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COMMUNICATIONS

SAVING OF ENERGY IN ROBOTICS BY USING MODIFIED VARIABLE–STRUCTURE CONTROL METHODS

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Saving of energy by modifying a very simple variable-structure control (VSC) method containing the sign function has been tested on a circular robot tool tip trajectory. Parameters of the modified variable-structure controllers are adjusted according to these three principles: desired tracking error, no chattering and minimum of energy. The results of computer simulations show the energy reducing of 3.7 times compared with the original variable-structure control method.

K e y w o r d s: robotics, energy, variable-structure control, simulation

1 INTRODUCTION

While analyzing the defined robot tool tip trajectory, it is very important to ensure the desired tracking error and to take care about the used energy.

The desired tracking error of the robot tool tip trajectory can be achieved using a very simple variable-structure control method containing the sign function in the control law introduced by Chen *et al* [1] and Morgan *et al* [5].

The energy used for robot moving along the desired trajectory can be reduced by modifying the original VSC method in order to eliminate chattering of the control signal. The replacement of the sign function with a continuous control signal is proposed by Hashimoto *et al* [3] and with a saturation function is suggested by Myszkorowski [6]. Another modification of the original variable-structure control signal can be done by the exponential function. More detailed descriptions of these modified variable-structure control methods follow.

2 VARIABLE–STRUCTURE CONTROLLERS

Variable-structure control methods used in robotics usually use the error of each ith motor shaft angle $e_i(t)$ and speed $\dot{e}_i(t) = \mathrm{d}e_i(t)/\mathrm{d}t$ as system state variables:

$$e_i(t) = \theta_{r_i}(t) - \theta_i(t)\,, \quad 1 \le i \le n\,, \tag{1}$$

$$\dot{e}_i(t) = \dot{\theta}_{r_i}(t) - \dot{\theta}_i(t)\,, \quad 1 \le i \le n\,, \tag{2}$$

where n is the number of robot motors.

The following common error can be defined for state variables of the ith robot motor:

$$\sigma_i(e_i, \dot{e}_i) = K_{\sigma_i}[\lambda_i e_i(t) + \dot{e}_i(t)]\,, \quad 1 \le i \le n\,. \tag{3}$$

The goal of the variable-structure control method is elimination of motor errors $\sigma_i(t)$:

$$\sigma_i(t) = \sigma_i(e_i(t), \dot{e}_i(t)) = 0\,, \quad 1 \le i \le n\,. \tag{4}$$

For determining the control law for the ith robot motor, which ensures that the system is operating in the sliding mode, the following Liapunov function can be used:

$$V_L(t) = \frac{\boldsymbol{\sigma}^\top(t) \cdot \boldsymbol{\sigma}(t)}{2}\,, \quad 1 \le i \le n\,. \tag{5}$$

The controlled system will be asymptotically stable, if the following condition is fulfilled for all motors $1 \le i \le n$:

$$\dot{V}_L(t) = \boldsymbol{\sigma}^\top(t) \cdot \dot{\boldsymbol{\sigma}}(t) \le 0\,. \tag{6}$$

The last condition can be fulfilled, if the following selection of $\mathrm{d}\sigma_i(t)/\mathrm{d}t$, by using equations (3) and (4), is made for all robot motors $1 \le i \le n$:

$$\dot{\sigma}_i(t) = -\gamma_i \operatorname{sgn}(\sigma_i(t))\,. \tag{7}$$

From equations (7) and (3), the following simple control law can be derived for the ith robot motor, as can be seen in Fig. 1:

$$u_{R_i}(t) = u_{1_i}(t) + u_{2_i}(t)\,, \quad 1 \le i \le n\,, \tag{8}$$

$$u_{1_i}(t) = \gamma_i \operatorname{sgn}\{K_{\sigma_i}[\lambda_i\, e_i(t) + \dot{e}_i(t)]\}\,, \ 1 \le i \le n\,, \tag{9}$$

$$u_{2_i}(t) = K_{\sigma_i}\, \lambda_i\, \dot{e}_i(t)\,, \quad 1 \le i \le n\,. \tag{10}$$

To reduce the chattering of the control signal, the first part of the control law (9) which contains the sign function (the 1st VSC method, it is polyline no. 1 in Fig. 2):

$$\gamma_i \operatorname{sgn}[\sigma_i(t)] = \gamma_i \frac{\sigma_i(t)}{|\sigma_i(t)|}\,, \quad 1 \le i \le n\,, \tag{11}$$

is replaced by a continuous control signal (the 2nd VSC method in Fig. 2, polyline no. 2):

$$\gamma_i \frac{\sigma_i(t)}{|\sigma_i(t)| + \delta_i}\,, \quad 1 \le i \le n\,, \tag{12}$$

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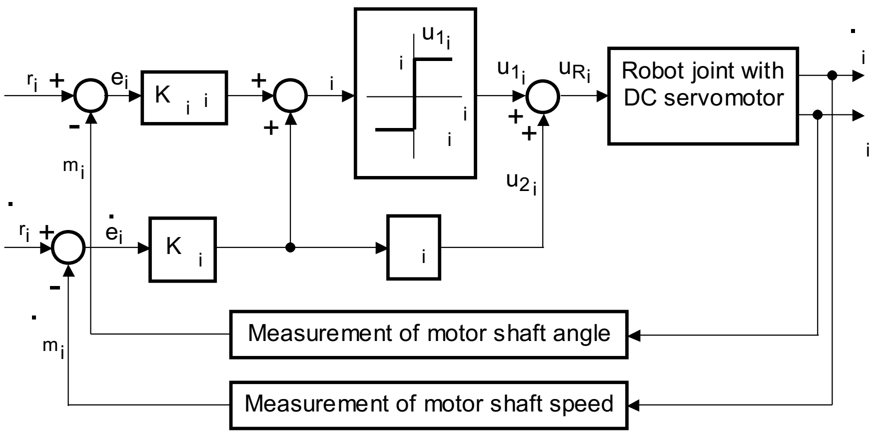

Fig. 1. The original variable-structure control scheme.

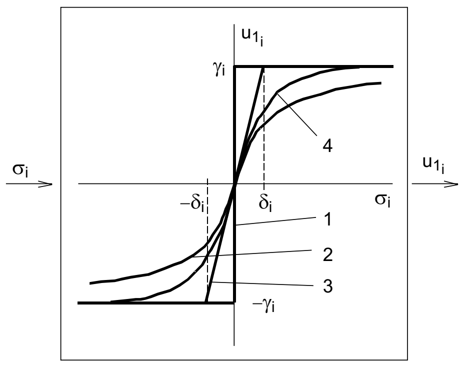

Fig. 2. The first part of the control law in the 1st (polyline no. 1), the 2nd (polyline no. 2), the 3rd (polyline no. 3) and the 4th (polyline no. 4) VSC method.

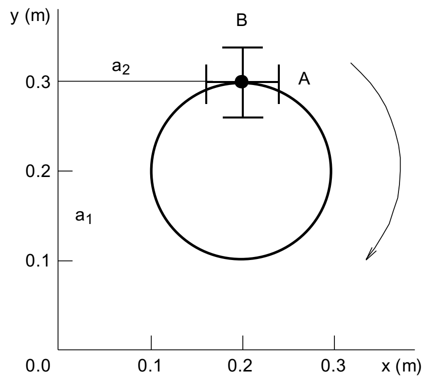

Fig. 3. The reference circular trajectory.

and by a continuous approximation with a high gain, *ie*, saturation function, which is presented in Fig. 2 by polyline no. 3 (the 3rd VSC method) $(1 \le i \le n)$:

$$\gamma_i \,\mathrm{sat}[\sigma_i(t)] = \begin{cases} -\gamma_i\,, & \sigma_i < -\delta_i \\ \gamma_i \frac{\sigma_i}{\delta_i}\,, & -\delta_i < \sigma_i < \delta_i \\ \gamma_i, & \sigma_i > \delta_i \end{cases} . \tag{13}$$

The mentioned signal can also be replaced by the following exponential function shown in Fig. 2 by polyline no. 4 (the 4th VSC method) $(1 \le i \le n)$:

$$\gamma_i \, m \,\mathrm{sgn}[\sigma_i(t)]\big(1 - e^{-|\sigma_i|/\delta_i}\big) = \gamma_i \, m \frac{\sigma_i(t)}{|\sigma_i(t)|}\big(1 - e^{-|\sigma_i|/\delta_i}\big)\,. \tag{14}$$

In Fig. 2 and in equations (12)–(14) δ_i denotes the thickness of the boundary layer for ith robot motor, $1 \le i \le n$.

3 SIMULATION RESULTS

The proposed control methods have been tested by computer simulations in programming language C in the case of moving the tool of a three-axis electric driven articulated planar robot [7] along the circular trajectory shown in Fig. 3, *ie*, between the start point A and the end point B, with a change of the tool roll angle by $-\pi/2$ (rad).

The lengths of robot segments are: $a_1 = 0.3$ m, $a_2 = 0.2$ m and the distance between the tool tip and the working plane is $d_3 = 0.1$ m (*ie* length of the third segment). The masses of segments are: $m_1 = 1$ kg, $m_2 = 0.7$ kg, $m_3 = 0.3$ kg.

The computer simulations have been performed by using the following dynamic model of robot [4]:

$$\begin{aligned}\tau_1 = {} & \Big[\Big(\frac{m_1}{3} + m_2 + m_3\Big) m a_1^2 + \Big(\frac{m_2}{3} + m_3\Big) m a_2^2 \\ & + (m_2 + 2m_3)\, m a_1 a_2 \cos(q_2) + J_{m1}\, m N_{r1}^2\Big]\, \ddot{q}_1 \\ & + \Big[\Big(\frac{m_2}{3} + m_3\Big) m a_2^2 + \Big(\frac{m_2}{2} + m_3\Big) m a_1 a_2 \cos(q_2)\Big]\, m \ddot{q}_2 \\ & - (m_2 + 2m_3)\, m a_1 a_2 \sin(q_2)\, m \Big(\dot{q}_1 \dot{q}_2 + \frac{\dot{q}_2^2}{2}\Big) \\ & + g_0 \Big[\Big(\frac{m_1}{2} + m_2 + m_3\Big) m a_1 \cos(q_1)\end{aligned}$$

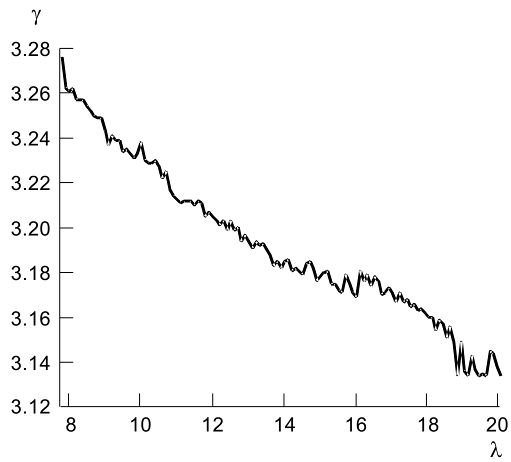

Fig. 4. Mutual dependence of parameters λ and γ for total error fixed to 0.5 mm in the case of the 1st VSC method.

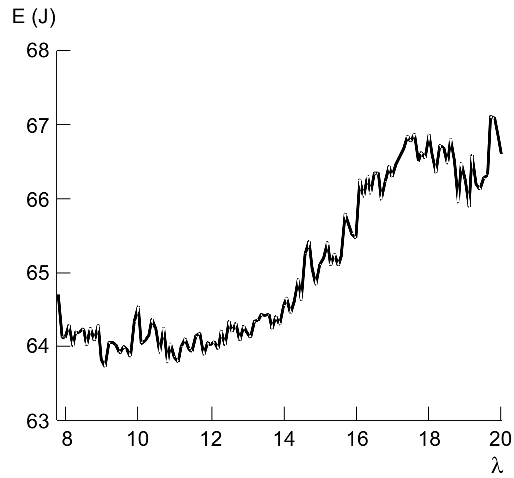

Fig. 5. Total energy of all motors as a function of parameter λ in the case of the 1st VSC method.

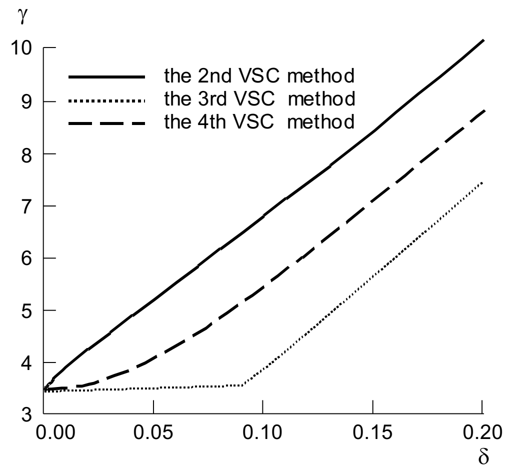

Fig. 6. Mutual dependence of parameters δ and γ for total error fixed to 0.5 mm in the case of modified VSC methods.

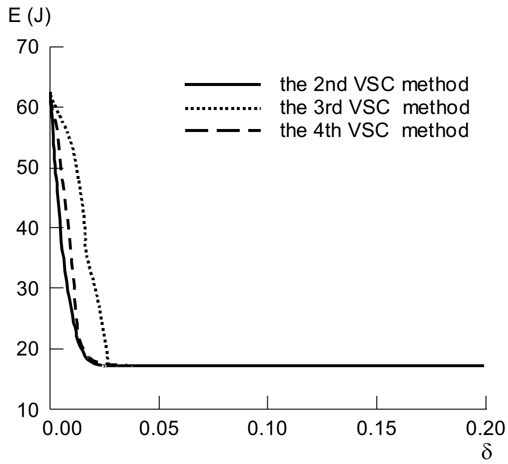

Fig. 7. Total energy of all motors as a function of parameter δ in the case of modified VSC methods.

$$+\left(\frac{m_2}{2}+m_3\right) m a_2 \cos(q_1+q_2)\Big] + b_1(\dot{q}_1) \tag{15}$$

$$\begin{aligned}\tau_2 = &\left[\left(\frac{m_2}{3}+m_3\right) m a_2^2 + \left(\frac{m_2}{2}+m_3\right) m a_1 a_2 \cos(q_2)\right] m\ddot{q}_1 \\ &+ \left[\left(\frac{m_2}{3}+m_3\right) m a_2^2 + J_{m2}\, m N_{r2}^2\right] m\ddot{q}_2 \\ &+ \left(\frac{m_2}{2}+m_3\right) m a_1 a_2 \sin(q_2)\, m\dot{q}_1^2 \\ &+ g_0\, m\left(\frac{m_2}{2}+m_3\right) m a_2 \cos(q_1+q_2) + b_2(\dot{q}_2)\end{aligned} \tag{16}$$

$$\tau_3 = J_{m3}\, N_{r3}^2\, \ddot{q}_3 + b_3(\dot{q}_3) \tag{17}$$

where q_i, $\dot{q}_i$ and $\ddot{q}_i$ are the ith joint variable, velocity and acceleration respectively ($1 \le i \le 3$), τ_i is the ith actuator torque, J_{mi} is the moment of inertia for the ith motor, N_{ri} is the ith gear ratio, g_0 is gravitational constant and $b_i(\dot{q}_i)$ denotes friction opposing the motion of the ith joint.

This realistic dynamic robot model contains the following viscous, dynamic and static joint and motor frictional forces ($1 \le i \le 3$):

$$b_i(\dot{q}_i) = b_{v_i}\, m\dot{q}_i + \left[b_{d_i} + (b_{s_i} - b_{d_i})\, m e^{-|\dot{q}_i|/\varepsilon_i}\right] \operatorname{sgn}(\dot{q}_i)\,, \tag{18}$$

where $\dot{q}_i$ is the velocity for joint i; b_{v_i}, b_{d_i}, b_{s_i} are the coefficients of viscous, dynamic and static friction, respectively, for joint i; e_i is a small positive parameter.

The following friction coefficients have been used in simulations: viscous motor friction coefficients $b_{vm1} = b_{vm2} = b_{vm3} = 0.00003855\,\mathrm{kgm^2/s}$; viscous joint friction coefficients $b_{v1} = 0.5\,\mathrm{kgm^2/s}$, $b_{v2} = 0.25\,\mathrm{kgm^2/s}$, $b_{v3} = 0.2\,\mathrm{kgm^2/s}$, dynamic joint friction coefficients $b_{d1} = 0.2\,\mathrm{kgm^2}$, $b_{d2} = 0.1\,\mathrm{kgm^2}$, $b_{d3} = 0.05\,\mathrm{kgm^2}$; static joint friction coefficients $b_{s1} = 2\,\mathrm{kgm^2}$, $b_{s2} = 1\,\mathrm{kgm^2}$, $b_{s3} = 0.5\,\mathrm{kgm^2}$; small constants

Other robot motor parameters are: resistances of armature winding $R_{a1} = R_{a2} = R_{a3} = 8.2\,\Omega$, inductances of armature winding $L_{a1} = L_{a2} = L_{a3} = 0.0000165\,\mathrm{H}$, torque constants $K_1 = K_2 = K_3 = 0.0394\,\mathrm{Nm/A}$, moments of inertia $J_{m1} = J_{m2} = J_{m3} = 0.00000268\,\mathrm{kgm^2}$, maximal armature currents $I_{am1} = I_{am2} = I_{am3} = 0.745\,\mathrm{A}$, maximal output controller voltage $U_{Rm1} = U_{Rm2} = U_{Rm3} = 10\,\mathrm{V}$, amplifier coefficients $K_{A1} = K_{A2} = K_{A3} = 2.4$, gear ratios $N_{r1} = 291$, $N_{r2} = 388$, $N_{r3} = 582$.

In all computations of the total energy along the trajectory the feedback of electrical energy into the network or into a storage battery is not considered [2]:

$$E = \sum_{i=1}^{3} \Big(\int_0^{T_s} U_{a_i} I_{a_i} \, \mathrm{d}t \,, \quad U_{a_i} I_{a_i} > 0 \Big) \,, \tag{19}$$

where T_s denotes trajectory traverse time, U_{ai} is armature voltage and I_{ai} is armature current for the ith motor $(1 \le i \le 3)$. The trajectory traverse time, with defined limits of joint accelerations and velocities, is $T_s = 8\,\mathrm{s}$.

At the beginning, gains $K_{\sigma i}$ for all robot motors $1 \le i \le 3$ are chosen to achieve the execution of the desired moving along the whole trajectory as follows: $K_{\sigma i} = 0.01$.

The maximal allowed total tracking error along the circular robot tool tip trajectory is set to $0.5\,\mathrm{mm}$. The goal is adjusting the parameters λ and γ of the 1st variable-structure controller (Fig. 1) to achieve the desired error with the minimum of energy. Therefore, the influence of parameters λ and γ on the total tracking error and total energy has been analyzed and the results are shown in Figs. 4 and 5. From these figures it can be seen that the minimum of energy along the desired trajectory $(E = 63.73\,\mathrm{J})$ can be achieved by choosing the following values of parameters: $\lambda_i = 9.1\,\mathrm{s}^{-1}$, $1 \le i \le 3$. The values of parameters γ are set to achieve the maximal allowed trajectory tracking error as follows: $\gamma_i = 3.237$, $1 \le i \le 3$.

The achieved minimal amount of energy is big because of chattering and it can be reduced by using modified variable-structure controllers, which are shown in Fig. 2. The values of parameters λ_i for all robot motors remain the same as in the case of the 1st VSC method. Parameters δ and γ of these controllers are adjusted according to the following principles: maximal allowed tracking error $(0.5\,\mathrm{mm})$, no chattering and minimum of energy consumption.

The results of computer analysis are shown in Figs. 6 and 7. From these figures, the parameters of all modified VSC methods are set as follows: $\delta_i = 0.04$ for all modified VSC methods, $\gamma_i = 4.655$ for the 2nd VSC method, $\gamma_i = 3.2665$ for the 3rd VSC method, $\gamma_i = 3.624$ for the 4th VSC method, $1 \le i \le 3$.

The simulation results in Fig. 7 show that the biggest amount of total energy needed for trajectory tracking $E = 63.73$ J exists in the case of thickness of boundary layer equals to zero for all modified VSC methods. This maximal amount of energy consumption is equal to the minimum of energy of the original 1st VSC method, as can be seen in Fig. 5.

In Fig. 7 it is shown that, with the same thickness of boundary layer in all modified VSC methods, the smallest amount of energy during the chattering can be obtained by using 2nd VSC method and the biggest in the 3rd VSC method. The same conclusion can be done from Fig. 2 because saturation function (polyline no. 3) has the largest slope.

The results presented in Fig. 7 show that the minimal amount of energy achieved by the mentioned settings of modified VSC parameters is $E = 17.2$ J. This energy is 3.7 times smaller in comparison with the minimum of energy consumption in 1st VSC method ($E = 63.73$ J in Fig. 5).

4 CONCLUSION

The results of parameter adjustment proved the efficiency of the new proposed modified VSC method, as well as the other two modified controllers, in reducing of chattering and energy saving in comparison with the original VSC method.

References

[1] CHEN, Y. F.—MITA, T.—WAKUI, S.: A New and Simple Algorithm for Sliding Mode Trajectory Control of the Robot Arm, IEEE Transactions on Automatic Control **35** No. 7 (1990), 828–829.

[2] DESOYER, K.: Geometry, Kinematics and Kinetics of Industrial Robots, In: Proceedings of the International Summer School CIM and Robotics, Krems 1994.

[3] HASHIMOTO, H—MARUYAMA, K.—HARASHIMA, F.: A Microprocessor-Based Robot Manipulator Control with Sliding Mode, IEEE Transactions on Industrial Electronics **IE-34** No. 1 (1987), 11–18.

[4] KOVAČIĆ, Z.—BOGDAN, S.—KRAJČÍ, V.: Fundamentals of Robotics (Osnove robotike), Graphis, Zagreb, 2002. (in Croatian)

[5] MORGAN, R.G.—ÖZGUNER, U.: A Decentralized Variable Structure Control Algorithm for Robotic Manipulators, IEEE Journal of Robotics and Automation **RA-1** No. 1 (1985), 57–65.

[6] MYSZKOROWSKI, P. Comments on "A New and Simple Algorithm for Sliding Mode Trajectory Control of the Robot Arm": IEEE Transactions on Automatic Control **37** No. 7 (1990), 1088.

[7] SCHILLING, R.J.: Fundamentals of Robotics: Analysis and Control, Prentice-Hall, New Jersey, 1990.

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