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ITERATIVE WEIGHTED LEAST SQUARES IDENTIFICATION AND PSRM CONTROL DESIGN

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This paper presents an iterative refinement scheme of successive closed-loop identifications and control law designs that aim to improve iteratively the achieved closed-loop performances. The key to this approach is to account for the evaluated modelling error in the control design and to let the closed-loop controller requirements determine the identification criteria. The first is achieved by the partial state reference model (PSRM) control criterion with frequency weighting filters that reflect the mismatch between the actual and the designed closed loop systems. The PSRM approach has been motivated by its suitable tracking capability. The least-squares identification is performed on closed-loop data with a filter, which improves the model accuracy at those frequencies where the robust performance dictates that a better model is needed. The effectiveness of the proposed iterative design strategy is illustrated by an example.

K e y w o r d s: partial state reference model control, closed-loop identification, least squares estimation, LQG cost function

1 INTRODUCTION

The design of linear control systems is frequently based on a model of the plant under consideration. When a reduced complexity of a model is used, the controller generally works better with the model than with the modelled plant due to the modelling error. It has been shown [1] that this difference between the performances can be minimized by selecting a model that is accurate at the closed loop relevant frequencies. Such a model can be obtained by performing the plant model identification in a closed loop and with an appropriate data filter. Since the data weighting filter is a function of an optimal controller, this approach can only be implemented by replacing the filter by its approximation, thus leading to the iterative control and identification method. The iterative aspect is essential because a model is needed for controller design and knowledge of the controller is needed in order to identify a good model.

The idea of combining the identification and the control in a mutually supportive way has been developed by many authors. The concept of optimal identification design for control was introduced by Gevers and Ljung in [1]. The idea of closed loop identification with a performance robustness enhancement data filter was first advocated in [2]. The dual idea of incorporating model error information into the LQG criterion was proposed in [3]. Different iterative control/identification schemes varying in the identification criteria and techniques, the identification data weighting filter design, the model structures that are used, the control design criteria have been proposed by several authors [6, 7, 8]. A survey of various approaches can be found for example in [4, 5].

An iterative control design procedure proposed in this paper is based on the approach developed in [8] for LQG control design. It consists of repeated application of a system identification procedure operating on closed loop data together with successive refinements of the designed controller. The control synthesis is using the partial state reference model (PSRM) control method and is carried out using the frequency weighted LQG cost function with filters that reflect the mismatch between the actual and the designed closed loop systems. The closed loop identification is performed using the modified least-squares (LS) method, where signals incoming to identification are filtered with a filter which improves model accuracy at those frequencies where the robust performance dictates that a better model is needed.

A detailed description of the iterative procedure is in Section 2. The control system performances are illustrated using a simple example in Section 3. Several concluding remarks end the paper.

2 PROBLEM STATEMENT

Consider that the input-output behaviour of the plant may be approximated by the ARMAX model

$$\begin{aligned} A(\theta,q^{-1})y(t) &= q^{-d}B(\theta,q^{-1})u(t)+v(t)+w(t) \\ D(q^{-1})v(t) &= C(q^{-1})\xi(t) \end{aligned} \tag{1}$$

with

$$\begin{aligned} A(\theta,q^{-1}) &= 1+a_1q^{-1}+\cdots+a_{na}q^{-na} \\ B(\theta,q^{-1}) &= b_0+b_1q^{-1}+\cdots+b_{nb}q^{-nb}\,;\ b_0\neq 0 \end{aligned}$$

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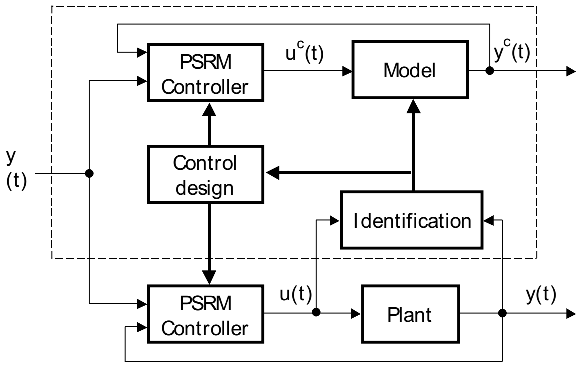

Fig. 1. Iterative PSRM control design

$$\begin{aligned} C(q^{-1}) &= 1 + c_1 q^{-1} + \cdots + c_{nc} q^{-nc} \\ D(q^{-1}) &= 1 + d_1 q^{-1} + \cdots + d_{nd} q^{-nd} \\ \theta^\top &= [a_1 \ \dots a_{na} \ b_0 \ \dots \ b_{nb}] \end{aligned}$$

where $u(t)$ is the control variable, $y(t)$ is the measured plant output, d denotes the minimum plant model delay in sampling periods, $v(t)$ and $w(t)$ represent the external disturbances and modelling errors, respectively, $\{\xi(t)\}$ is assumed to be a sequence of widely spread pulses of unknown magnitude or independent random variables with zero mean values and finite variances and θ denotes the vector of plant model parameters. The transfer function $z^{-d-1}B(\theta, z^{-1})/A(\theta, z^{-1})$ would represent the control model, while the transfer function $D(z^{-1})/C(z^{-1})$ denotes the internal model of the state disturbances.

The proposed iterative control design procedure consists of two parts (Fig. 1):

- a local PSRM design using the model of the plant and the plant/model mismatch information,
- a local LS identification performed on the closed loop data with a specific data filter.

2.1 PSRM control design

To ensure the suitable tracking capability, the PSRM approach has been used to design the controller [9]. The plant model (1) may be given the following partial state representation

$$\begin{aligned} A(\theta, q^{-1})D(q^{-1})x(t) &= q^{-(d+1)}D(q^{-1})u(t) \\ y(t) &= B(\theta, q^{-1})x(t) \end{aligned} \tag{2}$$

where $x(t)$ denotes the partial state. The ideal deadbeat partial state reference model control objective is

$$x(t) - x^*(t) = 0 \tag{3}$$

where $x^*(t)$ is the desired partial state, which can be specified as an output of asymptotically stable system

$$x^*(t) = \beta\, y^*(t) \tag{4}$$

$$A_m(q^{-1})y^*(t+d+1) = B_m(q^{-1})u^*(t)\,, \quad \beta = \frac{1}{B(1)}$$

where $B_m(z^{-1})/A_m(z^{-1})$ is the reference model transfer function, $\{u^*(t)\}$ is a bounded set-point sequence and β is a scalar introduced to get a unitary closed loop static gain.

Substituting (4) into (2–3) yields

$$\begin{aligned} A(\theta, q^{-1})D(q^{-1})\beta y^*(t) &= D(q^{-1})u(t-d-1) \\ y(t) &= B(\theta, q^{-1})\beta y^*(t) \end{aligned} \tag{5}$$

This allows to restate the ideal PSRMC objective (3) as follows

$$e_y(t) = D(q^{-1})e_u(t) = 0 \tag{6}$$

where

$$\begin{aligned} e_y(t) &= y(t) - B(\theta, q^{-1})\beta\, y^*(t) \\ e_u(t) &= u(t) - A(\theta, q^{-1})\beta\, y^*(t+d+1) \end{aligned} \tag{7}$$

In the non-ideal case, when the system output can be influenced by state disturbances and/or unmodelled dynamics, it is advisable to relax the ideal PSRMC objective (6) using the frequency weighted LQG criterion

$$J^C = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} \Big\{ \big[F_1(z^{-1})e_y(t)\big]^2 + \lambda^2 \big[F_2(z^{-1})D(z^{-1})e_u(t)\big]^2 \Big\} \tag{8}$$

where λ is a non-negative control weighting scalar and $F_1(z^{-1})$ and $F_2(z^{-1})$ denote user specified input and output frequency weighting, respectively.

The weighting functions are chosen as the following ratios of the estimated spectra

$$\begin{aligned} F_1(z^{-1}) &= \frac{w_{yn}(z^{-1})}{w_{yd}(z^{-1})} = \frac{\hat{\Phi}_{ey}}{\hat{\Phi}_{eyc}} \\ F_2(z^{-1}) &= \frac{w_{un}(z^{-1})}{w_{ud}(z^{-1})} = \frac{\hat{\Phi}_{eu}}{\hat{\Phi}_{euc}} \end{aligned} \tag{9}$$

where $\hat{\Phi}_{ey}$, $\hat{\Phi}_{eu}$, $\hat{\Phi}_{eyc}$, $\hat{\Phi}_{euc}$ denote spectra of signals $e_y(t)$, $e_u(t)$, $e_y^c(t)$, $e_u^c(t)$ that can be estimated from data using low order AR models.

$$\begin{aligned} e_y^c(t) &= y^c(t) - B(\theta, q^{-1})\beta\, y^*(t) \\ e_u^c(t) &= u^c(t) - A(\theta, q^{-1})\beta\, y^*(t+d+1) \end{aligned} \tag{10}$$

are signals obtained from the simulated closed loop with the designed controller where the true plant is replaced by the plant model (see Fig. 1).

The effect of frequency weightings can be interpreted as follows: if at some frequency $\hat{\Phi}_{ey}$ is larger than $\hat{\Phi}_{eyc}$, this means that at that frequency the model fit is poor with the consequence that the achieved performance is worse than expected from the designed system. Hence more emphasis should be put on the weighting at that frequency at the next control design stage, which is reflected by the weighting being larger than 1. If at some frequency $\hat{\Phi}_{ey}$ is smaller than $\hat{\Phi}_{eyc}$, this also means that at that frequency the model fit is poor but in such a way that the controller actually achieves a better performance on the true plant than on the model. The penalty at that frequency should therefore be decreased at the next control design stage to provide scope for improvement at other frequencies.

The PSRM design can be derived from the plant model reparametrization

$$\bar{A}(\theta, q^{-1}) e_y^f(t) = \bar{B}(\theta, q^{-1}) e_u^f(t) + \bar{C}(q^{-1})\xi(t) \quad (11)$$

with

$$\begin{aligned} \bar{A}(\theta, q^{-1}) &= A(\theta, q^{-1}) D(q^{-1}) w_{yd}(q^{-1}) w_{un}(q^{-1}) \\ \bar{B}(\theta, q^{-1}) &= B(\theta, q^{-1}) w_{yn}(q^{-1}) w_{ud}(q^{-1}) \\ \bar{C}(q^{-1}) &= C(q^{-1}) w_{yn}(q^{-1}) w_{un}(q^{-1}) \\ e_y^f(t) &= F_1(z^{-1}) e_y(t), \quad e_u^f(t) = F_2(z^{-1}) e_u(t) \end{aligned} \quad (12)$$

The resulting controller can be implemented in the general linear RST form

$$\begin{aligned} S(\theta, q^{-1}) D(q^{-1}) u(t) + R(\theta, q^{-1}) y(t) \\ = T(q^{-1}) y^*(t + d + 1) \end{aligned} \quad (13)$$

with $\quad P_c(\theta, q^{-1}) = A(\theta, q^{-1}) D(q^{-1}) S(\theta, q^{-1}) + B(\theta, q^{-1}) R(\theta, q^{-1})$.

The corresponding tracking and regulation dynamics are given by

$$\begin{aligned} y(t) &= B(\theta, q^{-1}) \beta y^*(t) \\ P_c(\theta, q^{-1}) y(t) &= S(\theta, q^{-1}) C(q^{-1}) \xi(t) \end{aligned} \quad (14)$$

As can be seen from equation (14), the PSRM concept allows to separate the desired tracking behaviour and desired regulation dynamics. The latter is closely related to the characteristic polynomial $P_c(\theta, q^{-1})$ which can be factored as follows

$$P_c(\theta, q^{-1}) = P_f(\theta, q^{-1}) P_0(\theta, q^{-1}) \quad (15)$$

where $P_f(\theta, q^{-1})$ and $P_0(\theta, q^{-1})$ denote the associated feedback control characteristic polynomial and the state observer respectively.

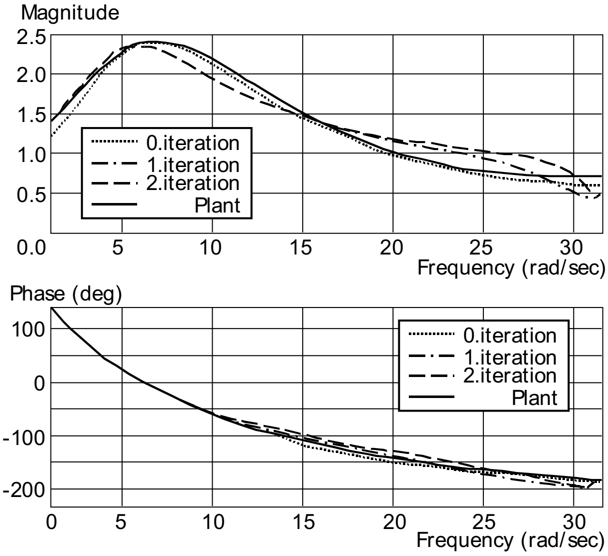

Fig. 2. Plant model Bode plots

2.2 Closed-loop identification

The identification component of our iterative approach is based on the recursive least-squares algorithm

$$\begin{aligned} \varepsilon(t, \hat{\theta}(t-1)) &= y(t) - \phi^\top(t)\hat{\theta}(t-1) \\ K(t) &= \frac{P(t-1)\phi(t)}{1 + \phi^\top(t) P(t-1)\phi(t)} \\ P(t) &= P(t-1) - K(t)\,\phi^\top(t) P(t-1) \\ \hat{\theta}(t) &= \hat{\theta}(t-1) + P(t)\phi(t)\varepsilon(t, \hat{\theta}(t-1)) \\ &= \hat{\theta}(t-1) + K(t)\varepsilon(t, \hat{\theta}(t-1) \end{aligned} \quad (16)$$

with

$$\begin{aligned} \phi(t) &= [-y(t-1) \dots - y(t-na)\, u(t-1) \dots u(t-nb)]^\top \\ \hat{\theta}(t)^\top &= \left[\hat{a}_1(t) \dots \hat{a}_{na}(t)\ \hat{b}_0(t) \dots \hat{b}_{nb}(t)\right] \end{aligned}$$

In our case the identification criterion is modified by introducing the data weighting filter

$$\begin{aligned} J^I(\theta) &= \frac{1}{N}\sum_{t=1}^{N} [\varepsilon_f(\theta, t)]^2 \\ &\triangleq \frac{1}{N}\sum_{t=1}^{N} \left\{ L(\theta, z^{-1})[y(t) - \hat{y}(\theta, t)] \right\}^2 \end{aligned} \quad (17)$$

where the transfer function $L(\theta, z^{-1})$ denotes the transfer function of a stable linear data filter operating on the prediction error which allows to favour some "interesting" data against the other "less interesting" for parameter estimation

$$\left|L(\theta, z^{-1})\right|^2 = \frac{\left|D(z^{-1})\right|^2 \left|C(z^{-1})\right|^2 \left[\left|S(\theta, z^{-1})\right|^2 + \lambda \left|R(\theta, z^{-1})\right|^2\right]}{\left|P_c(\theta, z^{-1})\right|^2} \quad (18)$$

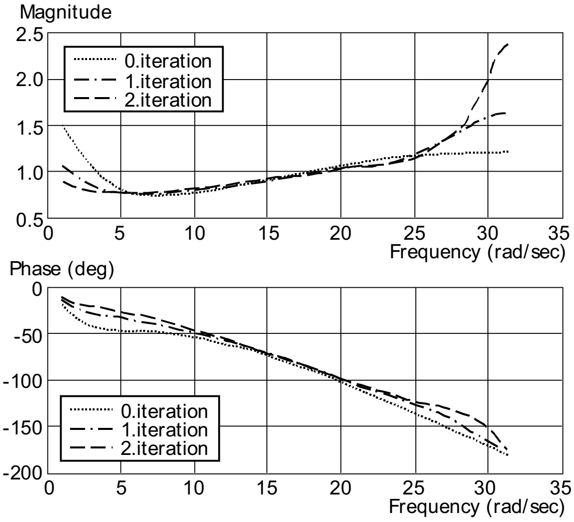

Fig. 3. Identification filter Bode plot

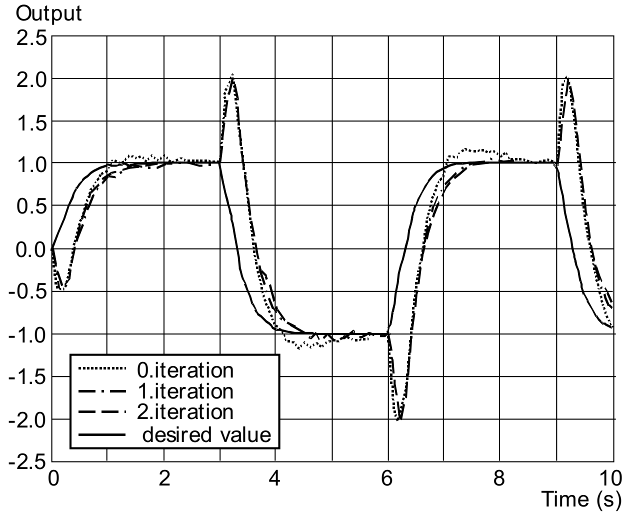

Fig. 4. True plant output response

where $C\left(z^{-1}\right)$ is an observer polynomial and $R\left(\theta, z^{-1}\right)$ and $S\left(\theta, z^{-1}\right)$ are the polynomials of the PSRM controller implemented in the form (13). The identification data filter $L\left(\theta, z^{-1}\right)$ can be calculated using the spectral factorization [10].

The identification hence aims to minimize the error between the real plant and the plant model emphasized in the frequency ranges important for control.

2.3 Iterative PSRM control design

We can now summarize our iterative control design algorithm:

Initial identification. Select $L\left(\theta, z^{-1}\right)= 1$ or any nonconstant filter if a priori information about the true plant is available. Perform an experiment in open loop or with a pre-existing controller to obtain an initial model $\hat{P}_0\left(\theta, z^{-1}\right)$. Set $i = 0$.

Initial control design. Choose $F_1(\theta, z^{-1}) = F_2(\theta, z^{-1}) = 1$. With plant model $\hat{P}_0(\theta, z^{-1})$ design controller $C_0(\theta, z^{-1})$ via minimization of the local control criterion and perform a closed loop experiment with controller $C_0(\theta, z^{-1})$.

Step 1 *Compute identification filter* $L_i(\theta, z^{-1})$.

Step 2 *Model identification.* Using the least-squares identification method with filter $L_i(\theta, z^{-1})$ and closed loop input-output data compute plant model $\hat{P}_{i+1}(\theta, z^{-1})$.

Step 3 *Real plant and model experiment.* With controller $C_i(\theta, z^{-1})$ acting in feedback with real plant generate closed-loop signals $\{y(t)\}$, $\{u(t)\}$ and with the same controller acting in feedback with plant model $\hat{P}_{i+1}(\theta, z^{-1})$ simulate closed-loop signals $\{y^c(t)\}$ and $\{u^c(t)\}$.

Step 4 *PSRM controller design.* Compute the frequency weighting filters included in local control criterion and design the controller $C_{i+1}(\theta, z^{-1})$. Set $i = i + 1$.

Repeat steps 1 to 4 until the desired performances are obtained. Note that significant improvement in performance is usually obtained during the first two or three iterations and not much improvement is obtained by doing further iterations. Users should by aware that this iterative identification/control scheme can sometimes diverge, but it is not our intention to analyze the reasons for such divergence in this paper. Nevertheless, the sufficient condition for the convergence of this algorithm is to reject each model that does not improve the performances.

The proposed iterative control design is focused directly on performance enhancement. There is no implied quality of robust stability. However the stability robustness test can be introduced into the iterative procedure in which the robust stability of the plant with a new controller obtained from a model is evaluated before this new controller is applied to the plant.

3 EXAMPLE

In order to illustrate the effectiveness of this design strategy consider the 5^{th} order true plant

$$P(\theta, z^{-1}) = \left(z^{-1} - 1.2z^{-2} - 0.3z^{-3} + 0.156z^{-4} + 0.0845z^{-5}\right)/\left(1 - 1.25z^{-1} + 0.4575z^{-2} + 0.0279z^{-3} - 0.0491z^{-4} + 0.0077z^{-5}\right) \quad (19)$$

whose output is affected by white noise signal with variance equal to 0.01 filtered by true noise transfer function

$$H\left(z^{-1}\right)= \frac{2}{1 + 0.6121z^{-1}} \quad (20)$$

The plant model to be identified is assumed to be of the third order. Figures 2 to 4 show the plant model Bode plots, identification filter Bode plots and true plant output time responses to the desired value from the initial to the 2^{nd} iteration.

In the initial step (denoted as 0^{th} iteration) the open loop identification of the true plant has been performed. With this initial plant model the initial PSRM controller has been designed and the closed loop experiment has been performed. The data weighting filter for identification has been calculated as a function of the initial controller and the identification has been performed on the data obtained in this closed-loop experiment. Figure 2 indicates that the Bode plot of the open-loop identified plant model (dotted line) has been much closer to the true plant (solid line) than the first iteration closed-loop identification model (dash-dotted line). Using the first step plant model the first step PSRM controller has been designed and used in the closed-loop experiment. Comparing the output plots of the initial (dotted line) and first iteration (dash-dotted line), see Fig. 4, it may be concluded that the controller designed with the plant model identified in the closed-loop has produced an improvement of the tracking capability and noise rejection. The second closed loop identification (dashed line) produced slight changes of the plant model only in the high-frequency range. The performances achieved in the second iteration (dashed line) seemed to be approximately the same as those achieved in the first iteration and the iterative procedure has been stopped.

4 CONCLUSION

In this paper a procedure for improving the performance of an existing controller using closed loop data information has been presented. It consists of repeated application of the system identification procedure operating on the closed loop data together with successive refinements of the designed controller. The closed loop identification is performed using the recursive least squares method with a data weighting filter that aims to improve the model accuracy at those frequencies where robust performance dictates that a better model is needed. The control design is based on the partial state reference model LQG method that allows separating the desired tracking and regulation dynamics. The simulation results exhibit that the performance enhancement has been obtained and confirm that the best model for control design is definitely not the best open loop model.

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