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RELIABILITY ANALYSIS OF DYNAMIC BEHAVIOR OF MULTI–STATE SYSTEMS

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A lot of systems (operating systems, database systems, distributed systems, information systems) can be described, form the point of view of reliability, like a multi-state system. The multi-state system is investigated in this paper as an object of reliability analysis. In this case, the system and its component may experience more than two states of reliability as opposed to the binary system (the system and its components are allowed to have only two possible states: completed failure and perfect functioning). The mathematical model of the multi-state system is improved. We examine the system to allow a different number of discrete states for the system and for each component we propose to apply dynamic reliability indices for investigation of this system. These indices estimate the influence upon the multi-state system reliability.

K e y w o r d s: distributed systems, multi-state systems, reliability analysis, dynamic reliability

1 INTRODUCTION

There are same mathematical models in the reliability analysis. Firstly, it is the Binary System: the system and its components are allowed to have only two possible states (completed failure and perfect functioning). Secondly, it is the Multi-State System (MSS). In a MSS, both the system and its components may experience more than two states, for example, completely failed, partially functioning and perfect functioning. The reliability analysis of the binary system has served as a foundation for the mathematical treatment of the reliability theory. Many problem of the binary system have been settled. But this approach fails to describe many situations where the system can have more than two distinct states [1, 2]. A MSS reliability analysis is a more flexible approach to evaluate the system reliability.

The MSS is investigated by the structure function tool in this paper. The minimal cut or minimal path sets is one of the major and fundamental tools for evaluating MSS reliability. Different indices are obtained over there. These indices are important topics in the planning, design and control of the engineering system. But it is well known that the structure function tool has a disadvantage. It does not allow to investigate the dynamic behavior of MSS.

Here it is necessary to emphasize some decisions of this problem. Firstly, it is a combination of the Markov processes and the structure function tool. The basic concept of this approach was submitted in [3]. Secondly, the interesting decision of this problem was offered in [4]. R. Boedigheimer and K. Kapur in [4]. They proposed the customer's structure function methodology that has permitted to analyse the changing of system states conditioned by a change of the component states. Thirdly, the results that were presented in [4] were developed using the logical differential calculus in [5-7].

In [5] the authors had substantiated the basic concept to apply first the direct partial logic derivatives (part of logical differential calculus) for the reliability analysis of MSS. These derivatives of the structure function allow determining the boundary states of MSS. These states cause the system failure or, in another case, of the system renewal if the system component state changes.

In [6] a new class of reliability indices was determined and was named Dynamic Reliability Indices (DRI). Two groups of DRI were obtained. There are the group of deterministic and the group of probabilistic indices. The deterministic indices define the sets of the boundary states of MSS. The other group of indices reveals the probability of system failure or its repairing. Another group of DRI was proposed in paper [7]. Indices of this group examine the influence of modifications of every system component upon the system reliability.

However it is necessary to note that the MSS was investigated in [5 - 7] when the levels of the component state are equal. This lack is removed in this paper and DRI are defined for MSS to allow a different number of discrete states for the system and for each component.

The general model of the MSS is considered in this paper, it is k-out-of-n system. The k-out-of-n MSS with n components works if at least k components work. Both series and parallel systems are a special case of k-out-of-n system: a series system is n-out-of-n system and a parallel system is 1-out-of-n system.

2 MATHEMATICAL MODEL OF MSS

2.1 Direct Partial Logic Derivative for MSS model

MSS is a system which consist of n components denoted as $x_i (i = 1, \ldots, n)$. Each component in the component set can take on either of states: from 0 (it is the

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complete failure) to $m_i - 1$ (it is the perfect functioning). The dependence of the system reliability (system state) on its components state is defined by the structure function identically $\phi(x_1, \dots x_2) = \phi(\mathbf{x})$, [2, 8, 9]:

$$\phi(\mathbf{x}) = \begin{cases} 0 \text{ if } \mathbf{x} \text{ is a failed system state} \\ s \text{ if } \mathbf{x} \text{ is an functioning system state} \end{cases}$$
$$s \in \{1, \dots, M-1\} \text{ and } x_i \in \{0, \dots, m_i - 1\}.$$

The structure function is determined as

$$\phi(\mathbf{x}) : \{0, \dots, m_1 - 1\} \times \dots \times \{0, \dots, m_n - 1\} \to \{0, \dots, M-1\}, \quad m_i \neq m_j \neq M \quad \text{in general.} \tag{1}$$

As a rule, the MSS in reliability analysis is a coherent system, therefore the structure function (1) has assumptions [2, 4, 9]:

a) the structure function is non-decreasing and so: $\phi(s, \dots, s) = s, \phi(0, \dots, 0) = 0$ and $\phi(\mathbf{x}) \leq \phi(\mathbf{x}')$ if $\mathbf{x} \leq \mathbf{x}'$;
b) the behavior of each component is mutually s-independent;
c) the every component is relevant to the system.

Table 1. Example of the structure function of the MSS 2-out-of-3

x_1, x_2, x_3	$\phi(\mathbf{x})$	x_1, x_2, x_3	$\phi(\mathbf{x})$
0 0 0	0	2 0 0	0
0 0 1	0	2 0 1	1
0 0 2	0	2 0 2	1
0 1 0	0	2 1 0	1
0 1 1	1	2 1 1	2
0 1 2	1	2 1 2	2
1 0 0	0	3 0 0	0
1 0 1	1	3 0 1	1
1 0 2	1	3 0 2	1
1 1 0	1	3 1 0	1
1 1 1	1	3 1 1	2
1 1 2	2	3 1 2	2

In previous papers [2 - 4, 8, 9] the structure function is used for the estimation of the probability of the different system state: $\Pr\{\phi(\mathbf{x}) = l\}, l \in \{0, \dots, M-1\}$. The mathematical approach for investigation of the dynamic behavior of MSS is designed in this paper and the probability of change of system state $\mathrm{P}\{\phi(\mathbf{x}) = j \to \phi(\mathbf{x}) = l\}$ is calculated in this paper for estimation of the system reliability. This approach was first suggested in [5 - 7]. It was founded on the logical differential calculus and the direct partial logic derivative in particular. But the direct partial logic derivative that was used in [5 - 7] is defined for a function whose variables belong to the same set: $m_i = m_j = M$ and $x_i, \phi(\mathbf{x}) \in \{0, \dots, M-1\}$. So, we need to extend the direct partial logic Derivative for function $\phi(\mathbf{x})$ where $x_i \in \{0, \dots, m_i - 1\}$ and $m_i \neq m_j \neq M$, in other words, for the system to allow a different number of discrete states for the system and for each component the direct partial logic derivative $\partial\phi(j \to k)/\partial x_i(a \to b)$ of function $\phi(\mathbf{x})$, (1) with respect to variable x_i reflects the fact of changing of function from j to l when the value of variable x_i is changing from a to b:

$$\partial\phi(j \to k)/\partial x_i(a \to b) = \\ = \phi(x_1, \dots, x_{i-1}, a, x_{i+1}, \dots, x_n) \bullet \\ \phi(x_1, \dots, x_{i-1}, b, x_{i+1}, \dots, x_n) \tag{2}$$

where $j, l \in \{0, 1, \dots, M-1\}$ and $a, b \in \{0, 1, \dots, m_i - 1\}$; "$\bullet$" is the symbol of a comparison operation:

$$\frac{\partial\phi(j \to k)}{\partial x_i(a \to b)} = \begin{cases} m-1, \text{ if: } & \begin{matrix} \phi(x_1, \dots, x_{i-1}, a, x_{i+1}, \dots, x_n) = j \\ \phi(x_1, \dots, x_{i-1}, b, x_{i+1}, \dots, x_n) = l \end{matrix} \\ 0, \text{ otherwise} \end{cases}$$

So, the direct partial logic derivative of the structure function allows to examine the influence of the i-th component state change into the system reliability. In other words this derivative discovers the system states that are transformed as a result of the change of the component state. Consequences of the direct partial logic derivatives are of interest for reliability analysis of MSS:

$$\frac{\partial\phi(1 \to 0)}{\partial x_i(a \to b)} \quad \text{for } a \in \{1, \dots, m_i - 1\} \text{ and } b \in \{0, \dots, m_i - 2\}, b < a$$

$$\frac{\partial\phi(0 \to l)}{\partial x_i(c \to d)} \quad \text{for } l, d \in \{1, \dots, m_i - 1\} \text{ and } c \in \{0, \dots, m_i - 2\}, c < d$$

The first derivative is a mathematical model of the system failure if i-th component state changes from a to b. Because the structure function $\phi(\mathbf{x})$ is non-decreasing, this derivative is $\partial\phi(1 \to 0)/\partial x_i(a \to a-1)$, where $a \in \{1, \dots, m_i - 1\}$. Another assumption for structure function (1) and experimental investigations in [10] conditioned on the next direct partial logic derivative for modelling of the system failure: $\partial\phi(1 \to 0)/\partial x_i(1 \to 0)$.

The second derivative permits the mathematical description of the system renewal. There are two variants of investigation for the system repairing. First it is the system repairing by the replacement of the failure component. This situation is determined by the direct partial logic derivative $(\partial\phi(0 \to l)/\partial x_i(0 \to m_i - 1)$. Second, it is the increase of component state that is described as $\partial\phi(0 \to 1)/\partial x_i(c \to c+1)$. However, the first variant is more important for application. Because the structure function of the MSS is non-decreasing, this derivative can be assigned as $\partial\phi(0 \to 1)/\partial x_i(0 \to m_i - 1)$.

So, for analysis of the MSS dynamic behavior there is a need to use the direct partial logic derivatives

$$\frac{\partial\phi(1 \to 0)}{\partial x_i(1 \to 0)}, \quad \frac{\partial\phi(0 \to 1)}{\partial x_i(0 \to m_i - 1)}$$

2.2 Example of MSS model for investigation of its dynamic behavior

For example, we show the MSS 2-out-of-3, where the structure function $\phi(\mathbf{x})$ depends of three variables ($n = 3$) and has $m_1 = 4$, $m_2 = 2$, $m_3 = 3$, $M = 3$. Computed direct partial logic derivatives $\frac{\partial\phi(1\to0)}{\partial x_i(1\to0)}$, $(i = 1, \ldots, n)$ of this function $\phi(\mathbf{x})$ for analysis of the system failure are in Table 1.

Table 2. Example of the direct partial logic derivatives

x_1, x_2, x_3	$\frac{\partial\phi(1\to0)}{\partial x_1(1\to0)}$	$\frac{\partial\phi(1\to0)}{\partial x_2(1\to0)}$	$\frac{\partial\phi(1\to0)}{\partial x_3(1\to0)}$
0 0 0	0	0	0
0 0 1	0	0	0
0 0 2	0	0	0
0 1 0	0	0	0
0 1 1	0	2	2
0 1 2	0	2	0
1 0 0	0	0	0
1 0 1	2	0	2
1 0 2	2	0	0
1 1 0	2	2	0
1 1 1	0	0	0
1 1 2	0	0	0
2 0 0	0	0	0
2 0 1	0	0	2
2 0 2	0	0	0
2 1 0	0	2	0
2 1 1	0	0	0
2 1 2	0	0	0
3 0 0	0	0	0
3 0 1	0	0	2
3 0 2	0	0	0
3 1 0	0	2	0
3 1 1	0	0	0
3 1 2	0	0	0

According to (2), for this function it is necessary to analyse the values of the function, when $x_i = 0$ and $x_i = 1$:

$$\frac{\partial\phi(1\to0)}{\partial x_1(1\to0)} = \phi(1, x_2, x_3) \bullet \phi(0, x_2, x_3)$$

$$\frac{\partial\phi(1\to0)}{\partial x_2(1\to0)} = \phi(x_1, 1, x_3) \bullet \phi(x_1, 0, x_3)$$

$$\frac{\partial\phi(1\to0)}{\partial x_3(1\to0)} = \phi(x_1, x_2, 1) \bullet \phi(x_1, x_2, 0)$$

Therefore, there elements of the direct partial logic derivative $\frac{\partial\phi(1\to0)}{\partial x_i(1\to0)}$ are equal if both $\phi(\mathbf{x}) = 0$ and $\phi(\mathbf{x}) = 1$ for specified variables only (Table 2).

So, this MSS 2-out-of-3 is a failure in states $x_1x_2x_3$ (Table 2):

a) 101, 102, 110 — if the first component is a breakdown;
b) 011, 012, 110, 210, 310 — if the second component is a failure;
c) 011, 101, 201, 301 — if the third component is not functioning.

The states for repairing the system are calculated by a similar method. For example, Table 3 shows the direct partial logic derivative $\frac{\partial\phi(0\to1)}{\partial x_i(0\to m_i-1)}$ and the analysis of this derivative permits to obtain states x1x2x3 of the system failure for which the replacement of the broken component restores the system:

a) 001, 002, 010 — if the first component is replaced;
b) 001, 002, 100, 200, 300 — if the second component is replaced;
c) 010, 100, 200, 300 — if the third component is replaced.

Direct partial logic derivatives allow to analyse dynamic properties of MSS which is submitted as a structural function. A class of indices are proposed to use for reliability analysis of MSS below.

Table 3. Example of the direct partial logic derivative $\frac{\partial\phi(0\to1)}{\partial x_i(0\to m_i)}$

x_1, x_2, x_3	$\frac{\partial\phi(0\to1)}{\partial x_1(0\to3)}$	$\frac{\partial\phi(0\to1)}{\partial x_2(0\to1)}$	$\frac{\partial\phi(0\to1)}{\partial x_3(0\to2)}$
0 0 0	0	0	0
0 0 1	2	2	0
0 0 2	2	2	0
0 1 0	2	0	2
0 1 1	0	0	0
0 1 2	0	0	0
1 0 0	0	2	2
1 0 1	0	0	0
1 0 2	0	0	0
1 1 0	0	0	0
1 1 1	0	0	0
1 1 2	0	0	0
2 0 0	0	2	2
2 0 1	0	0	0
2 0 2	0	0	0
2 1 0	0	0	0
2 1 1	0	0	0
2 1 2	0	0	0
3 0 0	0	2	2
3 0 1	0	0	0
3 0 2	0	0	0
3 1 0	0	0	0
3 1 1	0	0	0
3 1 2	0	0	0

3 DYNAMIC RELIABILITY INDICES

3.1 Dynamic Deterministic Reliability Indices

DDRI evaluate the influence of a change of the component state upon system reliability. They are defined as sets of boundary states of the system. Here the boundary state of the system is the system state $s_1 \ldots s_i \ldots s_n$ when the modification of i-th component state from s_i into s_i' causes the system to fail or repair.

DEFINITION 1. DDRI are sets of boundary state of the system $\{G_f\}$ (for system failure) and $\{G_r\}$ (for system repairing) [5, 10]:

$$\{G_f\} = \{G_f|x_1\} \subset \{G_f|x_2\} \subset \cdots \subset \{G_f|x_n\}, \tag{3}$$

$$\{G_r\} = \{G_r|x_1\} \subset \{G_r|x_2\} \subset \cdots \subset \{G_r|x_n\}, \tag{4}$$

where subsets are

$$\{G_f|x_1\} \Longleftrightarrow \{G_f|\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)} \neq 0 \tag{5}$$

$$\{G_r|x_1\} \Longleftrightarrow \{G_r|\frac{\partial\phi(1\to 0)}{\partial x_i(1\to m_i-1)} \neq 0 \tag{6}$$

$\{G_f|x_1\}$ and $\{G_r|x_1\}$ are subsets of boundary states of the system for every system component x_i. Therefore, it is necessary to analyse every component state s_i and to check the fact of MSS failure or repairing after modification of this state. The direct partial logic derivatives (2) allow to formalize this procedure.

3.2 Component Dynamic Reliability Indices

CDRI are represent probability evaluation of the influence of the i-th system component on the possibility of failure or repairing of the system. From the point of view of system reliability, unstable components are determined by these indices. CDRI are calculated by DDRI.

Definition 2. CDRI are probabilities of MSS failure and repairing at modification of the state of the i-th system component [7, 10]:

$$P_f(i) = p(i)^{1\to 0}_{1\to 0}\, p_1(i) \tag{7}$$

$$P_r(i) = p(i)^{0\to 1}_{0\to m_i-1}\, p_0(i) \tag{8}$$

$p(i)^{1\to 0}_{1\to 0}$ is the structural probability of i-th component state modification from 1 to 0, where the system fail; $p_1(i)$ is the probability of state 1 of the i-th component; $p(i)^{0\to 1}_{0\to m_i-1}$ is the structural probability of the i-th component replacement for system repairing; $p_0(i)$ is the probability of state 0 of the i-th component failure.

$$p(i)^{1\to 0}_{1\to 0} = \frac{p(i)^{1\to 0}_{1\to 0}}{m_1, m_2 \dots m_n} \tag{9}$$

$$p(i)^{0\to 1}_{0\to m_i-1} = \frac{p(i)^{0\to 1}_{0\to m_i-1}}{m_1, m_2 \dots m_n} \tag{10}$$

$p(i)^{1\to 0}_{1\to 0}$ is the number of system states when the breakdown of the i-th component forces the system to fail and $p(i)^{0\to 1}_{0\to m_i-1}$ is the number of system states when system repairing is brought about by replacing the i-th component.

Note, numbers $p(i)^{1\to 0}_{1\to 0}$ and $p(i)^{0\to 1}_{0\to m_i-1}$ are obtained as values of the direct partial logic derivative $\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)}$ and $\frac{\partial\phi(0\to 01)}{\partial x_i(0\to m_i-1)}$ with respect to i-th variable which are not equal 0. In other words, numbers $p(i)^{1\to 0}_{1\to 0}$ and $p(i)^{0\to 1}_{0\to m_i-1}$ are cardinality of the set $\{G_f|x_i\}$ in (5) and the set $\{G_r|x_i\}$ in (6) accordingly.

3.3 Dynamic Integrated Reliability Indices

DIRI are generalization of DDRI and probability evaluation of a modification of MSS reliability at a change of the system components state. In particular, the probability of boundary of system states is estimated by these indices.

Definition 3. DIRI is the probability of system failure or repairing if one of system components fails or restores [6, 10]:

$$P_f = \sum_{i=1}^{n} P_f(i) \prod_{q-1,\, q\neq i}^{n} (1 - P_f(q)) \tag{11}$$

$$P_r = \sum_{i=1}^{n} P_r(i) \prod_{q-1,\, q\neq i}^{n} (1 - P_r(q)) \tag{12}$$

where $P_f(i)$ and $P_r(i)$ are determined in (7) and (8).

4 THE ALGORITHM FOR CALCULATION OF THE DYNAMIC RELIABILITY INDICES

DRI are calculated for the estimation of the dynamic reliability analysis of MSS. As mentioned above, DRI are calculated by the direct partial logic derivative of the structure function. It is used in the algorithms of DRI calculation.

The Algorithm of DRI calculation.

Step 1.0 DDRI $\{G_f\}$ and $\{G_r\}$ are calculated for the MSS.

Step 1.1 The derivatives $\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)}$ are calculated by (2) $(i = 1, \dots, n)$.

Step 1.2 The subsets $\{G_f|x_i\}$ are obtained in accordance with (5).

Step 1.3 The set of the boundary states of the system $\{G_f\}$ (3) is formed.

Step 1.4 The derivative $\frac{\partial\phi(1\to 0)}{\partial x_i(1\to m_i-1)}$ is calculated on account of (2) $(i = 1, \dots, n)$.

Step 1.5 The subsets $\{G_r|x_i\}$ are obtained by (6).

Step 1.6 The set (4) $\{G_r\}$ of the boundary states of the system is formed.

Step 2.0 CDRI Pf(i) and Pr(i) are calculated.

Step 2.1 The numbers $p(i)^{1\to 0}_{1\to 0}$ and $p(i)^{0\to 1}_{0\to m_i-1}$ are obtained. They are conformed to numbers of nonzero elements of the direct partial logic derivatives $\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)}$ and $\frac{\partial\phi(1\to 0)}{\partial x_i(1\to m_i-1)}$ that are calculated in step 1.2 and step 1.4.

Step 2.2 The structural probability $p(i)^{1\to 0}_{1\to 0}$ of the i-th component state modification from 1 to 0 where the system fails and the structural probability $p(i)^{0\to 1}_{0\to m_i-1}$ of the i-th component replacement for system repairing are calculated according to (9) and (10).

Step 2.3 CDRI (probabilities of MSS failure or repairing at a modification of a state of i-th system component) are obtained by (7) - (8).

Step 3.0 DIRI for MSS estimation of the probability of the system failure and the system repairing by (11) and (12).

5 EXAMPLES

DRI are calculated for the MSS 2-out-of-3 (Table 1). These indices are determined according to the Algorithm of DRI calculation.

Step 1.0 DDRI $\{G_f\}$ and $\{G_r\}$ are calculated for the MSS.

Step 1.1 The derivatives $\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)}$ are calculated by (2) $(i = 1, 2, 3)$ and are presented in Table 2.

Step 1.2 The subsets $\{G_f|x_i\}$ are obtained in accordance with (5)and are:

$$\{G_f|x_1\} = \{101, 102, 110\},$$

$$\{G_f|x_1\} = \{011, 012, 110, 210, 310\},$$

$$\{G_f|x_1\} = \{011, 101, 201, 301\}.$$

Step 1.3 The set of the boundary states of the system $\{G_f\}$, (3) is formed:

$$\{G_f\} = \{011, 012, 101, 102, 110, 201, 210, 301, 310\}$$

Step 1.4 The derivative $\frac{\partial\phi(1\to 0)}{\partial x_i(1\to m_i-1)}$ is calculated on account of (2) $(i = 1, 2, 3)$, (Table 3).

Step 1.5 The subsets $\{G_r|x_i\}$ are obtained by (6):

$$\{G_r|x_1\} = \{001, 002, 010\},$$

$$\{G_r|x_2\} = \{001, 002, 100, 200, 300\},$$

$$\{G_r|x_13\} = \{010, 100, 200, 300\}.$$

Step 1.6 Set $\{G_r\}$, (4) of the boundary states of the system is formed.

$$\{G_r\} = \{001, 002, 110, 100, 200, 300\}$$

The set $\{G_f\}$ and the set $\{G_r\}$ are important for the reliability analysis of dynamic behavior of MSS. But DDRI stipulate for problem for the MSS of large dimensionality. CDRI and DIRI are preferable for applications.

Step 2.0 CDRI $P_f(i)$ and $P_r(i)$ are obtained.

Step 2.1 The numbers $p(i)^{1\to 0}_{1\to 0}$ and $p(i)^{0\to 1}_{0\to m_i-1}$ are:

$$p(1)^{1\to 0}_{1\to 0} = 3, \quad p(2)^{1\to 0}_{1\to 0} = 5, \quad p(3)^{1\to 0}_{1\to 0} = 4;$$

$$p(1)^{0\to 1}_{0\to 3} = 3, \quad p(2)^{0\to 1}_{0\to 1} = 5, \quad p(3)^{0\to 1}_{0\to 2} = 4;$$

Step 2.2 The structural probability of i-th component are calculated according to (9) and (10):

$$p(1)^{1\to 0}_{1\to 0} = 0.125,\ p(2)^{1\to 0}_{1\to 0} = 0.208,\ p(3)^{1\to 0}_{1\to 0} = 0.167;$$

$$p(1)^{0\to 1}_{0\to 3} = 0.125,\ p(2)^{0\to 1}_{0\to 1} = 0.208,\ p(3)^{0\to 1}_{0\to 2/} = 0.167;$$

Step 2.3 CDRI are obtained by (7) - (8):

$$P_f(1) = 0.019, \quad P_f(2) = 0.144, \quad P_f(3) = 0.057;$$

$$P_r(1) = 0.052, \quad P_r(2) = 0.144, \quad P_r(3) = 0.083.$$

The probability of the component state is in Table 4.

Table 4. Component state probability

Component	State			
	0	1	2	3
x_1	0.20	0.15	0.23	0.42
x_2	0.31	0.69	—	—
x_3	0.16	0.34	0.50	—

CDRI relates the system states to the component states when a change of the i-th component forces a change in the system state. Therefore the analysis of CDRI shows:

a) The system has the maximum probability of failure when the second component is failure because its CDRI has the largest value $P_f(2) = 0.144$;

b) The system fails with minimum probability if the first component has failed $(P_f(1) = 0.019)$;

c) MSS repairs with maximum probability by replacement of the second component since CDRI $P_r(2) = 0.143$.

Step 3.0 DIRI for MSS estimation the probability of the system failure and the system repairing by (11) and (12).

DIRI permits to obtain the probability of the system failure if one of the system components is breakdown. It is $P_f = 0.196$. The probability of the system repairing is $P_r = 0.234$ if one of the failure components of the system is replaced.

DRI are calculated for series (3-out-of-3) and (1-out-of-3) systems. Basic data (the number of components, state levels of the component *etc.*) is similar as for MSS 2-out-of-3 that is investigated above.

The series system. DDRI are calculated by virtue of direct partial logic derivatives $\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)}$ and $\frac{\partial\phi(0\to 1)}{\partial x_i(0\to m_i-1)}$ as in previous time for MSS 2-out-of-3. The structure function for the series system is in Table 5.

Table 5. The structure function of the series system (MSS 3-out-of-3)

$x_1,\ x_2,\ x_3$	$\phi(\mathbf{x})$	$;x_1,\ x_2,\ x_3$	$\phi(\mathbf{x})$
0 0 0	0	2 0 0	0
0 0 1	0	2 0 1	0
0 0 2	0	2 0 2	0
0 1 0	0	2 1 0	0
0 1 1	0	2 1 1	1
0 1 2	0	2 1 2	1
1 0 0	0	3 0 0	0
1 0 1	0	3 0 1	0
1 0 2	0	3 0 2	0
1 1 0	0	3 1 0	0
1 1 1	1	3 1 1	1
1 1 2	1	3 1 2	1

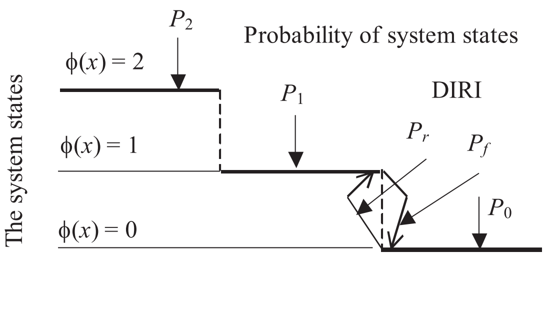

Fig. 1. DIRI and probability of system states

DDRI for series system are:

$$\{G_f\} = \{111, 112, 211, 212, 311, 312\},$$
$$\{G_r\} = \{011, 012, 101, 102, 110, 201, 202, 210, 301, 302, 310\}.$$

DDRI are not obvious. So the CDRI and DIRI are used in practice. The calculation of CDRI for a series system is presented in Table 6. So, the breakdown of the second component causes the maximum probability of the system failure $(P_f(2) = 0.173)$. The first component has an influence on the system failure least of all $(P_f(1) = 0.013)$. The system repairing is most probable by replacement of the second component $(P_r(2) = 0.173)$.

Table 6. CDRI calculation for series system

	Step 2.1		Step 2.2		Step 2.3	
	$p(i)^{1\to 0}_{1\to 0}$	$p(i)^{0\to 1}_{0\to m_i}$	$p(i)^{1\to 0}_{1\to 0}$	$p(i)^{0\to 0}_{1\to m_i}$	$P_f(i)$	$P_r(i)$
x_1	2	2	0.083	0.083	0.013	0.035
x_2	6	6	0.250	0.205	0.173	0.173
x_3	3	3	0.125	0.125	0.043	0.063

Table 7. The structure function of the parallel system (MSS 1-out-of-3)

$x_1,\ x_2,\ x_3$			$\phi(\mathbf{x})$	; $x_1,\ x_2,\ x_3$			$\phi(\mathbf{x})$
0	0	0	0	2	0	0	1
0	0	1	1	2	0	1	2
0	0	2	1	2	0	2	2
0	1	0	1	2	1	0	2
0	1	1	2	2	1	1	2
0	1	2	2	2	1	2	2
1	0	0	1	3	0	0	1
1	0	1	2	3	0	1	2
1	0	2	2	3	0	2	2
1	1	0	2	3	1	0	2
1	1	1	2	3	1	1	2
1	1	2	2	3	1	2	2

DIRI for series system are calculated in accordance to (15) and (16): $P_f = 0.208$ and $P_r = 0.233$.

The parallel system. The structure function of this system is in Table 7. The DDRI are calculated by the direct partial logic derivatives $\frac{\partial\phi(1\to 0)}{\partial x_i(1\to 0)}$ and $\frac{\partial\phi(0\to 1)}{\partial x_i(0\to m_i-1)}$. They are: $\{G_r\} = \{001, 010, 100\}$ and $\{G_r\} = \{000\}$.

Calculation of CDRI for the parallel system is presented in Table 8. The breakdown of the second component causes the maximum probability of the system failure $(P_f(2) = 0.029)$ as for previous system. The system repairing is most probable by the replacement of the second component $(P_r(2) = 0.029)$ too.

DIRI are probabilities of the change of the system reliability if the state of one of the system components is changed. The probability of the system failure, if one of the components breaks down, is $P_f = 0.049$ in accordance to (11). The probability of system repairing obtained by (12) and is $P_f = 0.064$ if one of the failed component of the system is replaced.

Table 8. CDRI calculation for parallel system (MSS 1-out-of-3)

	Step 2.1		Step 2.2		Step 2.3	
	$p(i)^{1\to 0}_{1\to 0}$	$p(i)^{0\to 1}_{0\to m_i}$	$p(i)^{1\to 0}_{1\to 0}$	$p(i)^{0\to 0}_{1\to m_i}$	$P_f(i)$	$P_r(i)$
x_1	1	1	0.042	0.042	0.006	0.018
x_2	1	1	0.042	0.042	0.029	0.029
x_3	1	1	0.042	0.042	0.014	0.021

These examples for series, parallel and 2-out-of-3 systems reveal the main point of dynamic indices CDRI and DIRI well. CDRI reflect the influence of the change of the specifically i-th component state upon the system reliability. In particular the system failure and system repairing depending on the i-th component state modification are examined in this paper. Since the component state probabilities are equal to the change of the system components, they have a similar influence on the system reliability in these examples. So the second component has the largest probability of system failure if this component breaks down. The first component has the least probability in analogous situation for series, parallel and 2-out-of-3 systems. And the second component has the largest probability for the system repairing in the case of replacement of its component.

DIRI describe the dynamic characteristic of the MSS and are different for series, parallel and 2-out-of-3 systems (Table 9). The probability of failure of the series system is highest possible $(P_f = 0.208)$ if one of the system components breaks down. The probability of the MSS failure P_f is minimum for the parallel system. But this probability has another meaning than the probability $P_l = Pr\phi(\mathbf{x}0 = l,\ l \in \{0, 1, \dots, m-1\}$, (Fig.1). It is clearly seen for the system repairing (the probability P_r

in Table 9). There are 11 boundary states for repairing of the series system by replacement of one of the failure system components. It is determined on account of $\{G_r\}$ for the series system. There is one state for parallel system only $\{G_r\} = \{000\}$. The failing parallel system is reestablished in three possible ways: replacement of the first component, of the second component or the third component.

Table 9. Reliability indices for series, 2-out-of-3 systems and parallel systems

Indices		The series system	The system 2-out-of-3	The parallel system
Static	P_0	0.750	0.292	0.042
	P_1	0.250	0.500	0.250
	P_2	0.000	0.208	0.708
Dynamic	P_f	0.208	0.196	0.049
(DIRI)	P_r	0.233	0.234	0.064

6 CONCLUSIONS

In this paper we presented a new measure for MSS reliability, which is calculated by the structure function of the MSS model of the network. This model allows to determine some level of system availability (there are not only two of them) in contrast to the Binary System. The MSS model is further improved: the system has a different number of discrete states for the system and for each component.

This measure, which is named DRI, involves probabilities of the changes of the system states that are assigned by changes of component states. We have shown two system changes: the system failure and system repair. The suggested method for reliability analysis of the MSS can be used to estimate also other changes in the system state.

In the next investigations we are planning to generalize this method to multi-input and multi-output networks.

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