Journal of ELECTRICAL ENGINEERING, VOL. 51, NO. 12/s, 2000, 17–20

GENERALIZATION OF THE PARTIAL SUMMATION PROCESS

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Two types of transformations of discrete random variables are presented. The first of them is a generalization of the partial summation mentioned in [1], [6] and [7]. Relations between probability generating functions and moments of the parent and descendant distributions are analyzed. It is shown that the Salvia-Bollinger distribution is invariant in regard to the considered transformations.

K e y w o r d s: discrete probability distributions, partial-sums distributions, the Salvia-Bollinger distribution

2000 *Mathematics Subject Classification*: 60E05, 62E10

1 $R(k,l)$–GENERATED DISTRIBUTIONS

Let R be the set of real numbers, Z_0^+ the set of non-negative integers and Z^+ the set of positive integers. Let X^* be a discrete random variable (*rv*) defined on the set Z_0^+ with a probability mass function (*pmf*) $\{P_i^*\}_{i=0}^{\infty}$.

Throughout the paper let us suppose any sum with the lower limit greater than the upper one to be equal to 0.

Definition 1. Let $k \in Z_0^+, l \in Z^+$. Let $X_{R(k,l)}$ be a discrete *rv* generated from X^* in the following way.

For $k = 0$:

$$_{R(0,l)}P_i = \frac{R(0,l)}{i+l} \sum_{j=i+l-1}^{\infty} P_j^*\,, \quad i = 0,1,2,\dots$$

For $k > 0$:

$$_{R(k,l)}P_i = \alpha_i, \quad \alpha_i \ge 0, i = 0,1,\dots,k-1\,; \sum_{i=0}^{k-1} \alpha_i \le 1\,;$$

$$_{R(k,l)}P_i = \frac{R(k,l)}{i-k+l} \sum_{j=i+l-1}^{\infty} P_j^*\,, \quad i = k, k+1, \dots$$

$R(k,l)\colon Z_0^+ \times Z^+ \to R$ is (for a fixed *pmf* $\{P_i^*\}_{i=0}^{\infty}$) a function adjusting the values of $\left\{\frac{1}{i-k+l} \sum\limits_{j=i+l-1}^{\infty} P_j^*\right\}_{i=k}^{\infty}$ taking into account the sequence $\{\alpha_i\}_{i=0}^{k-1}$ (if $k>0$) so that $\left\{{}_{R(k,l)}P_i\right\}_{i=0}^{\infty}$ is a *pmf* (i.e. $\sum\limits_{i=0}^{\infty} {}_{R(k,l)}P_i = 1$).

X^* will be called the $R(k,l)$-parent of $X_{R(k,l)}$; $X_{R(k,l)}$ the $R(k,l)$-descendant of X^*.

This summation is a generalization of the process $P_i = \frac{c}{i}\sum\limits_{j=i}^{\infty} P_j^*$, $i = 1,2,\dots$ (c being a proper constant), which is a result of theoretical explorations of the Bradford law [1]. It is also a mathematical model of law-like hypotheses in linguistics and musicology [7]. The process is analyzed in [6].

Lemma 1.

$$R(k,l) = \frac{1 - \sum\limits_{i=0}^{k-1} \alpha_i}{\sum\limits_{i=k}^{\infty} \sum\limits_{j=i+l-1}^{\infty} \frac{P_j^*}{i-k+l}}\,.$$

P r o o f .

$$1 = \sum_{i=0}^{\infty} P_i = \sum_{i=0}^{k-1} \alpha_i + R(k,l) \sum_{i=k}^{\infty} \frac{1}{i-k+l} \sum_{j=i+l-1}^{\infty} P_i^*$$

$$\Longrightarrow R(k,l) = \frac{1 - \sum\limits_{i=0}^{k-1} \alpha_i}{\sum\limits_{i=k}^{\infty} \sum\limits_{j=i+l-l}^{\infty} \frac{P_j^*}{i-k+l}}.$$

For the generated probabilities the following recurrence formula holds.

Lemma 2.

$$_{R(k,l)}P_{i+1} = {}_{R(k,l)}P_i - \frac{R(k,l)}{i-k+l} P_{i+l-1}^*, \qquad i = k, k+1, \dots$$

The proof is obvious and hence omitted.

Let $G^*(t) = \sum\limits_{i=0}^{\infty} P_i^* t^i$, $_{R(k,l)}G(t) = \sum\limits_{i=0}^{\infty} {}_{R(k,l)}P_i t^i$ be the probability generating functions (*pgf*'s) of X^* and $X_{R(k,l)}$, respectively.

Theorem 1.

$$_{R(k,l)}G(t) = \frac{R(k,l)}{t^{l-k}} \int_0^t \Bigg[\Big(1 - \sum_{i=0}^{k+l-2} P_i^*\Big) z^{l-1} - \Big(G^*(z) - \sum_{i=0}^{k+l-2} P_i^* z^i\Big) z^{-k+1}\Bigg] (1-z)^{-1}\, dz + \sum_{i=0}^{k-1} \alpha_i t^i\,.$$

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P r o o f .

$$_{R(k,l)}G(t)=\sum_{i=0}^{\infty}{}_{R(k,l)}P_it^i=\sum_{i=0}^{k-1}\alpha_it^i$$
$$+R(k,l)\Big[\frac{t^k}{l}\left(P^*_{k+l-1}+P^*_{k+l}+\ldots\right)+\frac{t^{k+1}}{l+1}\left(P^*_{k+l}+P^*_{k+l+1}+\ldots\right)+\ldots\Big]=\sum_{i=0}^{k-1}\alpha_it^i+\frac{R(k,l)}{t^{l-k}}\Big[\frac{t^l}{l}(P^*_{k+l-1}+P^*_{k+l}+\ldots)$$
$$+\frac{t^{l+1}}{l+1}\left(P^*_{k+l}+P^*_{k+l+1}+\ldots\right)+\ldots\Big]$$
$$=\sum_{i=0}^{k-1}\alpha_it^i+\frac{R(k,l)}{t^{l-k}}\Big[\Big(1-\sum_{i=0}^{k+l-2}P^*_i\Big)\int_0^t\frac{z^{l-1}}{1-z}\mathrm{d}z$$
$$-\int_0^t\Big(\sum_{i=0}^{\infty}P^*_{i+k+l-1}z^{i+l}\Big)(1-z)^{-1}\,\mathrm{d}z\Big]$$
$$=\sum_{i=0}^{k-1}\alpha_it^i+\frac{R(k,l)}{t^{l-k}}\int_0^t\Big[\Big(1-\sum_{i=0}^{k+l-2}P^*_i\Big)z^{l-1}$$
$$-\Big(G^*(z)-\sum_{i=0}^{k+l-2}P^*_iz^i\Big)z^{-k+1}\Big](1-z)^{-1}\,\mathrm{d}z\,.$$

A special case of Theorem 1 (for $k=l=1$ and $\alpha_0=0$) is in [6].

Let $x\in R$, $r\in Z^+$. Let us denote $x_{(r)}$ the r-th descending factorial of the number x, i.e. $x_{(r)}=x(x-1)\ldots(x-r+1)$. We only note that $x_{(0)}=1$.

Let $J^*(t)=\sum_{i=0}^{\infty}\frac{{}^*\mu'_{[i]}t^i}{i!}$, $_{R(k,l)}J(t)=\sum_{i=0}^{\infty}\frac{_{R(k,l)}\mu'_{[i]}t^i}{i!}$ be the generating functions of the descending factorial moments (see e.g. [3]) of the rv's X^* and $X_{R(k,l)}$, respectively. Let us suppose that these functions exist. It is easy to see that $_{R(k,l)}\mu'_{[1]}={}_{R(k,l)}\mu=E(X_{R(k,l)})$ and ${}^*\mu'_{[1]}={}^*\mu=E(X^*)$.

Theorem 2.

$$_{R(k,l)}\mu=R(k,l)\Big[{}^*\mu-\sum_{i=0}^{k+l-2}(i-k+l)P^*_i-(l-1)\Big(1-\sum_{i=0}^{k+l-2}P^*_i\Big)-k+1\Big]+\sum_{i=0}^{k-1}(i-k+l)\alpha_i+k-l$$

and for $r\geq1$ it holds

$$\sum_{i=0}^{r-1}\binom{r}{i}(r-i)(l-k)_{(i+1)}{}_{R(k,l)}\mu'_{[r-i-1]}+\sum_{i=0}^{r-1}\binom{r}{i}(r-i)(l-k)_{(i)}{}_{R(k,l)}\mu'_{[r-i]}$$
$$=R(k,l)\Big[\sum_{i=0}^{r}\binom{r}{i}(-k+1)_{(r-i)}{}^*\mu'_{[i]}-\sum_{i=0}^{k+l-2}(i-k+1)_{(r)}P^*_i-(l-1)_{(r)}\Big(1-\sum_{i=0}^{k+l-2}P^*_i\Big)\Big]+r\sum_{i=0}^{k-1}(i-k+l)_{(r)}\alpha_i\,.$$

P r o o f . The following equations hold (see e.g. [3]):

$$_{R(k,l)}\mu'_{[r]}={}_{R(k,l)}J^{(r)}(t)\big|_{t=0}\,,$$
$$_{R(k,l)}J(t)={}_{R(k,l)}G(t+1)\,.$$

Due to Theorem 1 we have

$$(t+1)^{l-k}{}_{R(k,l)}J(t)=R(k,l)\int_0^{t+1}\Big[\Big(1-\sum_{i=0}^{k+l-2}P^*_i\Big)z^{l-1}-\Big(G^*(z)-\sum_{i=0}^{k+l-2}P^*_iz^i\Big)z^{-k+1}\Big](1-z)^{-1}\mathrm{d}z+(t+1)^{l-k}\sum_{i=0}^{k-1}\alpha_i(t+1)^i$$

and computing derivative of both sides of the last equation we obtain

$$(l-k)t(t+1)^{l-k-1}J(t)+t(t+1)^{l-k}J'(t)$$
$$=R(k,l)\Big\{\Big[J^*(t)-\sum_{i=0}^{k+l-2}P^*_i(t+1)^i\Big](t+1)^{-k+1}-\Big(1-\sum_{i=0}^{k+l-2}P^*_i\Big)(t+1)^{l-1}\Big\}+\sum_{i=0}^{k-1}\alpha_i(i+l-k)t(t+1)^{i+l-k-1}.$$

The proof can be easily completed by substituting 0 for t into r-th derivative of the preceeding equation.

A special case of Theorem 2 (the mean of the $R(1,1)$-descendant if $\alpha_0=0$) can be found in [6].

2. $S(m,n)$–GENERATED DISTRIBUTIONS

Let X be a rv defined on the set Z_0^+ with a pmf $\{P_i\}_{i=0}^{\infty}$ and let $P_{m+s}\leq\frac{n+s-1}{n+s}P_{m+s-1}$ for $s=1,2,\ldots$

Definition 2. Let $m\in Z_0^+$, $n\in Z^+$, $P_m\neq0$. Let $X^*_{S(m,n)}$ be a rv generated from X in the following way.

For $(m,n)=(0,1)$:

$$_{S(0,1)}P^*_i=S(0,1)[(i+1)P_i-(i+2)P_{i+1}],\ i=0,1,2,\ldots$$

For $(m,n)\neq(0,1)$:

$$_{S(m,n)}P^*_i=\alpha^*_i\,,\quad \alpha^*_i\geq0\,,\ i=0,1,\ldots,m+n-2\,;\qquad \sum_{i=0}^{m+n-2}\alpha^*_i\leq1;$$
$$_{S(m,n)}P^*_i=S(m,n)\big[(i-m+1)P_{i-n+1}-(i-m+2)P_{i-n+2}\big],\ i=m+n-1,m+n,\ldots$$

$S(m,n)\colon Z_0^+\times Z^+\to R$ is (for a fixed pmf $\{P_i\}_{i=0}^{\infty}$) a function adjusting the values of $\{(i+n)(P_{i+m}-(i+n+1)P_{i+m+1})\}_{i=0}^{\infty}$ taking into account the sequence $\{\alpha^*_i\}_{i=0}^{m+n-2}$ (if $m+n\geq2$) so that $\{_{S(m,n)}P^*_i\}_{i=0}^{\infty}$ is a pmf.

X will be called the $S(m,n)$-parent of $X^*_{S(m,n)}$; $X^*_{S(m,n)}$ the $S(m,n)$-descendant of X.

Lemma 3.

$$S(m,n)=\frac{1-\sum\limits_{i=0}^{m+n-2}\alpha_i^*}{nP_m}\,.$$

P r o o f .

$$1=\sum_{i=0}^{\infty}{}_{S(m,n)}P_i^*=\sum_{i=0}^{m+n-2}\alpha_i^*+S(m,n)\Big(nP_m$$
$$-(n+1)P_{m+1}+(n+1)P_{m+1}-(n+2)P_{m+2}+\dots\Big)$$
$$\Longrightarrow\ S(m,n)=\frac{1-\sum\limits_{i=0}^{m+n-2}\alpha_i^*}{nP_m}\,.$$

Let $G(t)=\sum\limits_{i=0}^{\infty}P_it^i$, ${}_{S(m,n)}G^*(t)=\sum\limits_{i=0}^{\infty}{}_{S(m,n)}P_i^*t^i$ be the *pgf*'s of X and $X^*_{S(m,n)}$, respectively.

Theorem 3.

$${}_{S(m,n)}G^*(t)=S(m,n)\Big\{t^{m-1}(t-1)\Big[t^{n-m}\Big(G(t)$$
$$-\sum_{i=0}^{m}P_it^i\Big)\Big]'+nP_mt^{m+n-1}\Big\}+\sum_{i=0}^{m+n-2}\alpha_i^*t^i\,.$$

P r o o f .

$${}_{S(m,n)}G^*(t)=\sum_{i=0}^{\infty}{}_{S(m,n)}P_i^*t^i=\sum_{i=0}^{m+n-2}\alpha_i^*t^i+S(m,n)\{[nP_m$$
$$-(n+1)P_{m+1}]t^{m+n-1}+[(n+1)P_{m+1}-(n+2)P_{m+2}]t^{m+n}$$
$$+\dots\}=\sum_{i=0}^{m+n-2}\alpha_i^*t^i+S(m,n)\Big[nP_mt^{m+n-1}$$
$$+t^{m-1}(t-1)\Big(\sum_{i=1}^{\infty}P_{i+m}t^{i+n}\Big)'\Big]=\sum_{i=0}^{m+n-2}\alpha_i^*t^i$$
$$+S(m,n)\Big\{t^{m-1}(t-1)\Big[t^{n-m}\Big(G(t)-\sum_{i=0}^{m}P_it^i\Big)\Big]'$$
$$+nP_mt^{m+n-1}\Big\}.$$

Let $J(t)=\sum\limits_{i=0}^{\infty}\frac{\mu'_{[i]}t^i}{i!}$, ${}_{S(m,n)}J^*(t)=\sum\limits_{i=0}^{\infty}\frac{{}_{S(m,n)}{}^*\mu'_{[i]}t^i}{i!}$ be the generating functions of the descending factorial moments of X and $X^*_{S(m,n)}$, respectively. Let us suppose that these functions exist.

Theorem 4. *For $r=1,2,\dots$ we have*

$${}_{S(m,n)}{}^*\mu'_{[r]}=S(m,n)\Big[(n-m)\sum_{i=0}^{r-1}\binom{r}{i}(r-i)(n-2)_{(i)}\mu'_{[r-i-1]}$$
$$-\sum_{i=0}^{r-1}\binom{r}{i}(r-i)(n-1)_{(i)}\mu'_{[r-i]}-r\sum_{i=0}^{m}(i+n-m)(i+n$$
$$-2)_{(r-1)}P_i+n(m+n-1)_{(r)}P_m\Big]+\sum_{i=0}^{m+n-2}\alpha_i^*i_{(r)}.$$

P r o o f . Due to Theorem 3 we have

$${}_{S(m,n)}J^*(t)=S(m,n)\Big\{t(t+1)^{m-1}\Big[(t+1)^{n-m}\Big(J(t)$$
$$-\sum_{i=0}^{m}P_i(t+1)^i\Big)\Big]'+nP_m(t+1)^{m+n-1}\Big\}+\sum_{i=0}^{m+n-2}\alpha_i^*(t+1)^i.$$

By substituting $t=0$ into r-th derivative of the last equation we can easily complete the proof.

Corollary 1. *Let us denote* ${}_{S(m,n)}{}^*\mu=E\big(X^*_{S(m,n)}\big)$ *and* $\mu=E(X)$. *We have*

$${}_{S(m,n)}{}^*\mu=S(m,n)\Big[n-m-\mu-\sum_{i=0}^{m}(i+n-m)P_i$$
$$+n(m+n-1)P_m\Big]+\sum_{i=0}^{m+n-2}i\alpha_i^*\,.$$

3 THE RELATIONSHIP BETWEEN $R(k,l)$– AND $S(m,n)$–GENERATED DISTRIBUTIONS

Theorem 5. *Let $u\in Z_0^+$, $v\in Z^+$. Let X^* be a rv defined on the set Z_0^+ with a pmf $\{P_i^*\}_{i=0}^{\infty}$. Let $\sum\limits_{i=0}^{u-1}\alpha_i<1$ if $u\geq 1$. Let us construct the $S(u,v)$-descendant of $X_{R(u,v)}$ in such a way that ${}_{S(u,v)}P_i^*=P_i^*$ for $i=0,1,\dots,u+v-2$ if $u+v\geq 2$. Then $X_{R(u,v)}$ is the $S(u,v)$-parent of X^*.*

P r o o f . According to Definition 1 for $i=u,u+1,\dots$

$${}_{R(u,v)}P_i=\frac{R(u,v)}{i-u+v}\sum_{j=i+v-1}^{\infty}P_j^*$$

and due to Definition 2 for $i=u+v-1,u+v,\dots$

$${}_{S(u,v)}P_i^*=S(u,v)\big[(i-u+1)_{R(u,v)}P_{i-v+1}-(i-u$$
$$+2)_{R(u,v)}P_{i-v+2}\big]=R(u,v)S(u,v)\Big(\frac{i-u+1}{i-u+1}\sum_{j=i}^{\infty}P_j^*$$
$$-\frac{i-u+2}{i-u+2}\sum_{j=i+1}^{\infty}P_j^*\Big)=R(u,v)S(u,v)P_i^*.$$

Consequently,

$$R(u,v)S(u,v)\sum_{i=u+v-1}^{\infty}P_i^*=\sum_{i=u+v-1}^{\infty}{}_{S(u,v)}P_i^*$$
$$=1-\sum_{i=0}^{u+v-2}{}_{S(u,v)}P_i^*=1-\sum_{i=0}^{u+v-2}P_i^*=\sum_{i=u+v-1}^{\infty}P_i^*$$
$$\Longrightarrow\ R(u,v)S(u,v)=1\,.$$

For $i=0,1,2,\dots$ the equation

$${}_{S(u,v)}P_i^*=P_i^*$$

has been proved.

R e m a r k 1 . Analogously it can be proved that if X is a *rv* with a proper *pmf* $\{P_i\}_{i=0}^{\infty}$ then $X^*_{S(u,v)}$ is the $R(u,v)$-parent of X.

4 INVARIANT DISTRIBUTIONS

Let $k \in Z_0^+$. $\{P_i^*\}_{i=0}^\infty$ has the k-displaced Salvia-Bollinger distribution (see [4], [5]) if

$$P_i^* = (-1)^{i-k}\binom{\alpha}{i-k+1}, \quad i = k, k+1, \dots;\ 0 < \alpha < 1$$

and $P_i^* = 0$ for $i = 0, 1, \dots, k-1$ if $k \geq 1$.
The k-displaced Salvia-Bollinger distribution has the *pgf* $G^*(t) = [1-(1-t)^\alpha]t^{k-1}$.

Theorem 6. *Let $k \in Z_0^+$, $l = 1$. Let us choose ${}_{R(k,l)}P_i = 0$ for $i = 0, 1, \dots, k-1$ if $k \geq 1$. Then $X^* = X_{R(k,1)}$ if and only if X^* has the k-displaced Salvia-Bollinger distribution.*

Proof. Let $X^* = X_{R(k,1)}$. Then for $n = 1, 2, \dots$ we have

$$P_{k+n-1}^* = \frac{R(k,1)}{n}\left(P_{k+n-1}^* + P_{k+n}^* + P_{k+n+1}^* + \dots\right),$$
$$P_{k+n}^* = \frac{R(k,1)}{n+1}\left(P_{k+n}^* + P_{k+n+1}^* + P_{k+n+2}^* + \dots\right)$$

and we obtain

$$P_{k+n}^* = \frac{n - R(k,1)}{n+1}P_{k+n-1}^* \implies P_{k+n}^* = \frac{(1-R(k,1))^{(n)}}{2^{(n)}}P_k^*,$$

$(1-R(k,1))^{(n)}$ being the n-th ascending factorial of $(1-R(k,1))$, i.e. $(1-R(k,1))(2-R(k,1))\dots(n-R(k,1))$. Consequently (${}_2F_1(a,b;c;d)$ is the Gaussian hypergeometric function, see e.g. [2]),

$$1 = \sum_{i=0}^\infty P_i^* = P_k^* \sum_{i=0}^\infty \frac{(1+R(k,1))^{(i)}}{2^{(i)}} = P_k^*\,{}_2F_1(1-R(k,1),1;2;1) \implies P_k^* = R(k,1).$$

So $P_{k+n}^* = (-1)^n\binom{R(k,1)}{n+1}$, $n = 0, 1, 2, \dots$, which is the k-displaced Salvia-Bollinger distribution.

Let X^* have the k-displaced Salvia-Bollinger distribution, i.e. its *pgf* is

$$G^*(t) = [1-(1-t)^\alpha]t^{k-1}.$$

According to Lemma 1 (for $\alpha_i = 0$, $i = 0, 1, \dots, k-1$ if $k \geq 1$)

$$\big(R(k,1)\big)^{-1} = \sum_{i=k}^\infty \sum_{j=i}^\infty \frac{P_j^*}{i-k+1} = (-1)^0\binom{\alpha}{1} + (-1)^1\binom{\alpha}{2} + (-1)^2\binom{\alpha}{3} + \dots + \frac{1}{2}\left[(-1)^1\binom{\alpha}{2} + (-1)^2\binom{\alpha}{3} + (-1)^3\binom{\alpha}{4} + \dots\right] + \frac{1}{3}\left[(-1)^2\binom{\alpha}{3} + (-1)^3\binom{\alpha}{4} + (-1)^4\binom{\alpha}{5} + \dots\right]$$
$$+ \dots = \sum_{i=0}^\infty \frac{(-1)^i}{i+1}\binom{\alpha}{i+1}\,{}_2F_1(i+1-\alpha,1;i+2;1)$$
$$= \frac{1}{\alpha}\sum_{i=0}^\infty (-1)^i\binom{\alpha}{i+1} = {}_2F_1(1-\alpha,1;2;1) = \frac{1}{\alpha} \implies R(k,1) = \alpha.$$

Due to Theorem 1

$${}_{R(k,l)}G(t) = R(k,1)t^{k-1}\int_0^t \frac{1 - z^{-k+1}G^*(z)}{1-z}\mathrm{d}z = \alpha t^{k-1}\int_0^t (1-z)^{\alpha-1}\mathrm{d}z = [1-(1-t)^\alpha]t^{k-1} = G^*(t).$$

The invariance of the 1-displaced Salvia-Bollinger distribution in regard to the $R(1,1)$-summation is proved in [6].

Remark 2. Due to Theorem 5 the k-displaced Salvia-Bollinger distribution is invariant also in regard to the $S(k,1)$-transformation.

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Received 23 May 2000

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