Journal of ELECTRICAL ENGINEERING, VOL. 55, NO. 9-10, 2004, 245–250

DIPOLE ARRAY EXCITED BY SLOTS IN A COAXIAL FEEDER

Dušan Černohorský — Zdeněk Nováček *

Technical analysis of a coaxial dipole array excited by periodically distributed slots in the shield of the coaxial feeder is presented. The lossy transmission-line theory is applied for determination of the current distribution on all parts of the system and the input impedance as well as the radiation pattern are deduced. The calculated results are compared with the measured ones.

K e y w o r d s: dipole array, slot excited dipole, two-point excited radiators

1 INTRODUCTION

Some communication systems require vertically polarized antennas being omni-directional in the horizontal plane, and holding certain required directivity in the elevation (in the vertical plane).

Such antennas can be realized like vertically situated linear arrays of vertical dipoles. However, the accomplishment of a sufficiently simple feeding system together with a suitable and mechanically compact construction of such arrays cause troubles in practice. The antenna, schematically drawn in Fig. 1, represents a relatively simple solution of this problem.

The common vertical coaxial feeder (the "main feeder" f-f in Fig. 1) has periodically distributed slots in its shield (the outer conductor). Conducting tubes ("sleeves") are drawn over on the cable. They are connected with opposite sides of each slot (Fig. 1). The outer surfaces of these tubes act like dipole arms of the array. The inner part of each tube together with the outer surface of the main feeder create a coaxial resonator whose reactance connects the ends of dipole arms with the outer surface of the main feeder. In case the resonators are exactly tuned to the operating frequency, the arm ends are insulated and the whole system operates like a linear dipole array. When the resonators are detuned, their reactances are finite and the outer surface of the main feeder (its part E_1-F_1, see Fig. 1) becomes an active (radiating) part of the antenna.

The described antenna is occasionally used in the decimetre-wave band [1]. Some practical recommendations for its design are known; however, more general theoretical works appear only sporadically. A simplified analysis was published in [2], [3]; the radiating surface has been treated there like a cascade of lossy transmission lines excited by slots in the shield of the main coaxial feeder.

In this paper, the published results are summarized, integrated and completed by several calculations and experimental data. The lossy transmission-line theory is applied for the solution in the same way as in [2] and [3].

2 THE DIPOLE ARRAY AND ITS EQUIVALENT SET OF TRANSMISSION LINES

A substantial part of the antenna system, which forms the radiation pattern, consists of several dipoles excited by slots; two of these dipoles are drawn in Fig. 1. This "dipole approach" will be left in the next explanation and the antenna part between two neighbouring slots (the part P-Q in Fig. 1) will be considered now as the basic array element being called the "antenna section". According to this idea, the whole array consists of several identical sections P-Q, of one incomplete section below (at the antenna bottom) and of one incomplete section above (at the antenna top).

Each "complete" section (the incomplete ones will be discussed later) consists of two separate transmission-lines. One of these lines is the main coaxial feeder f-f (between the cross-sections P and Q in Fig. 1 or Fig. 2). The second line is the outer surface of the system, which radiates; it has three parts (C_1-D_1, E_1-F_1, A_2-B_2) connected through the detuned resonators reactances X_1 and X_2. It is unsymmetrical one and its "back conductor" is the ground or an imaginary conducting surface in infinity. The inputs of both lines as well as their outputs are connected in series as it is obvious from Fig. 1.

In agreement with the idea described above, the equivalent scheme in Fig. 2 can be drawn. Both transmission lines (the inner one, *ie* the coaxial, and the outer one, *ie* the radiating surface) are drawn like two-wire lines. Wire 1 is the inner conductor of the coaxial, wire 2 is the inner surface of the coaxial shield. Wire 3 is the outer (radiating) antenna surface and wire 4 is the "ground" in infinity. The notation of individual points corresponds to that in Fig. 1. The relative independence of both transmission lines is reasonably accented by a greater separation between them. Actually, the "wires" between points

* Department of Radioelectronics, FEEC BUT Brno, Purkyňova 118, 612 00 Brno, Czech Republic, e-mail: novacek@feec.vutbr.cz

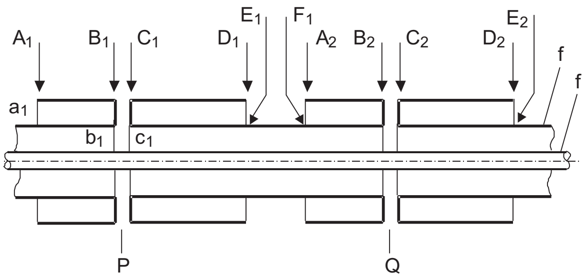

Fig. 1. The lengthwise-cross section of the analyzed array (schematic sketch). The system is vertically situated in reality.

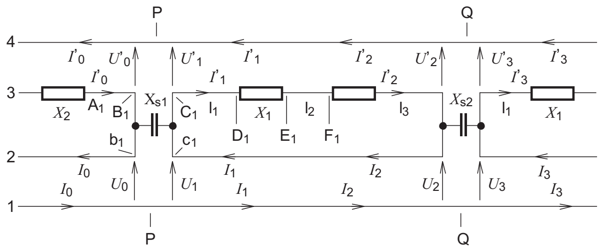

Fig. 2. Equivalent scheme of one complete section

b_1-B_1, c_1-C_1 *etc*, connecting the inner and the outer surfaces of the coaxial shield, are the opposite discs of each slot. They are (in a radial direction) very short with respect to the wavelength, so that the current at B_1 is equal to that in b_1, *etc*. However, the capacity between the opposite discs need not be negligibly small and its reactance is denoted as X_{s1} (and X_{s2}) in Fig. 2.

3 THE CURRENTS AND VOLTAGES IN THE ARRAY

3.1 The central part of the array. Sections P-Q

At first, the relations between the voltages and currents at the end and at the input of one section P-Q will be investigated. These relations will enable to connect an arbitrary number of sections into a cascade and then, using the lossy transmission-line theory, to calculate the current distribution on the antenna surface.

According to the model described above, the section P-Q is composed of two separate transmission-lines: of the main feeder and of the outer antenna surface. The main feeder (wires 1 and 2 in Fig. 2) is a homogeneous line. Between its input- and output voltages (U_1, U_2) and currents (I_1, I_2), the following well-known relation is valid:

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = [A] \cdot \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} . \tag{1}$$

The matrix

$$[A] = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_0 \sinh(\gamma l) \\ \sinh(\gamma l)/Z_0 & \cosh(\gamma l) \end{bmatrix} \tag{2}$$

where l is the line length, $\gamma = \beta + j\alpha$ is the propagation constant and Z_0 is the characteristic impedance of the line.

The second line in the model (the outer surface, wires 3 and 4) is not homogeneous. It consists of five elements connected in a cascade: three homogeneous transmission-lines (C_1-D_1, E_1-F_1, A_2-B_2) and two reactances (X_1, X_2). However, the outer surface as a whole is an electrically linear element and therefore its properties can be described by equations analogical with (1):

$$\begin{bmatrix} U'_1 \\ I'_1 \end{bmatrix} = \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} \cdot \begin{bmatrix} U'_2 \\ I'_2 \end{bmatrix} = [A'] \cdot \begin{bmatrix} U'_2 \\ I'_2 \end{bmatrix} \tag{3}$$

The quantities relevant to the outer (radiating) surface are differentiated by dashes. As matrix $[A]$ keeps similar properties like the transmitting matrix, the resulting matrix of a cascade connection of several elements is equal to the product of A- matrices of the isolated elements. Thus, the A-matrix of the outer surface $(3,4)$ is

$$[A'] = [A_1]\ [A_{x1}]\ [A_2]\ [A_{x2}]\ [A_3] = \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} \tag{4}$$

where $[A_1]$, $[A_2]$ and $[A_3]$ are the A-matrices of the type (1) of the homogeneous lines C_1-D_1, E_1-F_1, A_2-B_2 and

$$[A_{x1,2}] = \begin{bmatrix} 1 & jX_{1,2} \\ 0 & 1 \end{bmatrix} \tag{5}$$

are the matrices of series reactances $X_{1,2}$.

Firstly, the situation, when the capacitances C_s inside the slots are very small as a result of which the effect of reactances X_s can be neglected will be dealt with. In such a case, the following is evident from Fig. 2:

$$I_0 = I_1\,, \qquad I'_0 = I'_1\,, \tag{6a}$$

$$I_2 = I_3\,, \qquad I'_2 = I'_3\,, \tag{6b}$$

$$I'_1 = I_1\,, \qquad I'_2 = I_2\,. \tag{6c}$$

Let us denote further:

$$\begin{aligned} U_1 + U'_1 &= U_p\,, \\ U_2 + U'_2 &= U_q\,. \end{aligned} \tag{7}$$

Combining (6), (7) with (1b) and (3b), it can be found:

$$\begin{aligned} U_2 &= \frac{c'}{c+c'}\,U_B + \frac{d'-d}{c+c'}\,I_2\,, \\ U'_2 &= \frac{c}{c+c'}\,U_B - \frac{d'-d}{c+c'}\,I_2\,. \end{aligned} \tag{8}$$

The last relations are useful for numerical transformations of the current and voltage lengthwise the antenna array (from its end to its input). The quantities U_q and $I_3 = I_2$ at the cross-section Q follow from the analysis of

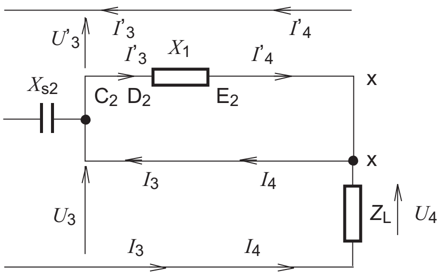

Fig. 3. The antenna top part

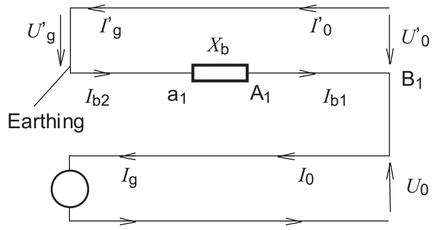

Fig. 4. The antenna bottom part

the antenna top part. The voltage U_q will be firstly decomposed between both transmission-lines (1,2; 3,4) using the equations (8). Then, the voltages and currents at the cross-section P will be obtained by the application of (1) and (3). Equations (8) also demonstrate that in case $C_s = 0$, the section P-Q can be treated like a two-port element: the output quantities are U_q and I_2 and the input ones are U_p and I_1.

Let us turn our attention to the case, when the effect of reactances X_s cannot be neglected, which makes the situation more complicated. The currents flowing through the reactances X_s cause coupling between both transmission-lines and the section P-Q must be treated like a single element — a four-port element. Voltages U_p and U_q lose their practical importance and all four quantities U, I, U' and I' must be considered as output or input quantities of the section.

Suppose a complete section consisting of the pair of transmission-lines $(U_2, U_2', \dots, U_1, U_1')$ and of the slot (the left one in Fig. 2). These two parts of the section are connected in cascade. The behaviour of the transmission-lines is described by equations (1) and (3). They can be easily integrated into one equation of the following form:

$$\begin{bmatrix} U_1' \\ I_1 \\ U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a' & b' & 0 & 0 \\ c' & d' & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{bmatrix} \cdot \begin{bmatrix} U_2' \\ I_2' \\ U_2 \\ I_2 \end{bmatrix}$$

or briefly

$$[UI]_1 = [A]\ [UI]_2\ . \tag{9}$$

The behaviour of the slot can be easily deduced from Fig. 2:

$$\begin{aligned} U_0' &= U_1' - jX_{s1}\left(I_1 - I_1'\right), \qquad & I_0' &= I_1', \\ U_0 &= U_1 - jX_{s1}\left(I_1 - I_1'\right), \qquad & I_0 &= I_1. \end{aligned} \tag{10}$$

In a matrix form, it is:

$$\begin{bmatrix} U_0' \\ I_0' \\ U_0 \\ I_0 \end{bmatrix} = \begin{bmatrix} 1 & jX_{s1} & 0 & -jX_{s1} \\ 0 & 1 & 0 & 0 \\ 0 & -jX_{s1} & 1 & jX_{s1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} U_1' \\ I_1' \\ U_1 \\ I_1 \end{bmatrix}$$

or briefly

$$[UI]_0 = [X]\ [UI]_1\ . \tag{11}$$

Finally, the following is valid for the complete section:

$$[UI]_0 = [X]_1\ [A]\ [UI]_2\ . \tag{12}$$

On the basis of the above mentioned, it is easy to complete the relations for a cascade connection of any number of sections. Only line parameters and reactances are contained in the square-matrices $[A]$ and $[X]$. The matrix elements are independent on voltages and currents and are constant for a given antenna. The product of matrices that may be cause troubles in an analytical form can be easily carried out numerically.

3.2 The top- and bottom parts of the array

To complete the analysis, the top- and bottom parts of the system have been taken into account. The simplified scheme of the antenna top is drawn in Fig. 3.

The coaxial line is terminated by an impedance Z_L and the antenna outer surface proceeds (like the "right" arm of the highest dipole) up to the reactance X_t of the internal cavity and, if need be, it proceeds by a part of the antenna construction. The corresponding matrix equation is

$$\begin{bmatrix} U_3' \\ I_3' \\ U_3 \\ I_3 \end{bmatrix} = \begin{bmatrix} a' & 0 & 0 & 0 \\ c' & 0 & 0 & 0 \\ 0 & 0 & 0 & aZ_L + b \\ 0 & 0 & 0 & cZ_L + d \end{bmatrix} \cdot \begin{bmatrix} U_4' \\ I_4' \\ U_4 \\ I_4 \end{bmatrix}$$

or briefly

$$[UI]_3 = [A_t]\ [UI]_4\ . \tag{13}$$

On the bottom part (see Fig. 4), the antenna outer surface proceeds downward up to the reactance X_b inside the "left" arm of the lowest dipole and then like the outer surface of the feeder up to the point, where the shield is earthed. When the upper (short-circuited) transmission-line in Fig. 4 is replaced by its input impedance Z', it has to be valid:

$$U_1' + Z'\,I_1' - jX_{s1}\,[I_1 - I_1'] = 0\,. \tag{14}$$

The equations (13) and (14) enable to finish the solution in the following way. The relations between voltages and currents at the antenna "output" (U_4', I_4', U_4, I_4, Fig. 4) are expressed. These relations depend upon the concrete arrangement of the antenna top, *eg* if the feeder is loaded by the impedance Z_L and the end of the outer surface

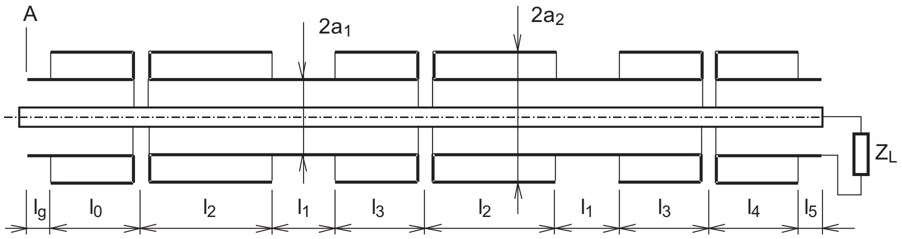

Fig. 5. The measured antenna

antenna is open circuited, $U_4/I_4 = Z_L$, $I_4' = 0$. Later on, the voltages and currents at the input of the first section are calculated:

$$\begin{bmatrix} U_1' \\ I_1' \\ U_1 \\ I_1 \end{bmatrix} = [X]\ [A]_1\ [X]_2\ [A]_2\ [X]_3\ [A]_t \cdot \begin{bmatrix} U_4' \\ 0 \\ U_4 \\ I_4 \end{bmatrix} = \\ = \begin{bmatrix} q_{11} & 0 & 0 & q_{14} \\ q_{21} & 0 & 0 & q_{14} \\ q_{31} & 0 & 0 & q_{34} \\ q_{41} & 0 & 0 & q_{44} \end{bmatrix} \cdot \begin{bmatrix} U_4' \\ 0 \\ U_4 \\ I_4 \end{bmatrix} . \qquad (15)$$

As mentioned above, the product of square matrices can be carried out in a numerical way as the matrix elements q_{ii} are numbers. Then, an arbitrary value of the current I_4 is chosen and the voltages and currents in the left-hand column matrix are expressed as a function of the voltage U_4' (Fig. 3). When substituting U_1', I_1' and I_1 into (14), the last unknown voltage U_4 can be found. The voltages and currents at the ends of individual parts of the outer antenna surface may be evaluated by a consecutive application of equations (13), (12) and (14). These values open the way to the current distribution and the antenna radiation.

The current distribution on individual parts of the antenna sections can be calculated from equations (1b) and (2). They must be applied to each part (C_1-D_1, E_1-F_1, A_2-B_2) of the outer surface separately and the total length l in (2) must be replaced by a variable coordinate measured from the end of each part backwards. The reactances X_1, X_2 may be passed using the matrices (5). Knowing the current distribution, the calculation of radiation pattern and of radiated power is a classical task. The theory of transmission-line enables also the determination of the approximate value of the antenna input impedance as well.

The attenuation constant β and the characteristic impedance of the transmission-lines have not been spoken about up to now. An applicable procedure for the determination of the constant β was proposed in [2]. For the initial (numerical) calculation, the constant β is chosen (estimated). After the evaluation of the antenna input-resistance R_{in} according to the transmission-line theory, the same quantity is calculated once more by the integration of the squared radiation pattern (from the radiated power). Both values are supposed to be identical. If they are not identical, another value of β must be chosen. It is easy to create an iteration procedure which presents suitable values sufficiently quickly.

The characteristic impedance Z_0 in (2) is the characteristic impedance of each part of the antenna outer surface (C_1-D_1, E_1-F_1, *etc*). These parts being cylindrical tubes are relatively thick with respect to their lengths. The approximate formula

$$Z_0 = 60\left(\ln 2l/a - 1\right) \qquad (16)$$

is often used for such tubes operating like unsymmetrical transmission-lines, (l is the tube length, $2a$ is its diameter). This formula was deduced with the help of a mean- potential method for a single (isolated) tube in the free space. It is taken for granted that the value of Z_0 is affected by the other tubes (other parts of the antenna surface) in our case. The mutual coupling need not be negligible and ought to be at least approximately determined. To keep a consistent approach, the mean-potential method is also applied for the estimation of mutual coupling between parts of the antenna surface. To simplify the result, let us suppose that only directly neighbouring tubes have a principal effect and identical currents flow on these tubes. After some simplifications, it can be found that due to the presence of one (*only one*) neighbouring part (tube), an additional term $+\ln(2) \doteq 0.69$ appears in the brackets of the formula (16). With this correction, it is

$$Z_0 = 60\left(\ln 2l/a - 0.31\right) \qquad (17a)$$

for the first and the last tube,

$$Z_0 = 60\left(\ln 2l/a + 0.38\right) \qquad (17b)$$

for the other parts (each having two adjoining tubes).

4 EXPERIMENTAL RESULTS

The method described above has been tested on the basis of a comparison between the calculated and measured radiation pattern and input impedance. The antenna built for this purpose is schematically drawn in Fig. 5. It has two identical sections (three dipoles). The lengths of dipole arms are equal one with another and with the "space" between neighbouring dipoles (l_1 in Fig. 5).

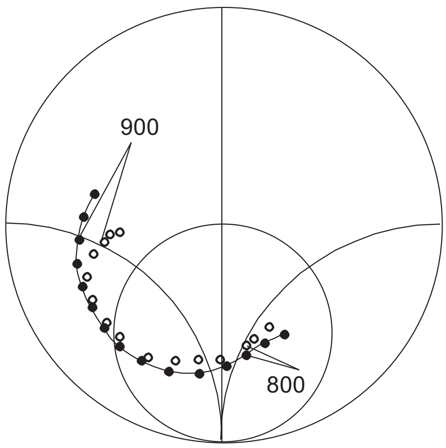

Fig. 6. The antenna input impedance from 780 MHz (lower end) up to 900 MHz (upper end). The curve is calculated; measured values are represented by small circles

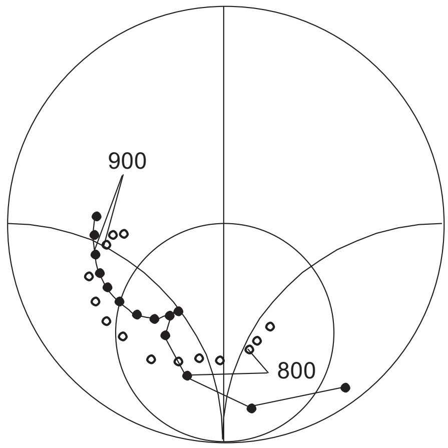

Fig. 7. The antenna input impedance when the length of cavities inside the dipole arms is shortened from 95 mm to 88 mm.

The most important data of the measured antenna are following:

lengths of arms: $l_0 = l_1 = l_2 = l_3 = 100$ mm;
top- and bottom parts: $l_5 = l_9 = 50$ mm;
diameters of tubes: $2a_1 = 10$ mm, $2a_2 = 22$ mm;
lengths of cavities inside arms: 95 mm;
characteristic impedance of the coaxial feeder:
$Z_0 = 85\,\Omega$;
phase velocity inside the feeder:
$v_f = 0.83\,c = \xi \times 10^8\,\mathrm{ms}^{-1}$;
capacity within slots: 1.5 pF (approx.);
top load: $Z_L = 0$.

The total length of one section is $l_c = l_1 + l_2 + l_3 = 300$ mm, which is also the geometrical length of the coaxial feeder inside. The electrical length of the feeder $(\alpha = 2\pi)$ is $2\pi\, l_c/(\lambda\,\xi)$. The electrical length at frequency $f_0 = 833$ MHz, is just 2π, and this frequency can be accepted as the nominal (basic) frequency of the antenna. A symmetric radiation pattern having only one dominant lobe (being perpendicular to the antenna axis) can be expected on this frequency.

Two most important antenna parameters have been measured and compared with the calculated ones: the input impedance and the radiation pattern. The input impedance was measured using the vector network analyzer. The values are related to the cross-section A in Fig. 5. The radiation pattern was measured in "relatively free" space (the open air, small obstacles were sporadically present) and therefore the measured pattern may be slightly disturbed. Both parameters were measured within the frequency band from 780 MHz up to 900 MHz.

The frequency dependence of input impedance is shown in Fig. 6. The measured values are denoted by small circles, the continuous line is calculated. When changing the length l_9, the impedance characteristic shifts around the diagram. It is interesting and even surprising that the formula (16) for the characteristic impedance of isolated parts of the antenna surface leads to better agreement between the calculation and experiment.

It is necessary to point out the fact that the impedance is considerably sensitive to some antenna dimensions. For instance, when the length of cavities inside the dipole arms is shortened by several millimeters, the impedance characteristics is changes dramatically (see Fig. 7).

The measured radiation pattern (small circles) and calculated ones (continuous line) on frequencies 780 MHz, 840 MHz and 900 MHz are shown in Fig. 8. The direction $\Theta = 90°$ is perpendicular to the antenna axis and $\Theta = 0$ is the direction downwards (to the feeding point). Although the antenna has many tuned elements, the radiation pattern remains nearly identical within a relatively wide frequency band: 100 MHz or a little more (see Fig. 8), which may found to be interesting.

Only the main lobe direction shifts by several degrees. The maximum value of directivity (the absolute gain) varies within the limits 5.2–4.2 (the radiation pattern is not practically affected by small alternations of antenna dimensions).

A brief summary of the main antenna properties is completed by the calculated current distribution on the outer surface of the antenna (see Fig. 9). The lowest frequency 780 MHz lies near to the cavity resonance frequency (790 MHz) and therefore the middle parts of each section are practically isolated, without perceptible current. The antenna behaves like a classical equiphase dipole array. The cavities are detuned significantly on the other frequencies and the whole outer surface radiates. It is not out of interest that the phase of current does not change its sign along the whole antenna.

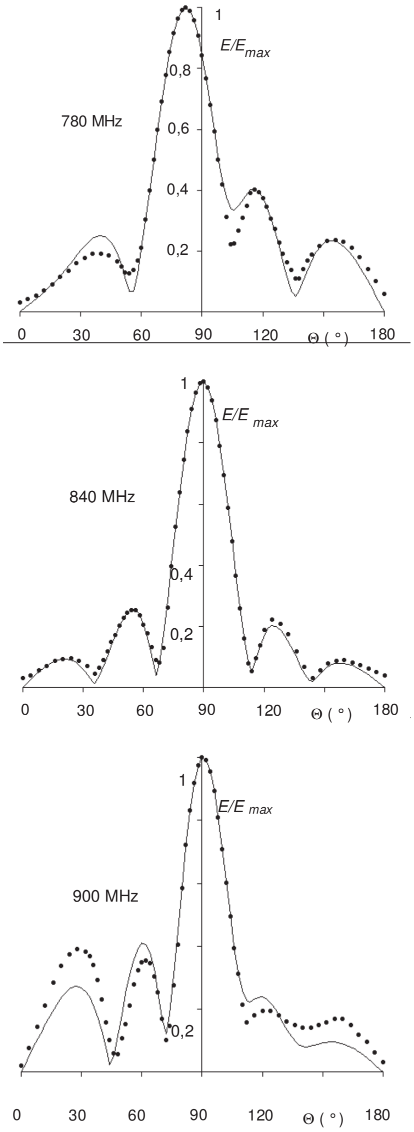

Fig. 8. The radiation pattern of the investigated antenna. Continuous line — calculated; circles — measured.

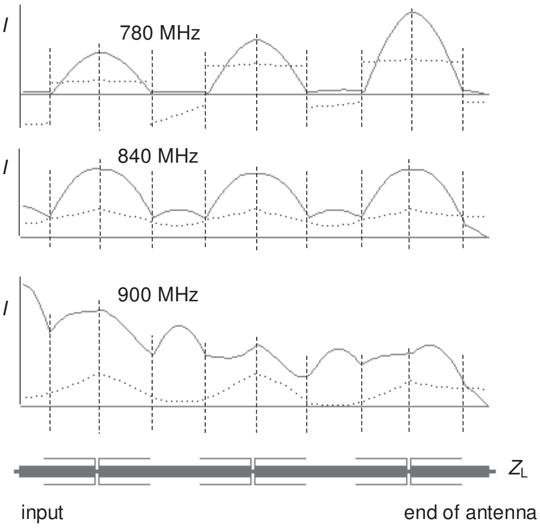

Fig. 9. The calculated current distribution. Continuous line — the amplitude; dashed line — the phase.

5 CONCLUSIONS

Two important conclusions follow from the above paragraphs.

1. The analyzed antenna array can be exploited like a moderate directional antenna (in a vertical plane). The application is not limited only to the narrow frequency band. By suitable choice of individual dimensions, miscellaneous forms of the radiation pattern can be reached (discussed in [3]). However, the pattern of an equiphase dipole array (Fig. 8) is likely to be the most often required form of the radiation pattern.

2. The agreement between the calculated and measured results demonstrates that the above-described analysis based on the lossy transmission-line theory brings sufficiently good results and can be applied in practice.

Acknowledgements

This project was supported by the research plan No. MSM 262200011 "Research of Electronic Communication Systems and Technologies".

References

[1] FUJIMOTO, K.—JAMES, J.R.: Mobile Antenna System Handbook, Artech House, London, 2001.

[2] ČERNOHORSKÝ, D.—NOVÁČEK, Z.: Dipole Array Excited by a Slots, Radioengineering **10** No. 4 (2001), 9–16.

[3] ČERNOHORSKÝ, D.—NOVÁČEK, Z.: Effect of Internal Capacitances in Slots Excited Dipole Array, in: Proceedings of the 12th. International Czech–Slovak Scientific Conference RADIOELEKTRONIKA 2002, Bratislava, 2002, pp. 278–281.

Received 30 May 2003

Dušan Černohorský was born in Prague, Czech Republic, 1930. He received his Ing (MSc) and the CSc (PhD) degrees from the Military Technical Academy, Brno, Czech Republic, in 1954 and 1965, respectively. In 1964, he spent a year at the Military Engineering College in Cairo, Egypt. Since 1970, he has been with the Department of Radioelectronics at the Brno University of Technology. From 1970 to 1991 he worked as an Associated Professor and in 1991 he became a Professor of this department. His research interests are EM field theory and antenna theory and practice. His main research areas include short wave and mobile antennas, adaptive antennas and the space-time signal processing.

Zdeněk Nováček received his Ing (MSc) and CSc (PhD) degrees from the Brno University of Technology, Czech Republic, in 1969 and 1980, respectively. Since 1969, he has been with the Department of Radioelectronics at the Brno University of Technology working firstly as a Senior lecturer and since 1997 as an Associated Professor. His research interests are EM field, antennas and the propagation of radio waves. His research areas include mobile antennas, antenna measurements and the space-time signal processing.