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# EQUIVALENT T–SUBOPTIMAL CONTROL OF A 3–SEGMENT LEG

**Ján Kardoš * — David Harvey ** — David Howard *****

A particular case of a chattering-free variable structure control (VSC) — the equivalent t-suboptimal control — was recently presented for a mechatronic system with one degree of freedom [1]. In this paper, the same principle reducing the drawbacks of conventional VSC design has been applied to the trajectory tracking control of a non-redundant rigid 3-segment leg dynamics. The efficacy and robustness of the control algorithm is demonstrated through numerical simulation.

Keywords: variable structure control, reachability (or reaching) condition, sliding mode, chattering-free, time suboptimality, 3-segment leg

## 1 INTRODUCTION

As part of a British Council funded Joint Slovak-UK Project, the applicability of a variable structure controller to the servo-control of a biped walking robot has been investigated. The partners in this joint project include the Slovak University of Technology (Slovakia), the University of Portsmouth (UK), and the University of Salford (UK).

A robust and chattering-free sliding mode controller has been developed for SISO position control systems [1]. This is a non-linear controller which produces a near time-optimal (t-suboptimal) response, and is robust to external disturbances and parameter uncertainty in the system.

A 3-segment leg (thigh, shank and foot) was represented by a two dimensional model in which each segment is considered to be rigid [2]. The dynamics of each segment was treated as being decoupled by representing the dynamic coupling between segments as external disturbances. By decoupling the dynamics of the three segments, it was possible to use three SISO sliding mode controllers, one for each joint motor (hip, knee and ankle).

A simulation program was developed and used to assess the performance of the sliding mode controller in this application.

## 2 THE DYNAMICS OF THE 3–SEGMENT LEG

The two dimensional 3-segment leg has dynamic equations of motion of the following form

$$\mathbf{J}(\boldsymbol{q})\frac{\mathrm{d}^2\boldsymbol{q}}{\mathrm{d}t^2} = \boldsymbol{\tau} - \mathbf{B}\frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}t} - \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{c}(\boldsymbol{q},\dot{\boldsymbol{q}}) \tag{1}$$

where $\mathbf{J}(\boldsymbol{q})$ is the $3\times 3$ inertia matrix, $\boldsymbol{q}$ is the $3\times 1$ vector of joint angles, $\boldsymbol{\tau}$ is the $3\times 1$ vector of joint torques, $\mathbf{B}$ is the $3\times 3$ constant damping matrix, $\boldsymbol{g}(\boldsymbol{q})$ is the $3\times 1$ vector of gravity terms, and $\boldsymbol{c}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is the $3\times 1$ vector of Coriolis and centrifugal terms where $\dot{\boldsymbol{q}}$ is the derivative of $\boldsymbol{q}$ with respect to time.

Assuming diagonal inertia and damping matrices, the decoupled equation of motion for a single segment can be written as

$$\frac{\mathrm{d}^2 q_i}{\mathrm{d}t^2} = \frac{1}{j_{ii}(\boldsymbol{q})}\Big(\tau_i - b_i\frac{\mathrm{d}q_i}{\mathrm{d}t} - disturbances\Big), \tag{2}$$

where the Coriolis and centrifugal terms, and the gravity terms are treated as disturbances. This can be rewritten in the following standard form

$$\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} = \frac{1}{T(\boldsymbol{q})}\Big(K\tau - \frac{\mathrm{d}q}{\mathrm{d}t} - disturbances\Big), \tag{3}$$

where $K$ is the system gain and $T(\boldsymbol{q})$ is the system time constant. Equation (3) is in a form that allows the Variable Structure Controller developed by Kardoš [1] to be applied to the position control of each segment in a decoupled manner.

## 3 THE VARIABLE STRUCTURE CONTROLLER

If a feedback position controller is applied to the SISO system described by equation (3), and it is assumed that parametric disturbances can be represented by variations in $T$ provided that $|K\tau| \geq |disturbances|$, then using equation (3) the system can be described in the error vector space by the phase canonical form

$$\frac{\mathrm{d}\boldsymbol{e}}{\mathrm{d}t} = \begin{bmatrix} \dot{e} \\ -\frac{1}{T}(Ku + \dot{e}) \end{bmatrix}, \tag{4}$$

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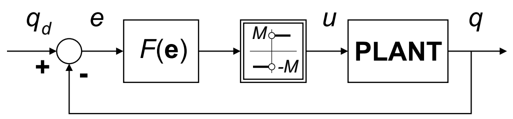


**Fig. 1.** Basic structure of the sliding-mode control system.

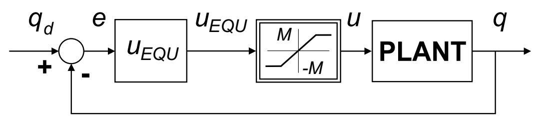


**Fig. 2.** Chattering-free equivalent control system.

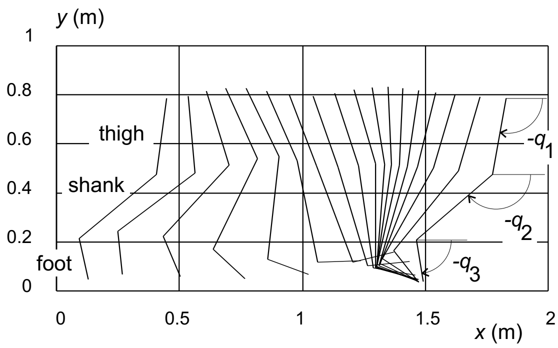


**Fig. 3.** Phases of a normal human walking.

where $K$ is the system gain, $T$ is the system time constant, $\boldsymbol{e}$ is the error vector with $e$ and $\dot{e}$ elements ($\dot{e}$ stands for the time derivative of $e$), and $u$ is the control effort, which in this case is the joint torque. The variations in $T$ are such that

$$T \in \langle T_{\min}, T_{\max} \rangle \,. \tag{5}$$

A robust and chattering-free variable structure controller has been developed for plants that can be described by equations (4) and (5). In summary, the principles underpinning this controller are as follows.

A basic sliding mode controller [3 and 4] can be devised (Fig. 1) which uses a switching function, $F(\boldsymbol{e})$, to set the control effort such that

$$u = M \operatorname{sgn}(F) \,, \tag{6}$$

where $M$ is the maximum absolute value of the control effort, and the switching function is given by

$$F(\boldsymbol{e}) = \dot{e} + \alpha e = 0 \,, \tag{7}$$

where $\alpha > 0$ ($\alpha = const.$) is chosen to minimize the response time whilst avoiding overshoot for any value of $T$ within the range (5) [1]. In the close neighbourhood of the switching line, $F(\boldsymbol{e}) = 0$, the system is in a sliding mode if the following reachability condition [4] is satisfied

$$F(\boldsymbol{e}) \frac{\mathrm{d}F(\boldsymbol{e})}{\mathrm{d}t} < 0 \,, \tag{8}$$

which means that the system's phase plane trajectory will converge on the switching line.

The problem with this simple sliding mode controller is that when it reaches the sliding mode, it will chatter as it moves along the switching line, $F(\boldsymbol{e}) = 0$. Here, this problem is solved by introducing a linear control band within which the control effort can change continuously (Fig. 2). Within the linear control band, the reachability condition (8) is guaranteed by imposing the following relationship ($k$ is a positive constant)

$$\frac{\mathrm{d}F(\boldsymbol{e})}{\mathrm{d}t} = -kF(\boldsymbol{e}) \,. \tag{9}$$

By combining equations (4), (7) and (9), the following equation for the equivalent control effort can be derived

$$u_{EQU} = \frac{1}{K} \left[ (\alpha T_u - 1)\dot{e} + kT_u(\dot{e} + \alpha e) \right] \,, \tag{10}$$

where $T_u$ is an unknown time constant originating from the plant's parameter uncertainty (5). It can be shown [1] that $T_u \geq T_{\max}$ if overshoot is to be avoided for all $T$ within the range (5).

Therefore the robust, chattering-free variable structure controller is defined as follows

$$u = \begin{cases} u_{EQU} & \text{for } \operatorname{abs}(u_{EQU}) < M \,, \\ M \operatorname{sgn}(u_{EQU}) & \text{for } \operatorname{abs}(u_{EQU}) \geq M \,. \end{cases} \tag{11}$$

The equivalent control effort, $u_{EQU}$, varies continuously within the linear control band and ensures that the system's phase-plane trajectory converges on the switching line, $F(\boldsymbol{e}) = 0$, and then slides along the switching line without chattering. To obtain a t-suboptimal response, $k$ must satisfy the conditions

$$k \gg 1 \quad \text{and} \quad k \gg \alpha \,.$$

Whilst developing this variable structure controller, new theorems and lemmas were established relating to the existence of a sliding mode, the description of the sliding mode behaviour, the uniqueness of the *equivalent control* description of the sliding mode, and the avoidance of overshoot [1].

## 4 SIMULATION RESULTS

The dynamic equations of motion for the 3-segment leg (1) and the variable structure controller, (10) and (11), have been incorporated into a simulation program [1 and 2]. Three separate controllers were used, one for

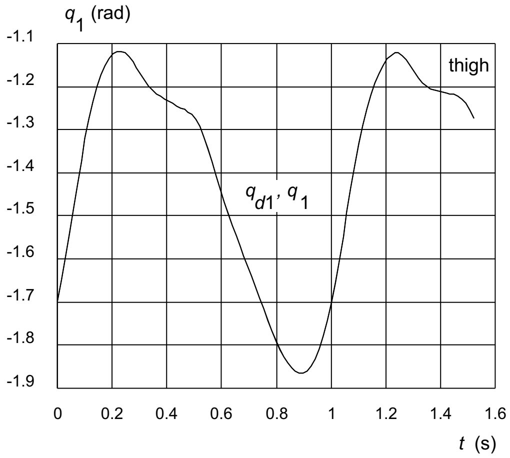


**Fig. 4.** The desired $q_{d1}$ and the actual $q_1$ trajectories for the thigh.

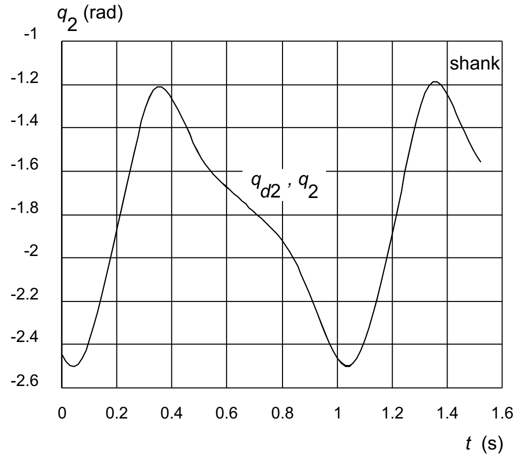


**Fig. 5.** The desired $q_{d2}$ and the actual $q_2$ trajectories for the shank.

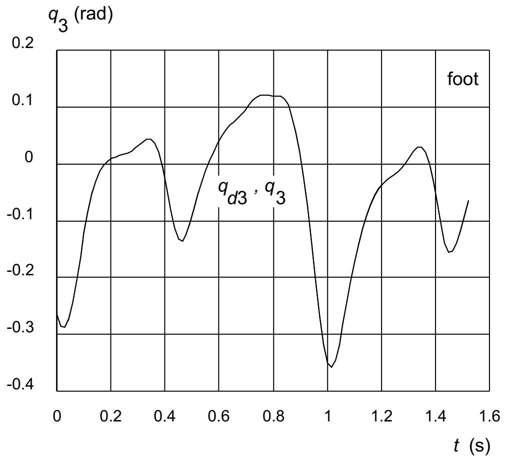


**Fig. 6.** The desired $q_{d3}$ and the actual $q_3$ trajectories for the foot.

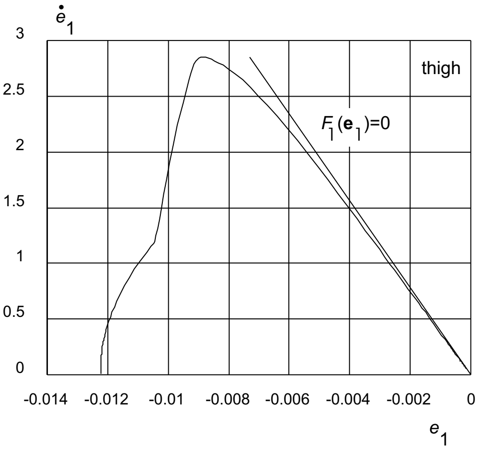


**Fig. 7.** Typical phase portrait in the error vector plane for the thigh

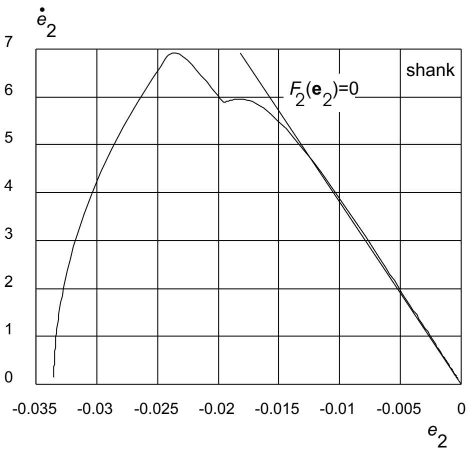


**Fig. 8.** Typical phase portrait in the error vector plane for the shank

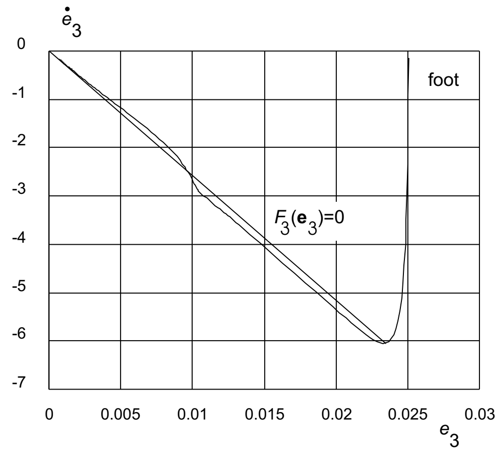


**Fig. 9.** Typical phase portrait in the error vector plane for the foot

each segment (joint motor). The simulation program has been used to simulate the controller's ability to follow a trajectory similar to that of a human leg during normal walking [5] (see Fig. 3).

Figures 4-6 show the desired trajectories and the actual trajectories for the thigh, shank and foot segments. It can be seen that excellent trajectory following is achieved provided that the set-point values are updated rapidly enough. In other words, the trajectories must be described by a series of desired positions that are sufficiently close in time. In this case, the set-points were updated every 0.0145 seconds.

Figures 7–9 illustrate a typical phase portrait between two set-points in the error vector plane for the hip, knee and ankle joints respectively. In this case, the corresponding sample time was 0.5075 sec. The interactions between leg segments are visible in the form of ripples of trajectories. Despite the presence of these disturbances, each trajectory converges on the switching line $F_i(\boldsymbol{e}) = 0$.

## 5 DISCUSSION

A robust and chattering-free sliding mode controller has been developed for SISO servo-systems. Based on the equivalent control approach, the non-linear control law produces a near time-optimal response and guarantees robustness to external disturbances and parameter uncertainty in the system model.

Three SISO sliding mode controllers, one for each joint motor, have been used to control a 3-segment leg represented by the two-dimensional model (1). The dynamics of each segment was treated as being decoupled by representing the dynamic coupling between segments as external disturbances.

Simulation results show that excellent trajectory following is achieved provided that the set-point values are updated rapidly enough.

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