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COMMUNICATIONS

AN ITERATIVE METHOD FOR HAMMERSTEIN–WIENER SYSTEMS PARAMETER IDENTIFICATION

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The paper deals with parameter identification of nonlinear dynamic systems using Hammerstein-Wiener models. Multiple application of a decomposition technique provides special expressions for the model description that are linear in parameters. This allows iterative estimation of all model parameters based on the measured input/output data and estimates of internal variables. The proposed algorithm is illustrated by identification of systems with polynomial characteristics.

K e y w o r d s: nonlinear systems, identification, Hammerstein-Wiener model

1 INTRODUCTION

The Hammerstein and Wiener systems are the simplest types of block-oriented nonlinear systems where the nonlinear block is static and follows or is followed by a linear dynamic block. These systems appear in many engineering applications and, therefore, the identification of Hammerstein and Wiener systems has been an active research area for many years (see [1] and [2] for a complete bibliography). However, there exists only few work reported in the literature on the so-called Hammerstein-Wiener system defined as a static nonlinear element in cascade with a linear dynamic system followed by another static nonlinear element, *eg*, [3–5] and even less on their identification [6–9].

In this paper a new approach to modeling and parameter identification of Hammerstein- Wiener systems is presented, where the previous results [10–14] are effectively applied and extended. The resulting mathematical model for this type of block-oriented systems contains explicit information on all the blocks of given system. An iterative algorithm is proposed for the estimation of model parameters, which is based on the measured input/output data and the estimates of all the internal variables resulting from the preceding estimates of the corresponding parameters. An illustrative example of Hammerstein-Wiener system parameter identification with polynomial nonlinear blocks is included.

2 DECOMPOSITION TECHNIQUE

In the previous work [10] a special decomposition technique has been proposed for composite mappings based on the so-called key term separation principle. This consists in separating a key term (variable) in the outer mapping and consequent half-substituting of the inner mapping for the key term only. Then the inner mapping appears both explicitly and implicitly in the outer one. Application of this approach to the block-oriented nonlinear dynamic models has led to useful simplifications because the original composite mappings were replaced by simpler ones more appropriate for identification and control purposes [11–14].

In the case of more complex composite mappings the additive form of this decomposition technique can be also applied sequentially more times. Let us assume there are mappings defined on proper sets $W_0, \ldots, W_n$ as:

$$w_n = \alpha_n(w_{n-1}) = \alpha_n(\alpha_{n-1}(\cdots \alpha_2(\alpha_1(w_0))\cdots)) \qquad (1)$$

for $w_i \in W_i$, $i = 0, 1, \ldots, n$, and the following additive forms of decomposition exist:

$$w_n = w_{n-1} + \alpha'_n(w_{n-1}) \qquad (2)$$

where $\alpha'_2(\cdot), \ldots, \alpha'_n(\cdot)$ are the 'remainders' of $\alpha_2(\cdot), \ldots, \alpha_n(\cdot)$, after separations of variables $w_1, \ldots, w_{n-1}$. Then the sequential half-substitutions of the corresponding (inner) mappings only for the separated (key terms) $w_1, \ldots, w_{n-1}$ will lead to

$$w_n = \alpha_1(w_0) + \sum_{i=2}^{n} \alpha'_i(w_{i-1}) \,. \qquad (3)$$

In this case, the composite mapping $\alpha_n \circ \alpha_{n-1} \circ \cdots \circ \alpha_1 \circ \alpha_0$ can be replaced by the sum of components (3), that may be of a simpler form than the original mapping, *eg*, may be linear in parameters. This approach was already applied in the case of special discontinuous nonlinear characteristics [11], [13]. In the following the above decomposition will be used to simplify the description of Hammerstein-Wiener systems and the estimation of the corresponding model parameters.

3 HAMMERSTEIN–WIENER SYSTEMS

The Hammerstein-Wiener system is given by the cascade connection of a static nonlinearity block N1 followed

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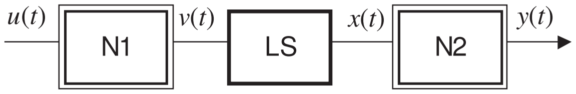

Fig. 1. Hammerstein-Wiener system

by a linear dynamic system LS which is followed by a static nonlinearity block N2. The noiseless case is shown in Fig. 1. The first nonlinear static block is characterized by the mapping $f(\cdot)$:

$$v(t) = f[u(t)] \tag{4}$$

where $u(t)$ and $v(t)$ are the inputs and outputs, respectively. The difference equation model of the linear dynamic block is:

$$x(t) = B(q^{-1})v(t) + [1 - A(q^{-1})]x(t) \tag{5}$$

where $v(t)$ and $x(t)$ are the inputs and outputs, respectively, $A(q^{-1})$ and $B(q^{-1})$ are scalar polynomials in the unit delay operator q^{-1}

$$A(q^{-1}) = 1 + a_1q^{-1} + \cdots + a_mq^{-m}\,, \tag{6}$$

$$B(q^{-1}) = b_0 + b_1q^{-1} + \cdots + b_nq^{-n}\,. \tag{7}$$

The second nonlinear static block is characterized by the mapping $g(\cdot)$:

$$y(t) = g[x(t)] \tag{8}$$

with inputs $x(t)$ and outputs $y(t)$. The system inputs $u(t)$ and outputs $y(t)$ are measurable, while the internal variables $v(t)$ and $x(t)$ are not.

The input-output description of the Hammerstein-Wiener system resulting from direct substitutions of the corresponding variables from (4) into (5) and then into (8) would be strongly nonlinear both in the variables and in the parameters, hence not very appropriate for parameter estimation. Therefore, the aforementioned decomposition will be applied with the aim to derive a simpler form of the model description.

The second nonlinear block can be decomposed and written as follows:

$$y(t) = g_1x(t) + g'[x(t)] \tag{9}$$

where the internal variable $x(t)$ is separated. The linear dynamic block equation can be written as

$$x(t) = b_0v(t) + [B(q^{-1}) - b_0]v(t) + [1 - A(q^{-1})]x(t) \tag{10}$$

where the internal variable $v(t)$ is separated. Now, to complete the sequential decomposition, the corresponding half-substitutions can be performed, *ie*: (i) from (4) into (10) only for $v(t)$ in the first term, and (ii) from (10) into (9) again only for $x(t)$ in the first term. The resulting output equation of the Hammerstein-Wiener model will be

$$y(t) = g_1\{b_0f[u(t)] + [B(q^{-1}) - b_0]v(t) + [1 - A(q^{-1})]x(t)\} + g'[x(t)]\,. \tag{11}$$

The model blocks characteristics are directly projected into the model description through $f(\cdot)$ for N1, $A(q^{-1})$ and $B(q^{-1})$ for LS, and through $g'(\cdot)$ for N2. Proper parametrizations of two nonlinear blocks descriptions can significantly simplify the model output equation and possibly lead to the linearity in all the model parameters.

As the Hammerstein-Wiener system consists in the cascade connection of three subsystems, the parametrization of model (11) is not unique because many combinations of parameters can be found [7]. Therefore, one parameter in at least two blocks has to be fixed in the mathematical model to be unique. Evidently, the choices $g_1 = 1$ and $b_0 = 1$ (more precisely $b_i = 1$, where b_i is the first nonzero parameter considered) in (11) will simplify the Hammerstein-Wiener system description.

Let us assume that the characteristics in both nonlinear blocks can be approximated by proper polynomials of known orders as follows:

$$v(t) = f[u(t)] = \sum_{k=1}^{p} f_ku^k(t)\,, \tag{12}$$

$$y(t) = g[x(t)] = \sum_{k=1}^{r} g_kx^k(t)\,. \tag{13}$$

Then the resulting output equation of the Hammerstein-Wiener model, considering $g_1 = 1$, $b_0 = 1$, is

$$y(t) = \sum_{k=1}^{p} f_ku^k(t) + [B(q^{-1}) - 1]v(t) + [1 - A(q^{-1})]x(t) + \sum_{k=2}^{r} g_kx^k(t)\,. \tag{14}$$

Now equations (5) and (12) defining the internal variables $x(t)$ and $v(t)$, respectively, and the output equation (14) represent the Hammerstein-Wiener model. The model is linear-in- parameters for given $u(t)$, $x(t)$ and $v(t)$ and can be written in the vector form

$$y(t) = \varphi^\top(t,\theta)\theta \tag{15}$$

where the vector of parameters and the vector of data are

$$\theta = [f_1, \ldots, f_p, b_1, \ldots, b_n, a_1, \ldots, a_m, g_2, \ldots, g_r]^\top \tag{16}$$

$$\varphi(t,\theta) = [u(t), \ldots, u^p(t), v(t-1), \ldots, v(t-n), -x(t-1), \ldots, -x(t-m), x^2(t), \ldots, x^r(t)]^\top \tag{17}$$

As all the model parameters in (14) are separated (*ie*, not cross-multiplied) the proposed form of Hammerstein-Wiener model is a 'parsimonious' model for the chosen parametrization with the least possible number of parameters to be estimated.

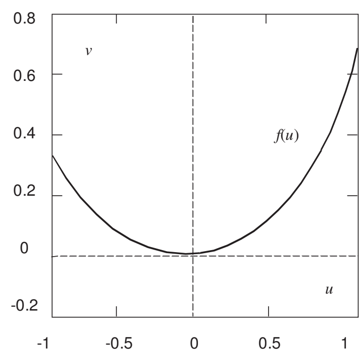

Fig. 2. Input nonlinearity

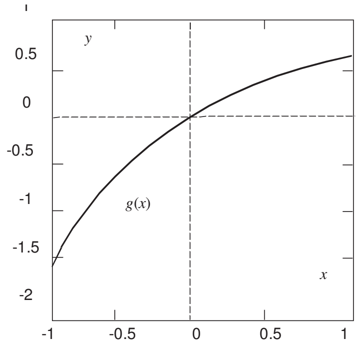

Fig. 3. Output nonlinearity

4 PARAMETER ESTIMATION

The Hammerstein-Wiener model contains two internal variables, which are generally unmeasurable; hence the parameter estimation cannot be performed directly on the basis of (15). It will be appropriate to apply the iterative approach with internal variable estimation proposed for Hammerstein and Wiener models in [10], as this can be simply extended to estimation of two or even more internal variables.

The iterative algorithm is based on the use of the preceding estimates of model parameters for the estimation of internal variables. Assigning the estimated variables in the s-th step as

$$ {}^{s}v(t) = \sum_{k=1}^{p} {}^{s}f_k u^k(t)\,, \tag{18} $$

$$ {}^{s}x(t) = {}^{s}v(t) + \sum_{i=1}^{n} {}^{s}b_i {}^{s}v(t-i) - \sum_{j=1}^{m} {}^{s}a_j {}^{s}x(t-j)\,, \tag{19} $$

the error to be minimized in the s-th step is gained from (15) as

$$ {}^{s}e(t) = y(t) - {}^{s}\varphi^{\top}(t, {}^{s}\theta)^{s+1}\theta \tag{20} $$

where ${}^{s}\varphi(t, {}^{s}\theta)$ is the data vector with the corresponding estimates of internal variables according to (18)–(19) and ${}^{s+1}\theta$ is the $(s+1)$-th estimate of the parameter vector. The steps in the iterative procedure may be now stated as follows:

a) Minimizing a proper criterion based on (20) the estimates of parameters ${}^{s+1}\theta$ are yielded using ${}^{s}\varphi(t, {}^{s}\theta)$ with the s-th estimates of internal variables.

b) Using (18) the estimates of ${}^{s+1}v(t)$ are evaluated by means of the recent estimates of corresponding model parameters ${}^{s+1}f_k$.

c) Using (19) with the estimates of ${}^{s+1}v(t)$ the estimates of ${}^{s+1}x(t)$ are evaluated by means of the recent estimates of parameters ${}^{s+1}b_i$ and ${}^{s+1}a_j$.

d) If the chosen estimation criterion (*eg*, MSE) is met the procedure ends, else it continues by repeating steps a)–c).

To illustrate the feasibility of the proposed identification technique, the following example shows the parameter estimation process for the Hammerstein-Wiener system, where the first nonlinear static block (Fig. 2) is given by the polynomial $v(t) = 0.1\,u(t) + 0.5\,u^2(t) + 0.1\,u^3(t)$, the linear dynamic system is given by the difference equation $x(t) = v(t-1) + 0.15\,v(t-2) + 0.2\,x(t-1) - 0.35\,x(t-2)$, and the second static nonlinearity (Fig. 3) is characterized by the polynomial $y(t) = x(t) - 0.5\,x^2(t) + 0.1\,x^3(t)$.

The least squares method was used for the repeated estimations of all the model parameters and the internal variables. The identification was carried out with 400 samples, using uniformly distributed random inputs with $|u(t)| \le 0.9$ and simulated outputs. The first estimation was performed for the quasi Hammerstein part only, *ie*, considering ${}^{0}x(t) = y(t)$ and ${}^{0}v(t) = u(t)$ in the linear dynamic block. The initial values of all the parameters were chosen zero.

Table 1. Parameter estimates

It.	f_1	f_2	f_3	b_2	a_1	a_2	g_2	g_3
1	0.1112	0.4258	0.0117	0.0050	−0.3483	0.3869	–	–
2	0.1028	0.4792	0.0742	0.2051	−0.2014	0.4081	−0.4635	−0.1321
3	0.1015	0.4826	0.0849	0.2212	−0.1445	0.3462	−0.4345	0.0372
4	0.1017	0.4989	0.0954	0.1751	−0.1900	0.3584	−0.5326	0.1171
5	0.1005	0.4967	0.0968	0.1679	−0.1837	0.3466	−0.4816	0.0881
6	0.1004	0.5005	0.0994	0.1528	−0.2001	0.3520	−0.5104	0.1068
7	0.1001	0.4993	0.0994	0.1533	−0.1965	0.3489	−0.4944	0.0964
8	0.1001	0.5002	0.1000	0.1499	−0.2007	0.3506	−0.5031	0.1020
t	0.1000	0.5000	0.1000	0.1500	−0.2000	0.3500	−0.5000	0.1000

Table 2. Mean square errors

It.	MSE-v	MSE-x	MSE-y
1	0.00104661264337	0.00149034182416	0.00032497006908
2	0.00008669414965	0.00022628460282	0.00022107294456
3	0.00004950774082	0.00004737481936	0.00001382008221
4	0.00000048904138	0.00000907355598	0.00000944961882
5	0.00000175642383	0.00000245677873	0.00000144911898
6	0.00000003724970	0.00000050761832	0.00000045908527
7	0.00000007892111	0.00000015136798	0.00000011369875
8	0.00000000818831	0.00000003828579	0.00000003241169

Table 3. Monte Carlo results

	True	Mean-100	STD-100	Mean-50	STD-50
f_1	0.1000	0.1000	0.0002	0.0999	0.0006
f_2	0.5000	0.5000	0.0005	0.5002	0.0007
f_3	0.1000	0.1000	0.0005	0.1005	0.0012
b_2	0.1500	0.1501	0.0013	0.1498	0.0015
a_1	−0.2000	−0.2001	0.0010	−0.2002	0.0010
a_2	0.3500	0.3502	0.0003	0.3499	0.0005
g_2	−0.5000	−0.5007	0.0037	−0.5009	0.0064
g_3	0.1000	0.1008	0.0054	0.0999	0.0110

The values of parameter estimates are included in Tab. 1 and the real values of the system parameters are written in the bottom row. The corresponding values of mean square errors for the internal variables $v(t)$ and $x(t)$ and the output $y(t)$ are in Tab. 2. The parameter estimates are almost equal to the real values after 8 iterations and the iterative process of parameter estimation shows good convergence.

The estimation process for the above example of Hammerstein-Wiener model was also tested with additive output noise and Monte Carlo simulation studies of more runs were performed. As the iterative processes may stop after distinct numbers of steps depending on the chosen estimation criterion, it is impossible to ensure the same conditions for the Monte Carlo analysis. For this reason the number of iterations was limited to 10. Normally distributed random noises with zero mean and different signal-to-noise ratios — SNR (the square root of the ratio of output and noise variances) were added to the simulated outputs. The results in 20 runs (with 1200 samples in each) and SNR = 100 and SNR = 50 are in Tab. 3. The estimated parameters are given by the mean values and the standard deviations.

5 CONCLUSION

The presented approach to the Hammerstein-Wiener system parameter identification is based on a new form of model description resulting from more consecutive decompositions of compound mappings describing this block-oriented model. An iterative parameter estimation algorithm with internal variable estimations has been proposed and illustrated on examples of simulated Hammerstein-Wiener systems with polynomial nonlinearities. Although there is no proof of convergence for this special modification of relaxation algorithm [1], the simulation results show that the proposed method is effective in the identification of Hammerstein-Wiener systems.

Finally, note that also other types of nonlinearities can be considered in the input block and/or in the output block of the Hammerstein-Wiener model (as the requirement to fix one parameter in the description of nonlinearity is not too severe), *eg*, multisegment piecewise-linear characteristics [14].

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