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PIIσ_FD CONTROLLER FOR LINEAR TIME INVARIANT SINGLE INPUT/SINGLE OUTPUT SYSTEMS

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The aim of this paper is to auto tune the length of the integration interval, σ, of the $PII_\sigma D$ controller in such a manner as to increase its accuracy using the fuzzy theory which generalizes 0 and 1 membership values of a crisp set to a membership function of a fuzzy sets which are classes with an un-sharp boundary. In other words, the length of integration in the proposed controller is a function of the error signal, and computed using a fuzzy logic controller. The modified controller will be denoted as $PII\sigma_F D$, which is a modification for $PII_\sigma D$ controller. In order to test the effectiveness of the proposed controller and to compare the performance of the proposed controller with PID and $PII_\sigma D$ controllers, numerical examples are presented. Using Ziegler-Nichols tuning rules, all simulation results prove that the performance of the $PII\sigma_F D$ controller is better than the performance of the others.

K e y w o r d s: fuzzy logic controller, PID, PII$_\sigma$D, Ziegler-Nichols, fuzzification, defuzzification

1 INTRODUCTION

The fuzzy controller design methodology primarily involves distilling human expert knowledge about how to control a system into a set of rules. Compared with the classical approach, fuzzy control utilizes more information from the expert domain than on the mathematical modelling describing the controlled system [1]. On one hand, fuzzy control theory can be heuristic and the expert knowledge that can control the plant is expressed as a fuzzy rule of the fuzzy logic controller (FLC). The rule base formed then becomes the heart of the FLC.

The FLC consists of R fuzzy rules; each fuzzy rule generates an output u_j, and the degree of membership $\mu_j \in [0,1]$ depends on the selected fuzzy input, $j = 1, 2, \dots, R$. The overall controller signal generated from the FLC is a function of both u_j and μ_j [2]. The j^{th} fuzzy rule of the FLC is of the following form:

$$\text{Rule } j: \quad \text{IF } \langle \text{premise } j \rangle \ \text{ THEN } \ u = u_j\,.$$

where $\langle$premise $j\rangle$ is the antecedent of the rule j with an input variable vector, y. In many cases, y will be the error and the change error signal; but it also can be a function of the state variable. The output of each fuzzy rule can be constant or any u_j real function. If the centre of average defuzzification method [3] is applied, the overall control signal is given by

$$u = \frac{\sum_{j=1}^{R} \mu_j(y) u_j(y)}{\sum_{j=1}^{R} \mu_j(y)} \tag{1}$$

and in the case of a symmetrical membership function the FLC will be bounded-input bounded-output (BIBO) stable.

Figure 1 shows a single-input single-output (SISO) FLC which consists of Fuzzification, which a mathematical procedure for converting an element in the universe of discourse into the membership value of the fuzzy set. Fuzzy Inference calculates the rules conclusion based on its matching degree. Rule Base contains the fuzzy rule that forms the heart of the FLC. Finally, Defuzzifiaction, which is a mathematical process, is used to convert a fuzzy set to a real number.

2 THE PROPOSED $PII\sigma_F D$ CONTROLLER

Recently, Radaideh and Hyajneh [4] proposed a modified PID controller, which was denoted as $PII_\sigma D$, and has the following form

$$u(t) = k_p\big(x_d(t) - x(t)\big) + k_d\big(\dot{x}_d(t) - \dot{x}(t)\big) + k_i \int_0^t \int_{\tau-\sigma}^{\tau} \big(x_d(\alpha) - x(\alpha)\big)\mathrm{d}\alpha \mathrm{d}\tau \tag{2}$$

where k_p, k_d and k_i are positive feedback gains, x_d is the desired time function of the system position x, and $\sigma > 0$. Although the $PII_\sigma D$ gives the ability to control feedback gains and length of integration interval σ, this control ability is done manually. The objective of this work is to auto tune the length of the integration interval σ using the theory of fuzzy logic controller to obtain a better performance than that of $PII_\sigma D$ and PID controllers. The proposed controller will be called $PII\sigma_F D$ and has the following form

$$u(t) = k_p\big(x_d(t) - x(t)\big) + k_d\big(\dot{x}_d(t) - \dot{x}(t)\big) + k_i \int_0^t \int_{\tau-\sigma(e)}^{\tau} \big(x_d(\alpha) - x(\alpha)\big)\mathrm{d}\alpha \mathrm{d}\tau\,. \tag{3}$$

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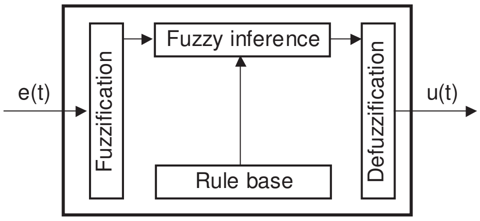

Fig. 1. SISO Fuzzy logic controller.

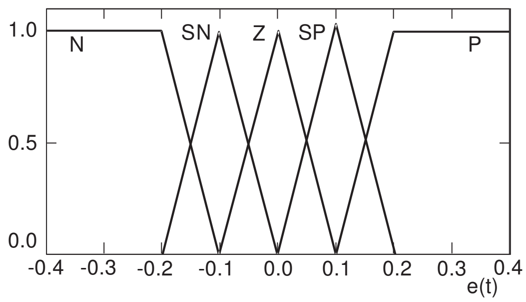

Fig. 2. Membership functions for $e(t)$.

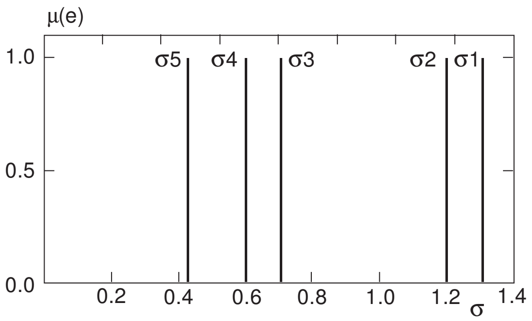

Fig. 3. Membership function for $\sigma(e)$ (example 1).

In this controller, the integrated difference $\int_{\tau-\sigma(e)}^{\tau}\big(x_d(\alpha)-x(\alpha)\big)\mathrm{d}\alpha$ is integrated over the time interval $\big(t-\sigma(e),t\big)$, where $\sigma(e)$ is the length of integration, which is a function in error computed by the fuzzy logic controller.

To auto-tune the length of integration, the fuzzy rules of the FLC are defined as follows.

IF $e(t)$ is N THEN $\sigma = \sigma_1$.
IF $e(t)$ is SN THEN $\sigma = \sigma_2$.
IF $e(t)$ is Z THEN $\sigma = \sigma_3$.
IF $e(t)$ is SP THEN $\sigma = \sigma_4$.
IF $e(t)$ is P THEN $\sigma = \sigma_5$.

Here N, SN, Z, SP, and P are "linguistic values". The membership functions for the premises are symmetrical and normal, and for consequence are singleton membership function as shown in Figs. 2 and 3. We can consider the FLC as multi level control, where the intermediate value is computed using fuzzy theory. The $\sigma(e)$ can be given as follow

$$\sigma(e) = \frac{\sum_{j=1}^{R}\mu_j(e)\sigma_j(e)}{\sum_{j=1}^{R}\mu_j(e)}\,. \tag{4}$$

Because of the use of a symmetrical membership function, the value of $\sigma(e)$ will range from $\sigma_{\min} \le \sigma(e) \le \sigma_{\max}$, where $\sigma_{\min}$ and $\sigma_{\max}$ denote the minimum and maximum length of integration, respectively.

In order to test the effectiveness (such as stability and tracking) of the proposed controller, a numerical simulation is done. For comparison, two examples have been simulated using PID, $PII_{\sigma}D$ and $PII\sigma_F D$ controllers as shown in Figs. 4 through 13.

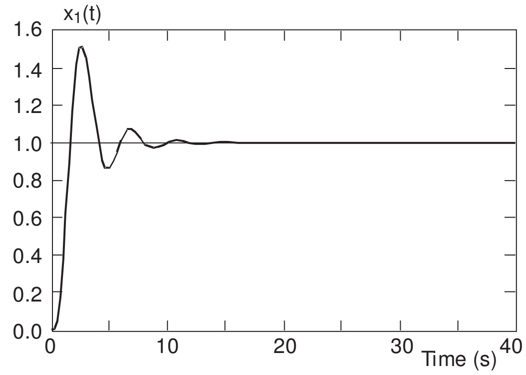

Fig. 4. Closed loop response, x_1, to unit step reference, $r(t)$, using PID controller: $G_c(s) = k_p + k_d s + \frac{k_i}{s}$, $k_p = 4.8$, $k_d = 2.18$, and $k_i = 2.65$.

3 SIMULATION RESULTS

EXAMPLE 1. Consider the controllable system that is governed by the following set of equations [5]

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= x_3(t)\\ \dot{x}_3(t) &= -x_1(t) - 3x_2(t) - 3x_3(t) + u(t)\end{aligned} \tag{5}$$

where $u(t)$ is the control input, $x_1(t)$ is the position of the system. The control objective is to minimize the steady state error tracking error, and the settling time for a step reference input. The optimal gains of the controllers according to Zeigler-Nichols rules [4] are $k_p = 4.8$. $k_d = 2.18$ and $k_i = 2.65$ and $\sigma = 0.8$. The initial values of all integrators used in the controllers are equal to zero. The simulations results for three controllers are shown in Figs. 4 through 9. The result, as shown in Fig. 4, clearly indicates that the percentage overshot of the step reference of the PID controller is very high, 52 %, and Fig. 5 shows the performance of $PII_{\sigma}D$ where the percentage overshot is also high, about 36.2 %. But as shown in Figs. 6 and 8, the proposed controller $PII\sigma_F D$, without changing other parameters and conditions, gives a more stable behaviour, *ie*, the transient response shows a smaller overshot (22 %) than for other controllers. Also, as shown in Fig. 8, the perfect tracking of the PID, $PII_{\sigma}D$ and $PII\sigma_F D$ controllers is achieved after 20 s, 5 s and 4.5 s, respectively. The variation of σ through FLC with $\sigma_1 = 1.32$, $\sigma_2 = 1.12$, $\sigma_3 = 0.7$, $\sigma_4 = 0.6$, $\sigma_5 = 0.42$ is shown in Fig. 7.

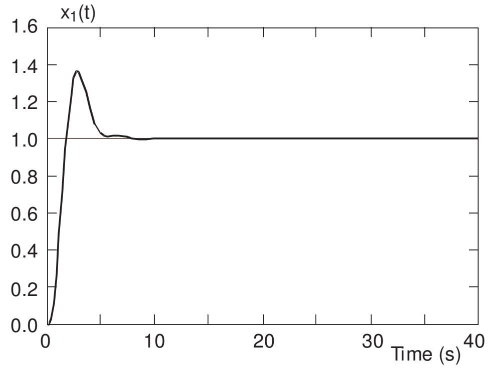

Fig. 5. Closed loop response, x_1, to unit step reference, $r(t)$, using $PII_\sigma D$ controller: $G_c(s) = k_p + k_d s + k_i \frac{1-e^{-\sigma s}}{s^2}$, $k_p = 4.8$, $k_d = 2.18$, $k_i = 2.65$ and $\sigma = 0.8$.

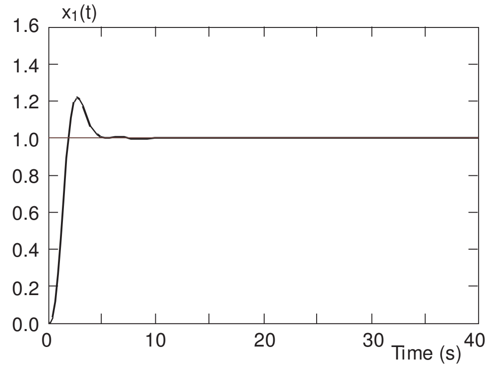

Fig. 6. Closed loop response, x_1, to unit step reference, $r(t)$, using $PII\sigma_F D$ controller: $G_c(s) = k_p + k_d s + k_i \frac{1-e^{-\sigma(e)s}}{s^2}$, $k_p = 4.8$, $k_d = 2.18$, $k_i = 2.65$ and $\sigma(e)$ is generated directly from FLC.

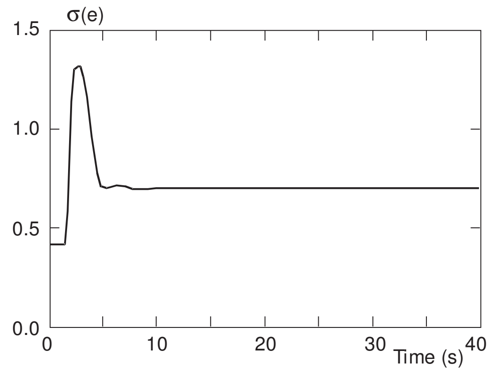

Fig. 7. The length of integration, $\sigma(e)$, as generated directly from FLC.

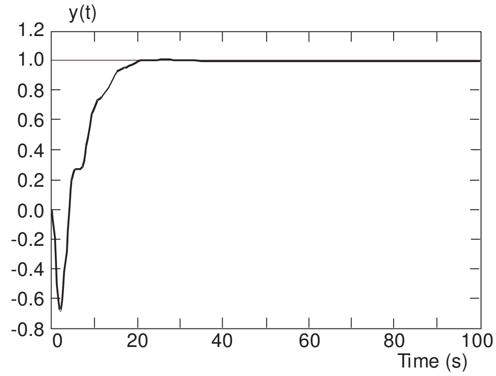

Fig. 8. Comparison between $PII\sigma_F D$, PID and $PII_\sigma D$ controllers.

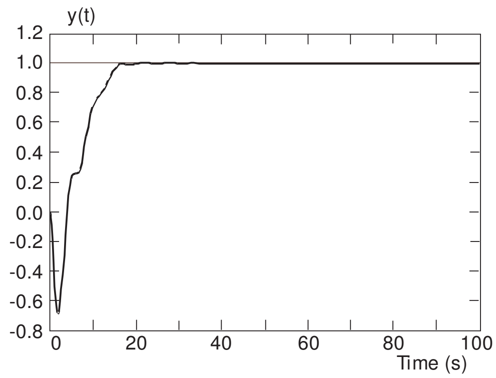

Fig. 9. Closed loop response, $y(t)$, to unit step reference, $r(t)$, using PID controller: $G_c(s) = k_p + k_d s + \frac{k_i}{s}$, $k_p = 3$, $k_d = 3.33$, and $k_i = 0.68$.

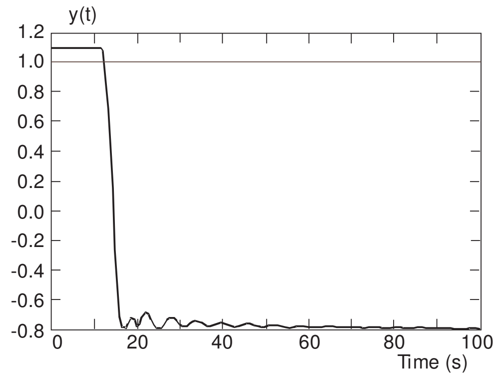

Fig. 10. Closed loop response, y, to unit step reference, $r(t)$, using $PII_\sigma D$ controller: $G_c(s) = k_p + k_d s + k_i \frac{1-e^{-\sigma s}}{s^2}$, $k_p = 3$, $k_d = 3.33$, $k_i = 0.68$ and $\sigma = 1.4$.

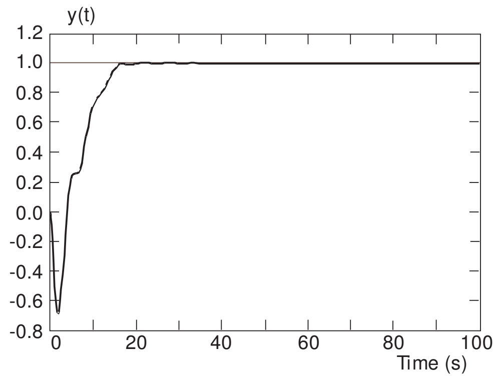

Fig. 11. Closed loop response, y, to unit step reference, $r(t)$, using $PII\sigma_F D$ controller: $G_c(s) = k_p + k_d s + k_i \frac{1-e^{-\sigma(e)s}}{s^2}$, $k_p = 3$, $k_d = 3.33$, $k_i = 0.68$ and $\sigma(e)$ is generated directly from FLC.

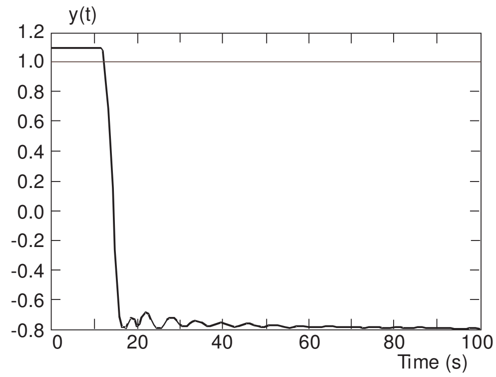

Fig. 12. The length of integration, $\sigma(e)$, as generated directly from FLC.

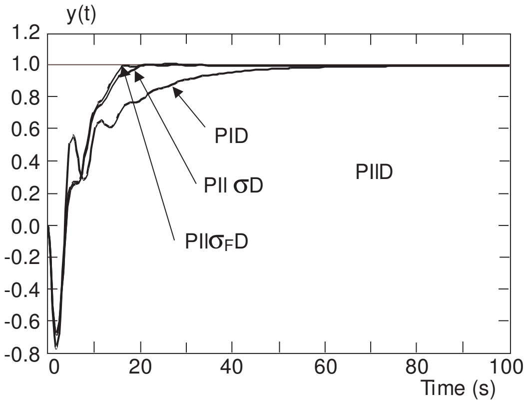

Fig. 13. Comparison between $PII\sigma_F D$, PID and $PII_\sigma D$ controllers.

EXAMPLE 2. Consider the controllable system that is governed by the following set of equations [5]

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= x_3(t)\\ \dot{x}_3(t) &= -x_1(t) - 3x_2(t) - 3x_3(t) + u(t)\\ y &= 0.1x_1(t) - 0.5x_2(t)\end{aligned} \tag{6}$$

where $u(t)$ is the control input, $y(t)$ is the position of the system. The optimal gains of the controllers according to Zeigler-Nichols rules [4] are $k_p = 3$. $k_d = 3.33$ and $k_i = 0.68$ and $\sigma = 1.4$. The initial values of all integrators used in the controllers are equal to zero. The simulation results for the three controllers are shown in Figs. 9 through 13. Although all the controllers achieved the output tracking as shown in Figs. 9–11, the proposed $PII\sigma_F D$ controller completely achieves the output tracking in 16 s while the PID and $PII_\sigma D$ achieve it in 60 s and 20 s, respectively. Also as shown in Fig. 12, the response of the proposed controller is more stable than the others. The variation of σ through FLC with $\sigma_1 = 1.45$, $\sigma_2 = 1.1$, $\sigma_3 = 0.5$, $\sigma_4 = 1.1$, $\sigma_5 = 1.45$ is shown in Fig. 13. Therefore, we believe that the proposed controller might be of some interest to the control theory community.

4 CONCLUSION

An auto-tuned $PII\sigma_F D$ controller has been presented in the paper. The proposed controller has better performance than PID and $PII_\sigma D$ controllers in terms of percent peak overshot and settling time. All simulation results indicate that the performance of the proposed controller is better than that of PID and $PII_\sigma D$ controllers.

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