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COMMUNICATIONS

SIMPLE RELATIONSHIPS FOR ESTIMATION OF THE PERFORMANCES OF SC INTEGRATORS WITH NON–IDEAL AMPLIFIERS

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Simple formulae relating the dc gain, the phase errors and the offset voltage errors of SC integrators with non-ideal opamps are presented. The validity of the analytical expressions is demonstrated for charge-differencing very-large-time-constant inverting SC integrators.

K e y w o r d s: operational amplifiers, SC integrators

1 INTRODUCTION

The most important frequency limitation in SC filters is imposed by the operational amplifier gain-bandwidth restrictions. A finite bandwidth reduces the speed of the operational amplifiers (opamps) by introducing a limit on the sampling frequency in order to assure a full charge transfer during the individual clock phase. But, in SC circuits the distortion introduced by the finite gain A is more pronounced than that of the finite bandwidth [1]. On the other hand a nonzero input-referred opamp offset voltage V_{os}, which can be considered constant, introduces an output offset voltage that may become a significant limitation to the permissible signal swing. To compare the SC integrators in terms of magnitude error $m(\omega)$, phase error $\theta(\omega)$ and offset voltage error γ, it is advantageous to use the known $\alpha\beta\gamma$ representation defined in [2]. Moreover the dc gain $H(0)$ of the integrators can also be an estimation for their performances.

In this paper simple relationships between $H(0)$, $\theta(\omega)$ and γ are derived. These analytical formulae can be used to compare the performances of different SC integrators.

2 THEORETICAL RESULTS

In the analysis it is assumed that the opamps have a finite dc gain $A = 1/\mu$ and an infinite bandwidth. This supposition is adequate for the analysis of SC circuits containing fast and relatively low-gain amplifiers.

The $\alpha\beta\gamma$ representation is based on writing the input/output relation of the integrators in the form [2]

$$v_o(n) = \mp\alpha k_{id} v_{in}(n-p) + \beta v_o(n-1) + \gamma V_{os}\,. \qquad (1)$$

Here, k_{id} is the gain of the ideal integrator, $\alpha = 1 + \Delta\alpha$ represents the change in gain, $\beta = 1 + \Delta\beta$ is the shifted pole frequency and γ is the suppression factor of the offset voltage V_{os}. In the ideal case, $\alpha = \beta = 1$, while $\gamma = 0$. The value of index p depends on the integrator considered. The upper sign $(-)$ is applied to inverting integrators, whereas the lower sign $(+)$ is applied to non-inverting integrators.

For all the integrators satisfying relation (1) the parameter β is positive and in most cases less than unity ($0 < \beta < 1$, $\Delta\beta < 0$) when the finite opamp gain is taken into account.

The output voltage of the non-ideal integrator, with the input grounded ($v_{in} = 0$) and $0 < \beta < 1$, is given by

$$v_{os}(n) = \gamma V_{os}(1+\beta+\beta^2+\cdots+\beta^{n-1}) = \gamma V_{os}\frac{1-\beta^n}{1-\beta}\,. \qquad (2)$$

The theoretical steady-state output voltage v_{oss}, its limited excursion being neglected, is

$$v_{oss} = V_{os}/\mu\,. \qquad (3)$$

From (2), when the number of clock periods n tends to infinity, the following expression for v_{oss} is obtained

$$v_{oss} = \gamma V_{os}/(1-\beta)\,. \qquad (4)$$

Comparing (3) and (4)

$$1 - \beta = \gamma\mu = -\Delta\beta \qquad (5)$$

is found. Then (2) can be rewritten in the form

$$v_{os}(n) = \frac{V_{os}}{\mu}\left[1 - (1-\gamma\mu)^n\right]\,. \qquad (6)$$

The z-transform of (1), for $V_{os} = 0$, is given by

$$H(z) = \mp\frac{k_{id}\alpha z^{-p}}{1-\beta z^{-1}}\,. \qquad (7)$$

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Table 1. Comparison of SC integrators in terms of $H(0)$ and α

Integr.	DC gain $H(0)$	α
Fig. 1	$-\frac{k(1-2\mu)}{\mu(5+6k)(1+\mu)}$	$1-4\mu$
Fig. 2	$-\frac{0.5k(1-2\mu)}{\mu(1+k)[1+(2.5+1.5k)\mu]}$	$1-(4+3k)\mu$
Fig. 3	$-\frac{0.5k}{\mu^2(2.5+5.5k)}$	$1-2k\mu$

Table 2. Comparison of SC integrators in terms of γ and $\theta(\omega)$

Integr.	γ	$\theta(\omega)$
Fig. 1	$\frac{k(2.5+3k)(1-\mu)}{(1+k)(1+1.5k)}$	$\frac{\mu k(1.25+1.5k)}{(1+k)(1+1.5k)\tan(\omega T/2)}$
Fig. 2	$\frac{k[1+0.5(1-k)\mu]}{1+1.5k}$	$\frac{0.5\mu k}{(1+1.5k)\tan(\omega T/2)}$
Fig. 3	$\frac{k(2.5+5.5k)\mu}{(1+k)(1+1.5k)}$	$\frac{0.5\mu^2 k(2.5+5.5k)}{(1+k)(1+1.5k)\tan(\omega T/2)}$

At dc the transfer function of the integrator is reduced to

$$H(z)\big|_{z=1} = H(0) = \mp\frac{k_{id}\alpha}{1-\beta} = \pm\frac{k_{id}\alpha}{\Delta\beta}\,. \tag{8}$$

The sensitivity of the integrator's output to the opamp offset voltage V_{os} (the offset caused drift) can be evaluated by the number of clock periods n_q for which the output voltage $v_{os}(n_q)$ reaches the value qV_{os} $(q<1)$ with the amplifier operating linearly. Thus, from (5), (6) and (8) we obtain

$$n_q = \frac{\ln(1-q)}{\ln\big(1\pm k_{id}\alpha/H(0)\big)} \approx \pm\frac{H(0)}{k_{id}\alpha}\ln(1-q) \tag{9}$$

for $\ln(1\pm x)\approx \pm x$.
The phase error of the integrator is given by [2]

$$\tan\theta(\omega) \approx -\frac{\Delta\beta}{2}\operatorname{cotan}(\omega T/2) \tag{10}$$

where T is the sampling period.
Combining (10) with (5) and (8) we have

$$\tan\theta(\omega) \approx \mp\frac{k_{id}\alpha}{2H(0)}\operatorname{cotan}(\omega T/2) = \frac{\gamma\mu}{2}\operatorname{cotan}(\omega T/2) \tag{11}$$

and

$$H(0)\gamma\mu/\alpha = \mp k_{id}\,. \tag{12}$$

This relationship which for $\alpha\approx 1$ is reduced to

$$H(0)\gamma\mu \approx \mp k_{id} \tag{13}$$

enables us to calculate the suppression factor γ if the dc gain $H(0)$ is preliminarily known.

Let us consider two SC integrators with the same gain k_{id} and with the same opamp dc gain $A=1/\mu$. Then, from (9) and (11) the following relationships are obtained

$$\frac{H_1(0)}{H_2(0)} = \frac{\alpha_1\tan\theta_2(\omega)}{\alpha_2\tan\theta_1(\omega)} = \frac{n_{q1}\alpha_1}{n_{q2}\alpha_2} = \frac{\alpha_1\gamma_2}{\alpha_2\gamma_1}\,. \tag{14}$$

For $\alpha_1\approx 1$, $\alpha_2\approx 1$ and $\theta \ll 1$, formula (14) can be rewritten as

$$\frac{H_1(0)}{H_2(0)} \approx \frac{\theta_2(\omega)}{\theta_1(\omega)} \approx \frac{n_{q1}}{n_{q2}} \approx \frac{\gamma_2}{\gamma_1}\,. \tag{15}$$

The above formulae suggest that the preliminary knowledge of the values $H_i(0)$ can be used to compare the performances of different SC integrators in terms of phase error, offset error and offset caused drift.

3 COMPARATIVE STUDY OF SOME INVERTING SC INTEGRATORS

In order to demonstrate the validity of relationships (12) and (15), three charge-differencing (CD) very-large-time-constant (VLT) inverting SC integrators are considered:

1. the basic uncompensated Lin-92 integrator [3];
2. the gain-and-offset compensated (GOC) Lin-92 integrator [3];
3. the GOC integrator proposed in [4].

Figures 1 to 3 show the above integrators. In addition to the clock phase 1 and 2, the integrator from Fig. 3 requires two non-overlapping clocks, e and o, shown in Fig. 3b. The output voltage V_o is sampled in phase $2o = 2+o$. The derived approximate expressions for $H(0)$, α, γ and $\theta(\omega)$ of the three integrators are summarized in Table 1 and Table 2, for $Ca_1 = 1.5$, $C_1 = Ca_2 = C_h = 1$ and $k = C_1/C_A$.
Substituting the corresponding expressions for $H(0)$, α and γ from these tables into (12), we get for $\mu \ll 1$ and $k<1$:

$$H_1(0)\gamma_1\mu/\alpha_1 \approx -k_{id}(1+6\mu^2) \approx -k_{id}\,, \tag{16a}$$

$$H_2(0)\gamma_2\mu/\alpha_2 \approx -k_{id}(1+k\mu) \approx -k_{id}\,, \tag{16b}$$

$$H_3(0)\gamma_3\mu/\alpha_3 \approx -k_{id}(1+2k\mu) \approx -k_{id}\,. \tag{16c}$$

From (15) and tables 1 and 2 we obtain

$$\frac{H_2(0)}{H_1(0)} \approx \frac{\theta_1(\omega)}{\theta_2(\omega)} \approx \frac{\gamma_1}{\gamma_2} \approx \frac{2.5+3k}{1+k} = 2.53\,, \tag{17a}$$

$$\frac{H_3(0)}{H_1(0)} \approx \frac{\theta_1(\omega)}{\theta_3(\omega)} \approx \frac{\gamma_1}{\gamma_3} \approx \frac{2.5+3k}{\mu(2.5+5.5k)} = 474\,, \tag{17b}$$

$$\frac{H_3(0)}{H_2(0)} \approx \frac{\theta_2(\omega)}{\theta_3(\omega)} \approx \frac{\gamma_2}{\gamma_3} \approx \frac{1+k}{\mu(2.5+5.5k)} = 187 \tag{17c}$$

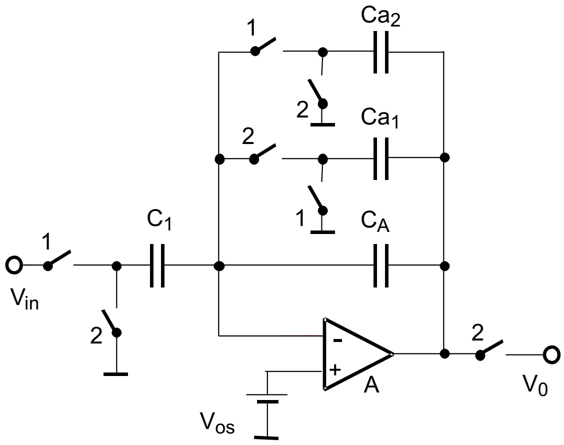

Fig. 1. Basic Lin-92 VLT integrator

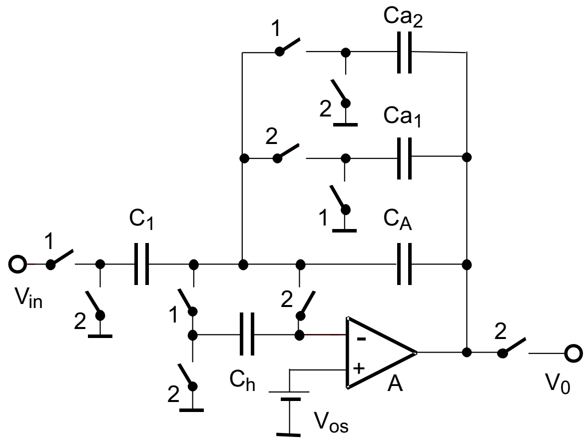

Fig. 2. Lin-92 GOC VLT integrator

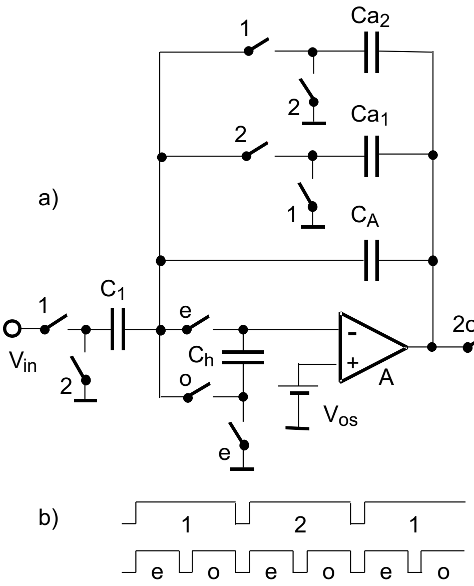

Fig. 3. Novel GOC VLT integrator and clocking scheme, a. GOC VLT integrator, b. Clocking scheme.

for $A = 500$ and $k = 1/16.77949$. These results confirm the validity of relationships (12) and (15).

4 CONCLUSION

Simple formulae relating the phase errors and the offset voltage errors of SC integrators with non-ideal opamps have been derived. It was shown that the dc gain of the integrators $H(0)$ could be used as a criterion for comparing their performances. On the other hand the dc gain $H(0)$ is more easily calculated than the other performance parameters. The validity of the analytical expressions has been demonstrated for CD VLT inverting SC integrators. The given approach can be applied to other types of SC integrators as well.

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