X_{tS'}$, R_{tR}, X_{tR}, are given by equations (13) and (7).

By solving of equations (16) we get

$$\mathbf{I}_S = \mathbf{U}_S \frac{(R_{tS'} + jX_{tS'})(R_{tR} + jX_{tR}) + X_{RS}^2 4x'^2}{\mathbf{D}} \qquad (17)$$

$$\mathbf{I}_{S'} = \mathbf{U}_S \frac{X_{RS}^2 2x'}{\mathbf{D}} \qquad (18)$$

$$\mathbf{I}_R = -\mathbf{U}_S \frac{jX_{RS}(R_{tS'} + jX_{tS'})}{\mathbf{D}} \qquad (19)$$

$$\mathbf{U}_{ZS'} = -\mathbf{U}_S \frac{X_{RS}^2 2x'}{\mathbf{D}}(R_{ZS'} + jX_{ZS'}) \qquad (20)$$

where $\mathbf{D} = (R_S + jX_S)(R_{tS'} + jX_{tS'})(R_{tR} + jX_{tR}) + X_{RS}^2(R_{tS'} + jX_{tS'}) + X_{RS}^2 4x'^2(R_S + jX_S)\,.$ (21)

3.3.2 Characteristics

Resistances and reactances R_S, X_S, $R_{S'}$, $X_{S'}$, R_R, X_R, X_{RS} were calculated considering different stator coils end-turns, the shortening of rotor coils end-turns and their connection in series. Resistances and reactances have in this case the values: $R_S = 57.92\,\Omega$, $X_S = 1126.44\,\Omega$, $R_{S'} = 64.05\,\Omega$, $X_{S'} = 1132.27\,\Omega$, $R_R = 47.42\,\Omega$, $X_R = 1224.49\,\Omega$, $X_{RS} = 792.89\,\Omega$, $R_{ZS'} = 50\,\mathrm{k}\Omega$, $X_{ZS'} = 0$. Rotor winding was short circuited.

Characteristics are calculated for terminal voltage $U_S = 72$ V. At this voltage the core and windings temperature rises were the same as these temperature rises of the reference linear transformer. Iron losses are $\Delta P_{\mathrm{Fe}} = 0.5156$ W. The calculated characteristics are in Table 4.

As follows from Table 4 the slope of the output volt-angle characteristic is $1.3222\,\mathrm{V}/1^\circ$, that is 1.46-multiple of the slope of the output volt-angle characteristic of the reference linear transformer. The error of function accuracy is under 0.5 %.

The weight of the brushless linear transformer with two rotor half pitch coils connected in series is 0.99-multiple of the weight of the brushless linear transformer with one full pitch rotor coil.

The slope of the output volt-angle characteristic can be increased by permanent connection of a condenser in the rotor circuit.

4 DISCUSSION

Application of two half pitch coils connected in series on the stator allows a better utilization of the core. At the same slope of the output volt-angle characteristic the weight of the linear transformer was reduced by 27 %. At the same weight of the linear transformer the slope increased 1.83 times. At the same time the error of function accuracy was much lower than 0.5 %, which is a value of a linear transformer with higher class of accuracy.

In the range of rotor turning angles $\alpha = -90^\circ \div -45^\circ$ and $\alpha = +45^\circ \div +90^\circ$ the output voltage is constant, not dependent on the rotor turning angle.

Application of two half pitch coils connected in series on the rotor of the brushless linear transformer allows to obtain the output volt-angle characteristic with required linearity. Error of function accuracy was under 0.5 %, which could not be obtained at the brushless linear transformer with a full pitch coil on the rotor. In addition, the slope of the output volt-angle characteristic of brushless linear transformer with two coils on the rotor connected in series was 1.46-times higher than in the reference linear transformer.

The output volt-angle characteristic of linear transformer EL40H11 is linear with error of function accuracy lower than 0.5 % in the range of rotor turning angles $\alpha = -85^\circ \div +85^\circ$. The rest of characteristic to $\pm 90^\circ$ considerably differs from the straight line as a consequence of the influence of slot openings. By the same reason the output volt-angle characteristics of the presented linear transformers are linear with required linearity only in the range $\alpha = -40^\circ \div +40^\circ$.

The exciting power of the presented linear transformers does not exceed the power input of the linear transformer EL40H11.

The input current (input impedance) of all presented linear transformers is constant, not dependent on the rotor turning angle.

5 CONCLUSION

There are presented linear transformers, the stator or rotor winding of which consist of two half pitch coils connected in series.

In the case of two coils connected in series on the stator the core is better utilized.

In the case of two coils connected in series on the rotor the output volt-angle characteristic of the brushless linear transformer with better linearity is obtained.

By the use of the half pitch coils, consumption of copper is reduced.

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