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ROBUST FEEDBACK LINEARIZATION AND GH_∞ CONTROLLER FOR A QUADROTOR UNMANNED AERIAL VEHICLE

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In this paper, a mixed robust feedback linearization with linear GH_∞ controller is applied to a nonlinear quadrotor unmanned aerial vehicle. An actuator saturation and constrain on state space output are introduced to analyse the worst case of control law design.The results show that the overall system becomes robust when weighting functions are chosen judiciously. Performance issues of the controller are illustrated in a simulation study that takes into account parameter uncertainties and external disturbances as well as measurement noise.

K e y w o r d s: UAV, GH, sensitivity, robust linearization

1 INTRODUCTION

The H_∞ control problem with continuous-time measurement output for linear systems has been studied in the last decade by many researchers such as Kwakernaak [1] and Grimble [2] in polynomial form and by van der Schaft [3] and Ball [4] for nonlinear systems. For the hovering control of helicopters, many control methods have been proposed including linear approaches such as *LQG* [5], H_∞ design [6],[7] and the nonlinear approaches such as sliding mode [8], backstepping technique [9], and input/output linearization [10]. Even tough the design of controllers to achieve a linear input-output response for nonlinear systems has been well researched [11], the conventional input-output linearization techniques will perform very poorly when it comes to output tracking as it will render the unstable internal dynamics unobservable [12]. Hence the nonlinear system must have stable zero dynamics for the input-output linearized system to be internally stable [11]. Following this context, classical feedback linearization may have poor robustness properties and cannot be easily combined with a H_∞ control law. So a robust nonlinear feedback is proposed to robustly control an uncertain nonlinear system around an operating point on using an appropriate approach for stability and robustness, *W–stability* [13]. Inertial Navigation System (INS) and GPS are used to calculate a position and orientation of the vehicle. They are especially suitable for guidance and navigation of an autonomous Unmanned Aerial Vehicle (UAV) [14].

This paper proposes an attempt to apply linear H_∞ outer control of helicopter quadrotor with plant uncertainty combined with a robust feedback linearization inner controller. The plant to be controlled is described by six-degree-of-freedom nonlinear dynamics with plant uncertainties due to the variation of moments of inertia and payload operation. Successful application of the autonomous quadrotor depends on its level of controllability and flying qualities. The overall inner outer controller should improve tracking performance and disturbance rejection capability. The process disturbance represents not only the uncertainty in the operating conditions, but the lack of precision in the system model. It degrades the robustness and performance of control systems and the estimation of unknown dynamics seems to be difficult. The approach that allows for worst-case disturbances is the H_∞ controller. Uncertainty bounds step can be used to define simple sensitivity and complementary sensitivity weights. These weights are chosen to maximize the disturbance attenuation properties. The disturbances attempt to maximize performance index, while the control attempts to minimize. The analysis of this problem has been primarily in the frequency domain. This analysis can be carried out in time domain, and naturally extends H_∞ theory to finite-time and non-linear systems [15]. In this work we mention the polynomial solution based on Diophantine equations [16].

2 DYNAMIC QUADROTOR

Using Newton law and referring to V. Mistler *et al* [17] [18], the general MIMO nonlinear system (Fig.1) can be represented in form:

$$\begin{aligned} \dot{x} &= F(x) + G_1(x)w + G_2(x)\bar{u} \\ y_1 &= H_1(x) + K_{12}(x)\bar{u} \\ y_2 &= H_2(x) + K_{21}(x)w \end{aligned} \tag{1}$$

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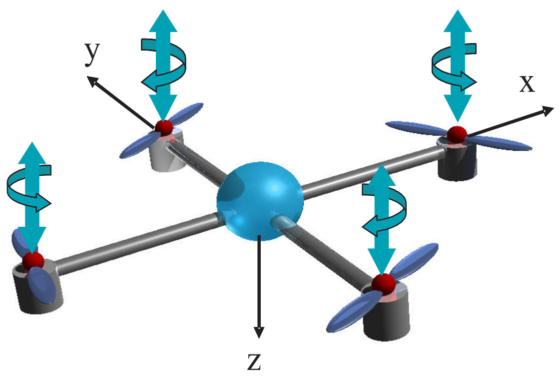

Fig. 1. The quadrotor helicopter

where $x \in \Re^n$ is the state vector, $\bar{u} \in \Re^m$ is the control input, $w \in \Re^n$ is the noise and unknown perturbation vector, $y_1 \in \Re^l$ is the controlled output, $y_2 \in \Re^p$ is the available measure vector. The following hypothesis hold:

(A1) the functions $F(x)$, $G_1(x)$, $G_2(x)$, $H_1(x)$, $H_2(x)$, $K_{12}(x)$, $K_{21}(x)$ are piecewise continuous.

(A2) $F(0) = 0$, $H_1(0) = 0$ and $H_2(0) = 0$ for almost every t.

(A3) $H_1^T(x)K_{12}(x) = 0$, $K_{12}^T(x)K_{12}(x) = I$, $K_{21}(x)G_1^T(x) = 0$, $K_{21}(x)K_{21}^T(x) = I$. where

$$w = \begin{bmatrix} w_b \\ w_p \end{bmatrix},$$

and where w_b is the noise vector of size 14. w_p is composed of aerodynamic forces disturbances $[A_x, A_y, A_z]^T$ and aerodynamic moment disturbances $[A_p, A_q, A_r]^T$. They act on the UAV and are computed from the aerodynamic coefficients C_i as $A_i = \frac{1}{2}\rho_{air} C_i W^2$ (ρ_{air} is the air density, W is the velocity of the UAV with respect to the air), (C_i depend on several parameters like the angle between airspeed and the body fixed reference system, the aerodynamic and geometric form of the wing). The rotor is the primary source of control and propulsion for the UAV. The Euler angle orientation to the flow provides the forces and moments to control the altitude and position of the system. The absolute position is described by three coordinates (x_0, y_0, z_0), and its attitude by Euler angles (ψ, θ, ϕ), under the conditions $(-\pi \le \psi < \pi)$ for yaw, $(-\frac{\pi}{2} < \theta < \frac{\pi}{2})$ for pitch and $(-\frac{\pi}{2} < \phi < \frac{\pi}{2})$ for roll. The state vector and other parameters are defined as:

$$x = (x_0, y_0, z_0, \psi, \theta, \phi, \dot{x}_0, \dot{y}_0, \dot{z}_0, \zeta_1, \xi, \dot{\psi}, \dot{\theta}, \dot{\phi})^T$$

ζ_1 and ξ are defined in (1)

$$F(x) = [f_1(x), .., f_{14}(x)]^T$$

$$G_1(x) = \begin{bmatrix} 0_{6\times 14} & 0_{6\times 3} & 0_{6\times 3} \\ 0_{3\times 14} & M_1 & 0_{3\times 3} \\ 0_{2\times 14} & 0_{2\times 3} & 0_{2\times 3} \\ 0_{3\times 14} & 0_{3\times 3} & P_1 \end{bmatrix}$$

$$G_2(x) = \begin{bmatrix} 0_{10\times 4} \\ P_4 \end{bmatrix}$$

$$H_1(x) = [0, 0, 0, 0, x_0, y_0, z_0, \psi]^T$$

$$H_2(x) = x$$

$$K_{12}(x) = \begin{bmatrix} I_{4\times 4} \\ 0_{4\times 4} \end{bmatrix}$$

$$K_{21}(x) = [I_{14\times 14}]$$

The real control signals (u_1, u_2, u_3, u_4) have been replaced by $(\bar{u}_1, \bar{u}_2, \bar{u}_3, \bar{u}_4)$ to avoid singularity in lie transformation matrices when using feedback linearization [17]. In that case u_1 has been delayed by a double integrator. The others control signals will keep unchanged.

$$\begin{aligned} u_1 &= \zeta_1 + mg \\ \dot{\zeta} &= \xi \\ \dot{\xi} &= \bar{u}_1 \\ u_2 &= \bar{u}_2 \\ u_3 &= \bar{u}_3 \\ u_4 &= \bar{u}_4 \end{aligned} \tag{2}$$

Let the state vector be written into the form:

$$x = (x_1, x_2, ..., x_{14})^T \tag{3}$$

so one can have:

$$f_1(x) = x_7,\ f_2(x) = x_8,\ f_3(x) = x_9$$
$$f_4(x) = x_{12},\ f_5(x) = x_{13},\ \ f_6(x) = x_{14}$$
$$f_7(x) = g_1^7 x_{10},\ \ f_8(x) = g_1^8 x_{10},$$
$$f_9(x) = g + g_1^9 (x_{10} + mg)$$
$$f_{10}(x) = x_{11},\ \ \ f_{11}(x) = 0$$

$$\begin{bmatrix} f_{12}(x) \\ f_{13}(x) \\ f_{14}(x) \end{bmatrix} = P_2 \begin{bmatrix} x_{12}^2 \\ x_{13}^2 \\ x_{14}^2 \end{bmatrix} + P_3 \begin{bmatrix} x_{12}x_{13} \\ x_{12}x_{14} \\ x_{13}x_{14} \end{bmatrix}$$

with

$$M_1 = \frac{1}{m} I_{3\times 3}$$

$$P_1 = \frac{1}{d} \begin{bmatrix} 0 & g_3^{12} & g_4^{12} d \\ 0 & g_3^{13} & g_4^{13} d \\ g_2^{14} & g_3^{14} & g_4^{14} d \end{bmatrix}$$

$$P_2 = \begin{bmatrix} p_{211} & 0 & 0 \\ p_{221} & 0 & 0 \\ p_{231} & p_{232} & 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} p_{311} & p_{312} & p_{313} \\ p_{321} & p_{322} & p_{323} \\ p_{331} & p_{332} & p_{333} \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & g_3^{12} & g_4^{12} \\ 0 & 0 & g_3^{13} & g_4^{13} \\ 0 & g_2^{14} & g_3^{14} & g_4^{14} \end{bmatrix}$$

$$g_1^7 = -\frac{1}{m}\left(Cx_6Cx_4Sx_5 + Sx_6Sx_4\right)$$
$$g_1^8 = -\frac{1}{m}\left(Cx_6Sx_5Sx_4 - Cx_4Sx_6\right)$$
$$g_1^9 = -\frac{1}{m}(Cx_5Cx_6)$$

$$g_2^{14} = \frac{d}{I_x}$$
$$g_3^{12} = \frac{dSx_6}{I_yCx_5};\; g_3^{13} = \frac{dCx_6}{I_y};\; g_3^{14} = \frac{dSx_5Sx_6}{I_yCx_5}$$
$$g_4^{12} = \frac{Cx_6}{I_zCx_5};\; g_4^{13} = -\frac{Sx_6}{I_z};\; g_4^{14} = \frac{Sx_5Cx_6}{I_zCx_5}$$

$$p_{211} = Sx_5Sx_6Cx_6\left(I_1 - I_3\right)$$
$$p_{221} = Sx_5Cx_5\left(Cx_6\right)^2\left(I_1 - I_3\right) - Cx_5Sx_6\left(I_1\right)$$
$$p_{231} = Sx_6Cx_6\left(Cx_5\right)^2\left(I_2 + I_3 - I_1\right) + Sx_6Cx_6\left(I_1 + I_3\right)$$
$$p_{232} = -I_2Sx_6Cx_6$$

$$p_{311} = T(x_5)\left(Cx_6\right)^2\left(I_1 - I_3\right) + T(x_5)\left(1 + I_3\right)$$
$$p_{312} = Sx_6Cx_6\left(I_3 - I_1\right)$$
$$p_{313} = \left(Cx_6\right)^2S_e(x_5)\left(I_3 - I_1\right) + S_e(x_5)(1 - I_3)$$
$$p_{321} = Sx_6Cx_6Sx_5\left(I_1 + I_3\right)$$
$$p_{322} = Cx_5\left(Cx_6\right)^2\left(I_3 - I_1\right) - Cx_5\left(1 - I_1\right)$$
$$p_{323} = Sx_6Cx_6\left(I_1 - I_3\right)$$
$$p_{331} = S_e(x_5)\left(1 + I_3\right) + \left(Cx_6\right)^2S_e(x_5)\left(I_1 - I_3\right) + \left(Cx_6\right)^2\left(2I_2 + I_3 - I_1\right) - Cx_5\left(I_2 + I_3\right)$$
$$p_{332} = Sx_6Cx_6Sx_5\left(-I_1 + I_3\right)$$
$$p_{333} = T(x_5)\left(Cx_6\right)^2\left(I_3 - I_1\right) + T(x_5)(1 - I_3)$$

$$I_1 = \frac{I_y - I_x}{I_z};\; I_2 = \frac{I_y - I_z}{I_x};\; I_3 = \frac{I_z - I_x}{I_y}$$

- g is the gravity constant ($g = 9.81\,\mathrm{ms}^{-2}$);
- d is the distance from the center of mass to the rotors;
- u_1 is the resulting thrust of the four rotors defined as $u_1 = (F_1 + F_2 + F_3 + F_4)$
- u_2 is the difference of thrust between the left rotor and the right rotor defined as $u_2 = d\left(F_4 - F_2\right)$
- u_3 is the difference of thrust between the front rotor and the back rotor defined as $u_3 = d(F_3 - F_1)$
- u_4 is the difference of torque between the two clockwise turning rotors and the two counter-clockwise turning rotors defined as $u_4 = C(F_1 - F_2 + F_3 - F_4)$
- C is the force to moment scaling factor
- I_x, I_y, I_z represent the diagonal coefficient of inertia matrix of the system.
- $S(.) = \sin(.)$, $C(.) = \cos(.)$, $T(.) = \tan(.)$, $S_e(.) = \sec(.)$.

3 ROBUST FEEDBACK LINEARIZATION (INNER CONTROLLER)

The robust feedback linearization method used in this context is based on Sobolev norm defined as

$$\|h\|_W = \left[\int_0^\infty h^T(t)h(t)dt + \int_0^\infty \dot{h}^T(t)\dot{h}(t)dt\right]^{\frac{1}{2}}. \quad (4)$$

It transforms a nonlinear system into its tangent linearized system around an operating point. Then, under state feedback

$$\bar{u}(x,v) = \alpha(x) + \beta(x)v$$

and change of coordinates

$$z = \phi(x)$$

defined by

$$\begin{aligned} \alpha(x) &= \alpha_c(x) + \beta_c(x)LT\phi_c(x) \\ \beta(x) &= \beta_c(x)R^{-1} \\ \phi(x) &= T^{-1}\phi_c(x) \end{aligned} \quad (5)$$

where $L = -\Delta.\frac{\partial \alpha_c}{\partial x}\mid_{x=0}, T = \frac{\partial \phi_c}{\partial x}\mid_{x=0}, R = \Delta^{-1}$, $\alpha_c(x) = -\Delta^{-1}(x)b(x)$, $\beta_c(x) = \Delta^{-1}(x)$ then the nonlinear system is transformed into a following one

$$\dot{z} = Az + B_2v + \left[\frac{\partial \phi}{\partial x}G_1(x)\right]_{x=\phi^{-1}(z)} \quad (6)$$

with $A = \frac{\partial F(x)}{\partial x}\mid_{x=0}, B_2 = G_2(0)$. Note that equation (4) satisfies $\frac{\partial \alpha}{\partial x}\mid_{x=0} = 0$, $\frac{\partial \phi}{\partial x}\mid_{x=0} = I_{14\times 14}$, $\beta(0) = I_{4\times 4}$. For the quadrotor helicopter the input-output decoupling problem is solvable for the nonlinear system by means of static feedback. The vector relative degree $\{r_1, r_2, r_3, r_4\}$ is given by

$$r_1 = r_2 = r_3 = 4;\; r_4 = 2$$

and we have

$$b(x) = \left[\, L_f^{r_1}h_1(x) \quad L_f^{r_2}h_2(x) \quad L_f^{r_3}h_3(x) \quad L_f^{r_4}h_4(x)\,\right]^T$$
$$\phi_c(x) = \left[\,\phi_{c1}(x), \phi_{c2}(x), \phi_{c3}(x), \phi_{c4}(x)\,\right]^T$$

$$\phi_{c1}(x) = \begin{bmatrix} h_1(x) = x_0 \\ L_fh_1(x) = x_7 = \dot{x}_0 \\ L_f^2h_1(x) = \frac{A_x}{m} + g_1^7x_{10} = \ddot{x}_0 \\ L_f^3h_1(x) = \dddot{x}_0 \end{bmatrix}$$

$$\phi_{c2}(x) = \begin{bmatrix} h_2(x) = y_0 \\ L_f h_2(x) = x_8 = \dot{y}_0 \\ L_f^2 h_2(x) = \frac{A_y}{m} + g_1^8(x_4, x_5, x_6)x_{10} = \ddot{y}_0 \\ L_f^3 h_2(x) = \dddot{y}_0 \end{bmatrix}$$

$$\phi_{c3}(x) = \begin{bmatrix} h_3(x) = z_0 \\ L_f h_3(x) = x_9 = \dot{z}_0 \\ L_f^2 h_3(x) = \frac{A_z}{m} + g + g_1^9 x_{10} = \ddot{z}_0 \\ L_f^3 h_3(x) = \dddot{z}_0 \end{bmatrix}$$

$$\phi_{c4}(x) = \begin{bmatrix} h_4(x) = x_4 \\ L_f h_4(x) = \dot{x}_4 \end{bmatrix}$$

$$\Delta(x) = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{14} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} & \Delta_{24} \\ \Delta_{31} & \Delta_{32} & \Delta_{33} & \Delta_{34} \\ \Delta_{41} & \Delta_{42} & \Delta_{43} & \Delta_{44} \end{bmatrix}$$

with

$$\begin{aligned} \Delta_{11} &= L_{g1} L_f^{r_1-1} h_1(x) \\ &= -\frac{1}{m}\left(Cx_6 Cx_4 Sx_5 + Sx_6 Sx_4\right) \\ \Delta_{12} &= L_{g2} L_f^{r_1-1} h_1(x) \\ &= \frac{d}{mI_x}\left(x_{10} Sx_6 Cx_4 Sx_5 - x_{10} Cx_6 Sx_4\right) \\ \Delta_{13} &= L_{g3} L_f^{r_1-1} h_1(x) = \frac{d}{mI_y}(-x_{10} Cx_4 Cx_5) \\ \Delta_{14} &= 0 \end{aligned}$$

$$\begin{aligned} \Delta_{21} &= L_{g1} L_f^{r_2-1} h_2(x) \\ &= -\frac{1}{m}\left(Cx_6 Sx_5 Sx_4 - Cx_4 Sx_6\right) \\ \Delta_{22} &= L_{g2} L_f^{r_2-1} h_2(x) \\ &= \frac{d}{mI_x}\left(x_{10} Sx_6 Sx_4 Sx_5 + x_{10} Cx_6 Cx_4\right) \\ \Delta_{23} &= L_{g3} L_f^{r_2-1} h_2(x) = \frac{d}{mI_y}(-x_{10} Sx_4 Cx_5) \\ \Delta_{24} &= L_{g4} L_f^{r_2-1} h_2(x) = 0 \end{aligned}$$

$$\begin{aligned} \Delta_{31} &= L_{g1} L_f^{r_3-1} h_3(x) = -\frac{1}{m}(Cx_5 Cx_6) \\ \Delta_{32} &= L_{g2} L_f^{r_3-1} h_3(x) = \frac{d}{mI_x}\left(x_{10} Sx_6 Cx_5\right) \\ \Delta_{33} &= L_{g3} L_f^{r_3-1} h_3(x) = \frac{d}{mI_y}(x_{10} Sx_5) \\ \Delta_{34} &= L_{g4} L_f^{r_3-1} h_3(x) = 0 \end{aligned}$$

$$\begin{aligned} \Delta_{41} &= L_{g1} L_f^{r_4-1} h_4(x) = 0 \\ \Delta_{42} &= L_{g2} L_f^{r_4-1} h_4(x) = 0 \\ \Delta_{43} &= L_{g3} L_f^{r_4-1} h_4(x) = \frac{d}{I_y}(Sx_6 S_e x_5) \\ \Delta_{44} &= L_{g4} L_f^{r_4-1} h_4(x) = \frac{1}{I_z}(Cx_6 S_e x_5) \end{aligned}$$

In fact the system in equation (5) is still nonlinear because of w vector. One seeks a controller which ensures the compensated system to be internally asymptotically stable and its output to tend asymptotically toward a desired trajectory even in the presence of external disturbance. In this context the linear GH_∞ is proposed.

4 H_∞ OPTIMAL CONTROL (OUTER CONTROLLER)

H_∞ synthesis methods take into account in an explicit manner some specification of robustness. The issue here is to take maximum guaranty for a synthesized control law on a chosen model to work effectively on the physical system. For that a transfer function family is considered where the nominal model $W_{nom} = A_0^{-1} B_0$ constitutes the "center". We assume that it is possible to choose these sets of transfer function contain the real system. Hence if the stability and performance of the closed loop system are obtained and demonstrated for all W_i elements then it will be also for the real system [19].

Let the transfer function of the uncertain system be

$$\tilde{W} = (A_0 + D_p \Delta_2 P_p)^{-1} (B_0 + D_p \Delta_1 F_p) \tag{7}$$

where $D_p \Delta_2 P_p$ and $D_p \Delta_1 F_p$ are modelling errors on A_0 and B_0. D_p, P_p and F_p are characterized by low and pass filter respectively and Δ_1, Δ_2 are the non structured uncertainty. It is assumed that disturbances are bounded and there exists a function V which verifies

$$\left\|V^{-1}\Delta_1\right\|_\infty^2 + \left\|V^{-1}\Delta_2\right\|_\infty^2 < 1 \tag{8}$$

Hence the minimization criterion is written as:

$$J_\infty = \left\|(P_p S + F_p M)^* \, \Phi_{ff} \, (P_p S + F_p M)\right\|_\infty \tag{9}$$

The cost function of GH_∞ lead to eigenvalues problem which lead to the minimization of $\left\|(P_p S + F_p M) A^{-1} D_f\right\|$ where $\Phi_{ff} = A^{-1} D_f D_f^* A^{*-1}$. The weighting functions can be represented as

$$P_p(z^{-1}) = P_d^{-1} P_n, \; F_p(z^{-1}) = P_d^{-1} F_n \tag{10}$$

where P_d is strictly schur and $P_n(0) \neq 0$. The polynomials P_n and F_n are chosen to assure that the polynomial

$$L_c = P_n B - F_n A \tag{11}$$

verifies $L_c L_c^* > 0$ on $|z| = 1$. If we can write $L_c = L_1 L_2$ with L_1 strictly minimal phase, L_2 a non-minimal phase and L_{2s} is Schur polynomial satisfying $L_{2s} = L_2^* z^{-n2}$ where $n_2 = deg(L_2)$, then the control law procedure is summarized as follow:

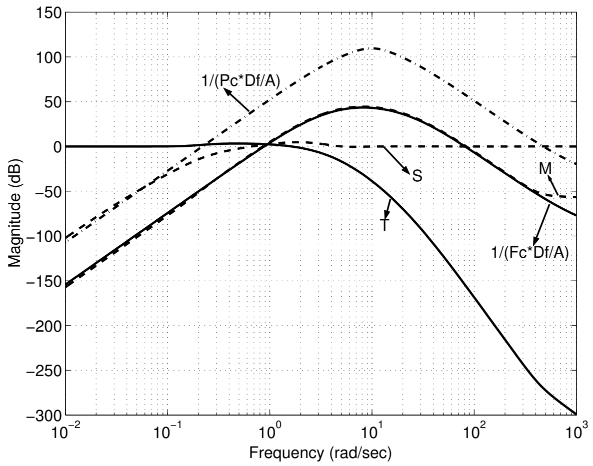

Fig. 2. Sensitivity S, M and T and inverse of weightings for x_0, y_0, z_0

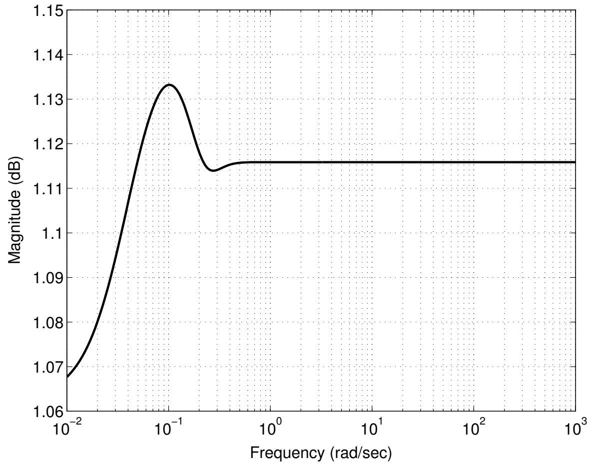

Fig. 3. Cost function for x_0, y_0, z_0

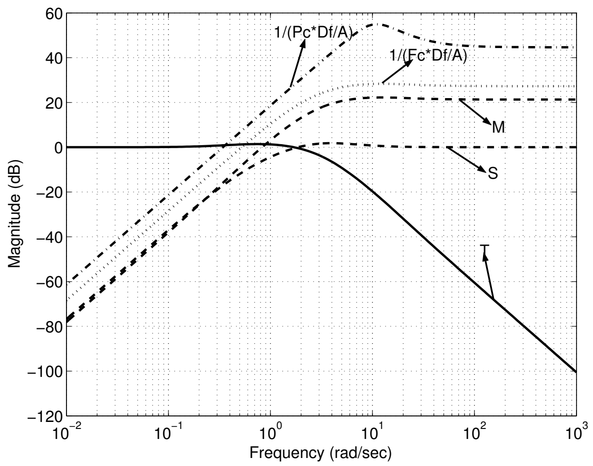

Fig. 4. Sensitivity S, M, T and inverse weightings for ψ

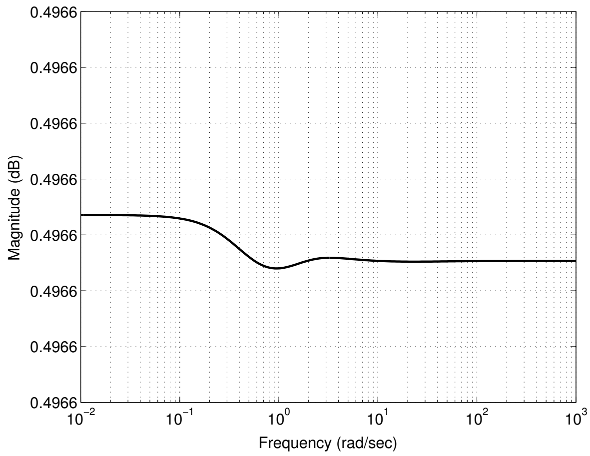

Fig. 5. Cost function for ψ

- compute (G_2, H_2, F_2) through Diophantine equations

$$F_2 A P_d + L_2 G_2 = L_{2s} P_n D_f \tag{12}$$

$$F_2 B P_d - L_2 H_2 = L_{2s} F_n D_f \tag{13}$$

- compute the eigenvalue/eigenvector equation $(N_1, F_1, F_{1s}, \lambda)$ with F_{1s} is schur polynomial satisfying $F_{1s} = F_1^* z^{-n1}$ where $n_1 = deg(F_1)$.

$$L_2 N_1 + F_1 \lambda L_{2s} = -F_{1s} F_2 \tag{14}$$

- compute the control law

$$C_0 = (H_2 + KB)^{-1} (G_2 - KA), \quad K = F_{1s}^{-1} N_1 P_d \tag{15}$$

The main difference between H_∞ and GH_∞ is that the first one uses iteration algorithm to compute control law whereas the second one uses eigenvalue/eigenvector problem to get solution which is easier to compute. The error sensibility S, the control sensibility M and the complementary sensibility T are defined as follow:

$$S = \frac{A(H_2 + KB)}{L_1 L_{2s} D_f}, M = \frac{A(G_2 - KA)}{L_1 L_{2s} D_f}$$
$$T = \frac{B(G_2 - KA)}{L_1 L_{2s} D_f}$$

Finally, the GH_∞ controller has been computed with the following constrain:

- Forces must be greater than or equal to zero and less than 10 $(0 \le F_i \le 10\,\mathrm{N})$ which systematically lead to $(u_1 \ge 0)$. This is due to actuator output limits.
- The altitude z_0 must be less than or equal zero $(z_0 \le 0)$ since the reference frame is upside down.

5 APPLICATION TO QUADROTOR

The nominal transfer matrix is computed for $I_{y0} = 1.2416; I_{z0} = 1.2416; I_{x0} = 1.2416; m_0 = 2; d = 0.1$ with

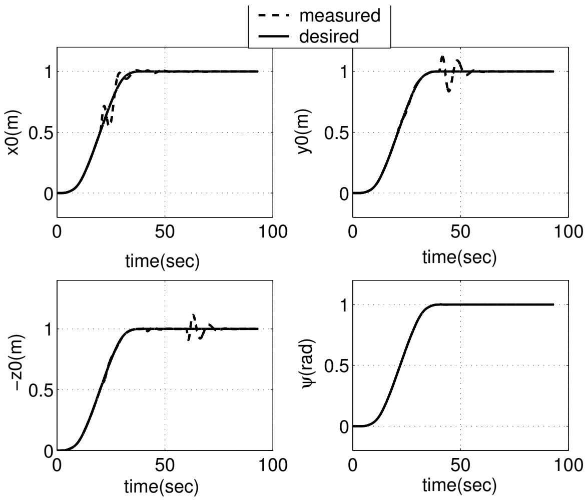

Fig. 6. Trajectories x_0, y_0, z_0 and ψ

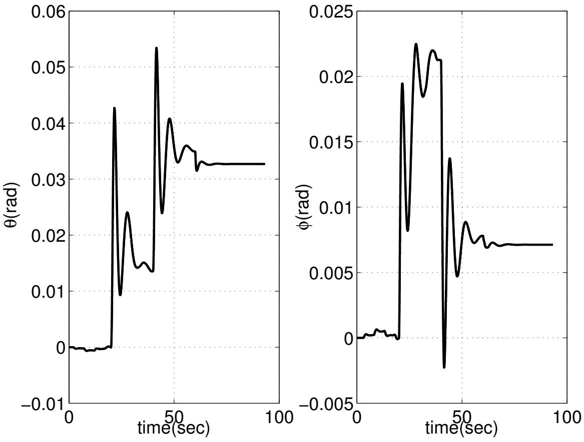

Fig. 7. Trajectories θ and ϕ

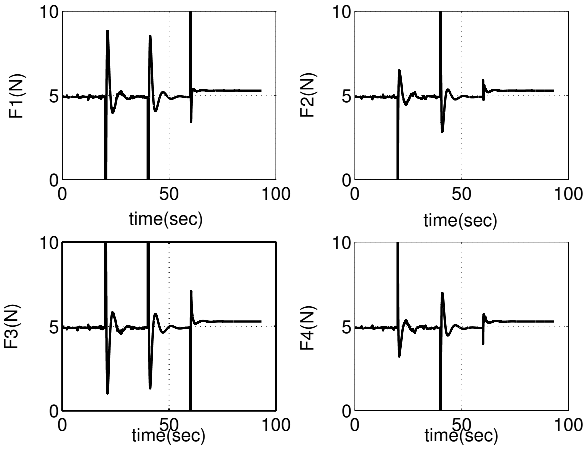

Fig. 8. Applied forces

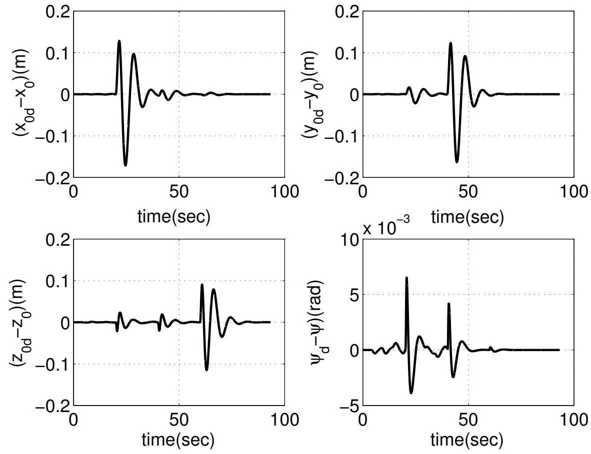

Fig. 9. Tracking errors

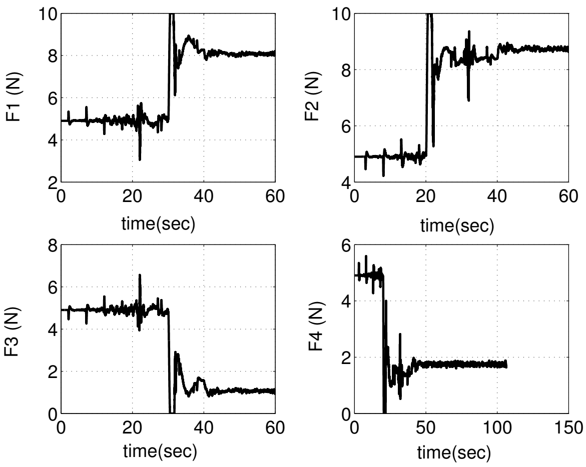

Fig. 10. Applied forces

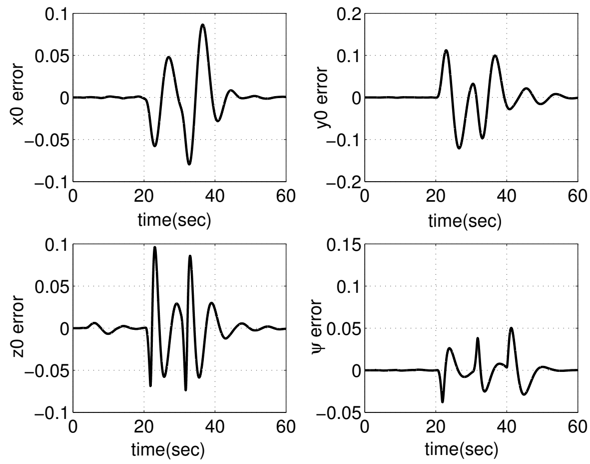

Fig. 11. Tracking errors

inputs v_1, v_2, v_3, v_4 and outputs x_0, y_0, z_0, ψ.

$$W_{nom} = \begin{bmatrix} 0 & 0 & \frac{-.7269652062}{s^4} & 0 \\ 0 & \frac{0.7269652062}{s^4} & 0 & 0 \\ \frac{-0.5}{s^4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{0.8054123711}{s^2} \end{bmatrix}$$

The reference trajectory is chosen as:

$$x_{0d} = y_{0d} = z_{0d} = 0.5a(t-2)^2\Gamma(t-2) + 0.75a(t-T-2)^2\Gamma(t-T-2) - 0.75a(t-2T-2)^2\Gamma(t-2T-2) - 0.5a(t-3T-2)^2\Gamma(t-3T-2) - 0.5a(t-4T-2)^2\Gamma(t-4T-2) - 0.75a(t-5T-2)^2\Gamma(t-5T-2) + 0.75a(t-6T-2)^2\Gamma(t-6T-2) + 0.5a(t-7T-2)^2\Gamma(t-7T-2)$$

$\psi_{0d} = 0.5b(t-2)^2\Gamma(t-2) + 0.75b(t-T-2)^2\Gamma(t-T-2) - 0.75b(t-2T-2)^2\Gamma(t-2T-2) - 0.5b(t-3T-2)^2\Gamma(t-3T-2) - 0.5b(t-4T-2)^2\Gamma(t-4T-2) - 0.75b(t-5T-2)^2\Gamma(t-5T-2) +$
$0.75(t-6T-2)^2\Gamma(t-6T-2) + 0.5b(t-7T-2)^2\Gamma(t-7T-2)$

With $\Gamma(t-\alpha T) = 0$ for $t \le \alpha T$ and $\Gamma(t-\alpha T) = 1$ for $t > \alpha T$; $T = 5$ s , $a = 2.222.10^{-3}\text{m/s}^2$ and $b = 2.222.10^{-3}\text{rad/s}^2$ so the steady state of the trajectories will be $x_{0d} = 1$ m $y_{0d} = 1$ m; $z_{0d} = 1$ m $\psi_d = 1$ rad.

With a chosen weighting function F_n, P_n and P_d, the controller C_0 is given. The sensitivity S, the control sensitivity M, the complementary sensitivity T and the cost function are represented:

- **Output:** x_0, y_0, z_0

Poles of C_0	Zeros of C_0
$-27.2518 \pm 4.2603i$	613.34
$-27.1501 \pm 4.2764i$	$-186.0725 \pm 394.6909i$
-20.6711	-353.7157
$-9.6508 \pm 4.1499i$	$-27.2518 \pm 4.2603i$
$-2.5778 \pm 4.2796i$	-3.5256
-3.5256	-2.2263
-2.2262	$-0.1676 \pm 0.1779i$
$-0.1676 \pm .1779i$	$-0.1301 \pm 0.1325i$
	-0.0388

$$|\lambda_{x0}| = |\lambda_{y0}| = |\lambda_{z0}| = 0.89617$$
$$\gamma_{x_0} = \gamma_{y_0} = \gamma_{z_0} = 1.1159$$

- **Output** ψ

Poles of $C_{0\psi}$	Zeros of $C_{0\psi}$
$-27.6616 \pm 2.2323i$	-24.6533
-20.0710	$-24.3742 \pm 0.4382i$
-17.4163	-23.7707
-6.6884	-3.6203
-3.6203	-1.8308
-1.8308	-0.8156
-0.8160	-0.3817

$$|\lambda_\psi| = 2.0135;$$
$$\gamma_\psi = \frac{1}{|\lambda_\psi|} = 0.49665$$

Case of Aerodynamic force disturbances:

An aerodynamic force disturbances has been taken for $A_x = 0.5$ N, $A_y = 0.5$ N and $A_z = 1.5$ N occurring at 20 s, 40 s and 60 s respectively, the results of (x_0, y_0, z_0, ψ) and (θ, ϕ) are shown in Fig. 5 and Fig. 6. The behavior of the applied forces and aerodynamic force disturbances is shown in Fig. 7. The tracking errors are shown in Fig. 8.

Case of Aerodynamic moment disturbances:

For the aerodynamic moments $A_p = 0.08$ Nm, $A_q = 0.08$ Nm and $A_r = 0.5$ Nm occurring at 20 s, 40 s and 60 s respectively the results of the applied forces are represented in Fig. 9. Tracking errors are represented in Fig. 10.

- It is noted from Fig. 5 to Fig. 8 that the system when subjected to aerodynamic force disturbance $[A_x, A_y, A_z]$ and 20% uncertainties on mass and inertia,the mixed inner-outer controller satisfying results even without block disturbances estimation. This can be shown from tracking error trajectories which vanished after a finite time with a perfect convergency. This is due not only to the robustness of the H_∞ controller but also to robust feedback linearization which preserve the good robustness properties. This is shown at the time 20 s on x_0, 40 s on y_0, and at the time 60 s on z_0 trajectory when the disturbances occurs. The robustness of the system can be confirmed by tracking errors trajectories (Fig. 8). The magnitude disturbances are limited by actuator saturation between 0 and 10 N. However despite the overshoots on forces in figure(Fig. 7) which lead to saturation the system remain stable.
- However, when subjected to aerodynamic moments disturbances $[A_P, A_q, A_r]$ and 20% uncertainties on mass and inertia, the results are shown in Fig. 9, Fig. 10. It is seen that the inner-outer controller shows efficiency to overcome easily disturbances on z and ψ, better than on x and y. The forces in Fig. 9 reflect perfectly the relation between control input $(\bar{u}_1, \bar{u}_2, \bar{u}_3, \bar{u}_4)$ and (F_1, F_2, F_3, F_4). The computed sensitivity S in Fig. 1 and Fig. 3 show a low gain at low frequency and a gain oscillating near 0 dB at high frequency, however the complementary sensitivity T shows a gain of 0 dB at low frequency and a low gain at high frequency.

This confirms that the resulting design is appropriate.

References

[1] KWAKERNAAK, H.: A polynomial approach to Hinfini optimization of control systems, Modelling, Robustness and Sensitivity Reduction (Ruth F. Curtain, ed.), Springer, Berlin, 1987, pp. 83-94.

[2] GRIMBLE, M. J.—JOHNSON, M. A.: H$_\infty$ robust control design- A tutorial review, IEE Comput. Contr. Eng. J., **2**, No. 6, 275-81, (1991).

[3] Van der SHAFT, A.J.:: L$_2$-gain analysis of nonlinear systems and nonlinear state feedback H$_\infty$ control, IEEE Trans. Automat. Contr. **37** No. 6 (June 1992), 770-784.

[4] BALL, J. A.—HELTON, J. W.—WALKER, M. L.: Hinfini control for nonlinear systems with output feedback, IEEE Trans. Automat. Contr. **38** No. 4 (April 1993), 546-559.

[5] MORRIS, J. C.—NIEUWSTADT, M. V.—BENDOTTI, P.: Identification and Control of a Model Helicopter in Hover, Proceedings of American Control Conference **2** (1994), 1238-1242.

[6] LIN, F.—ZHANG, W.—BRANDT, R. D.: Robust Hovering Control of a PVTOL Aircraft, IEEE Trans. on Contr. Sys. Tech. **7** No. 3 (1999), 343-351.

[7] BENDOTTI, P.—MORRIS, J. C.: Robust Hover Control for a Model Helicopter, Proceedings of American Control Conference **1** (1997), 682-687.

[8] PIEPER, J. K.: Application of SLMC: TRC Control of a Helicopter in Hover, Proceedings of American Control Conference **2** (1995), 1191-1195.

[9] MAHONY, R.—HAMEL, T.—DZUL, A.: Hover Control via Lyapunov Control for an Autonomous Model Helicopter, Proceedings of Conference on Decision and Control **4** (1999), 3490-3495.

[10] MAHONY, R.—LOZANO, R.: Almost Exact Path Tracking Control for an Autonomous Helicopter in Hover Maneuvers, Proceedings of IEEE International Conference on Robotics and Automation **2** (2000), 1245-1250.

[11] ISIDORI, A.: Nonlinear Control Systems - An Introduction, Springer-Verlag, New York, NY, second edition, 1989.

[12] BENVENUTI, L.—DIGIAMBERARDINO, P.—FARINA, L.: Trajectory Tracking for a PVTOL Aircraft: A Comparative Analysis, Proceedings of the 35th Conference on Decision and Control, Kobe, Japan December 1996.

[13] BOURLÈS, H.—COLLEDANI, F.: W-stability and local input-output stability results, IEEE Trans. Automat.Contr. **AC-40** (1995), 1102-1108.

[14] SASIADEK, J. Z.—HARTANA, P.: Sensor Fusion for Navigation of an Autonomous Unmanned Aerial Vehicle, Proceedings of the 2004 IEEE International Conference on Robotics & Automation New Orleans, LA, April 2004.

[15] CHICHKA, D. F.: A Disturbance Attenuation Approach to a Class of Uncertain Systems, PHD in Aerospace Engineering, University of California Los Angeles, 1994.

[16] KWAKERNAAK, H.: H_2 Optimization Theory and Applications to Robust Control Design, Annual Reviews in Control 26, pp.45-56, 2002.

[17] MISTLER, V.—BENALLEGUE, A.—M'SIRDI, N. K.: Exact Linearization and Noninteracting Control of a 4 Rotors Helicopter via Dynamic Feedback, IEEE 10th IEEE International Workshop on Robot-Human Interactive Communication Bordeaux and Paris (2001).

[18] MOKHTARI, A.—BENALLEGUE, A.: Dynamic feedback controller of Euler angles and wind parameters estimation for a quadrotor unmanned aerial vehicle, IEEE Int. Conf. on Robotics and Automation (ICRA'04) (2004).

[19] GROCOTT, S. C.—MAcMARTIN, D. G.—MILLER, D. W.: Experimental implementation of a multiple model technique for robust control of the MACE test article, In Proceedings of 3rd International Conference on Adaptive Structures, San Diego, CA (1992).

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