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COMMUNICATIONS

OFDM IS AN ERROR CONTROL CODE

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In this paper it is shown, that it is possible to interpret the OFDM as an error control code (for example Reed Solomon code) defined over real, complex or extended rational fields. The redundancy in OFDM is introduced by not selecting all frequencies in the IDFT or DFT block for transmission. It opens up an opportunity to increase the reliability of the transmitted information by decoding the received channel symbols as encoded using error-control code and so exploit for error control the redundancy introduced by the transmitter into OFDM by not selecting all possible frequencies.

K e y w o r d s: OFDM, Reed Solomon Code, complex numbers, real numbers, rational numbers

1 INTRODUCTION

In common OFDM based systems [1, 2], the reliability is usually increased by an additional Reed Solomon code, which is used (separately) in cascade with the IDFT and DFT block without exploitation the OFDM itself as an Reed Solomon Code defined over real or complex numbers. One can wish to use an error control code over complex field because computational devices that do real or complex arithmetic are widely available. To achieve a single processor that does both digital signal processing and error control, one may wish to do the error control computations with real arithmetic.

Another advantage is that Reed Solomon codes of every block-length N exist in the complex field. In the real field or complex field there may be some minor errors in every component of the received signal. In such situation the error-control code can be used to correct up to t major errors, which may be due to burst noise or impulsive noise in the channel. The correction will be successful even if all the components have minor errors [5]. However to date there has been no theoretical work quantifying how big the minor errors and computational noise can be before the decoding algorithms break down.

One of the main advantages is that the redundancy introduced by transmitter into OFDM, could be further exploited for increasing the reliability of the transmission using the decoding algorithms described in [5] Chapter 8.

2 OFDM IS A RS CODE

A linear block code could be described using its generator matrix G and the encoding of information vector $\boldsymbol{x} = (x_0, x_1, \dots, x_{K-1})$ into a codeword $\boldsymbol{y} = (y_0, y_1, \dots, y_{N-1})$ by the following equation:

$$\boldsymbol{y} = \boldsymbol{x}\mathsf{G} \tag{1}$$

The $(N-K) \times N$ parity check matrix H, of rank $N-K$ is defined as follows:

$$\mathsf{G}\mathsf{H}^H = \mathbf{0} \tag{2}$$

H^H denotes a Hermitian transpose (*ie* the combination of complex conjugation and ordinary matrix transposition) of H. Basic idea in construction of BCH and Reed Solomon codes is that all code words are missing certain prescribed frequency components [3]. For example the t-error correcting Reed Solomon code of block-length N is defined in frequency domain as the set of all vectors whose spectrum has $2t$ zero components.

On the other hand the IDFT used in most OFDM systems could be described by a following matrix [4]:

$$\mathsf{Q}^H = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \phi & \phi^2 & \dots & \phi^{N-1} \\ 1 & \phi^2 & \phi^4 & \dots & \phi^{2(N-1)} \\ 1 & \phi^3 & \phi^6 & \dots & \phi^{3(N-1)} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & \phi^{(N-1)} & \phi^{2(N-1)} & \dots & \phi^{(N-1)^2} \end{bmatrix};$$

$$\phi = e^{j\frac{2\pi}{N}}, \tag{3}$$

where Q^H denotes a Hermitian transpose of orthonormal or unitary DFT matrix Q. Any complex or real vector of length N say $\boldsymbol{v}$ can be expressed as a linear combination of rows of Q^H from (3). The j-th row of Q^H will be referred to as the j-th frequency component, the index j as its frequency and the coefficient V_j of the j-th component in the linear combination as the j-th Fourier coefficient (or the j-th RS codeword symbol in frequency domain). If any K rows of Q^H are selected, the set of all linear combinations of these rows forms a K-dimensional subspace of the N-dimensional space, which is spanned by all N rows of Q^H. Any of these linear combinations will be missing the frequency components corresponding

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to the $N-K$ rows, which were not selected. These $N-K$ components are equivalent to the zero components of a Reed Solomon codeword in frequency domain. They form the redundant symbols [5]. The K-dimensional subspace of the N-dimensional space is the code. The construction in general allows to obtain complex number and real number BCH or Reed Solomon Codes. The $\mathbf{G}$ matrix is obtained by the selection of the K rows from $\mathbf{Q}^H$ corresponding to the components of the vector $\boldsymbol{v}$, which in terms of linear code is equivalent to the "information" vector $\mathbf{x} = (x_0, x_1, \ldots, x_{K-1})$ in (1). The parity check matrix $\mathbf{H}$ will have the remaining rows.

A special case is a systematic code. In this case:

$$\boldsymbol{y} = (\boldsymbol{x} \mid \boldsymbol{p}) \tag{4}$$

where $\boldsymbol{p}$ is a parity vector with $N-K$ coordinates. Let $\mathbf{G}_S$ and $\mathbf{H}_S$ be matrices, which define a systematic code. They are related $\mathbf{G}$ and $\mathbf{H}$ as follows [6]. First it is necessary to form a matrix:

$$\left[\begin{array}{c} \mathbf{G} \\ \hline \mathbf{H} \end{array}\right] = \left[\begin{array}{c|c} \mathbf{A}_a & \mathbf{A}_b \\ \hline \mathbf{A}_c & \mathbf{A}_d \end{array}\right].$$

Then:

$$\mathbf{G}_S = [\mathbf{I}_K \mid \mathbf{P}],$$

where:

$$\mathbf{P} = \mathbf{A}_a^{-1}\mathbf{A}_b = -\mathbf{A}_c^H \left(\mathbf{A}_d^H\right)^{-1}.$$

The reader is referred to the references [6] for further details of the constructions of such codes. For decoding of the codes many different algorithms could be used, which are described for example in Chapter 8 in [5] or in [6].

3 CONCLUSION

In this paper, it was shown that OFDM is an error Control Code. There could be different reasons for increasing the error control capabilities of OFDM. For example in case of services with guaranteed QoS it may happen that the reliability needs to be increased. In this situation the error control capability could be increased at the costs of transmission rate of the OFDM. Specifically the K could be decreased and $N-K$ increased in IDFT block. The frequencies not selected for transmission will form the redundancy and the signal processing at the receiver could include some of the many different error control decoding algorithms described for example in [5].

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